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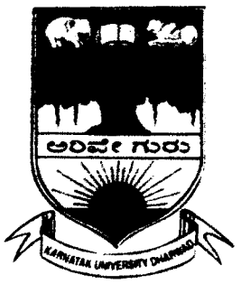
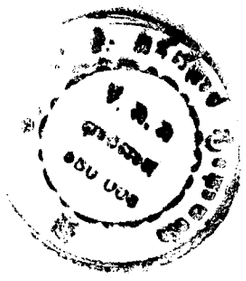
Some Special Contributions to Non-Parametric Inference





Thesis Submitted to the
Karnataka University, Dharwad
For the award of the degree of

Doctor of Philosophy
In
Statistics

By
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September 2008

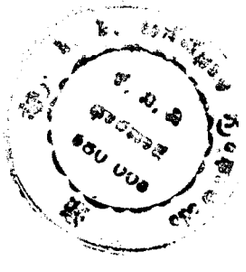
DECLARATION

I hereby declare that the thesis entitled "**Some Special Contributions to Non-Parametric Inference**" is a record of original research work undertaken by me in the P G Department of Studies in Statistics, Karnatak University, Dharwad – 03, under the guidance of Dr. I.D.Shetty, Professor of Statistics, P G Department of Studies in Statistics, Karnatak University, Dharwad – 03. I further declare that this work has not been submitted for the award of any degree or diploma to this or of any other university.

Place: Dharwad.
Date: September 2008

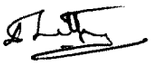
Keerthi.

Keerthi. M. Mathad



CERTIFICATE

This is to certify that the thesis entitled "**Some Special Contributions to Non-Parametric Inference**" submitted by Smt. Keerthi. M. Mathad for the award of Ph. D degree in Statistics involves the results of the original research work carried out by her in the P G Department of Studies in Statistics, Karnatak University, Dharwad – 03, under my supervision. This thesis or part of there of has not been submitted for any other degree or diploma to this or any other university previously.



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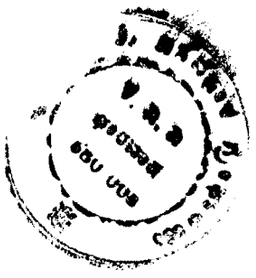

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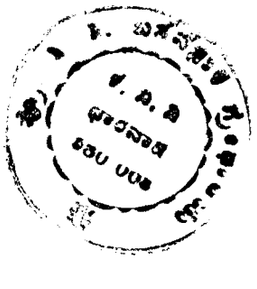

Lastly, I must thank all those who have extended their help directly and indirectly during my research work till its completion.

Place: Dharwad.
Date: September 2008

Keerthi. M. Mathad

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CHAPTER I

INTRODUCTION

Statistical Inference is that branch of statistics which is concerned with using probability concept to deal with uncertainty in decision-making. It refers to the process of selecting and using a sample statistic to draw inference about a population parameter based on a subset of it –the sample drawn from the population. Statistical Inference treats two different classes of problems.

1. Hypothesis testing that is to test some hypothesis about the parent population from which the sample is drawn.
2. Estimation that is to use the statistics obtained from the sample as estimate of the unknown parameter of the population from which the sample is drawn.

In both these cases the particular problem at hand is structured in such a way that inferences about relevant population values can be made from the sample data.

When sample descriptions are used to infer some information about the population, the subject is called inductive statistics or statistical inference. The entire classical statistical inference techniques is based on fairly specific assumptions regarding the nature of the underlying population distribution, usually its form and some parameter values must be stated.

An alternative set of techniques is available and the mathematical bases for these procedures may be classified as

distribution-free and non-parametric procedures. In a distribution free inference, whether for testing or estimation, the methods are based on functions of the sample observations whose corresponding random variable has a distribution which does not depend on the specific distribution function of the population from which the sample was drawn. Therefore assumptions regarding the underlying population are not necessary. Strictly speaking the term non-parametric test implies a test for a hypothesis which is not a statement about parameter values. The assumption most frequently required is simply that the population is continuous.

The theoretical foundations of non-parametric statistics were initialized by F.Wilcoxon(1945) and H.B.Mann and D.R.Whitney(1947). Some early contributions are cited in a survey paper by H.Scheffe(1943). Also some contributions such as the concept of asymptotic relative efficiency proposed by Pitman(1948) and the development of the theory of U-Statistics initiated by Hoeffding(1948).

Sequential analysis is a method of statistical inference whose characteristic feature is that the number of observations required by the procedure is not determined in advance of the experiment. The decision to terminate the experiment depends on different stages depending on the observations previously made.

The first idea of a sequential test procedure that is a test for which the number of observations is not determined in advance but

is dependent on the outcome of the observations as they are made, goes back to H.F.Dodge and H.G.Romig who constructed a double sample procedure. According to this scheme the decision whether or not a second sample should be drawn depends on the outcome of the observations in the first sample. Whereas this method allows for only two samples, Walter Bartky devised a multiple sampling scheme for the particular case of testing the mean of a binomial distribution. Later on Abraham Wald(1943) developed the sequential probability ratio test for the purpose of testing statistical hypothesis. The sequential probability ratio test frequently results in a saving of about 50 percent in the number of observations over the most efficient test procedure based on a fixed number of observations.

After a particular sequential test has been adopted, that is, a particular choice of the sets has been made, the probability that the process will terminate with the acceptance of the hypothesis H_0 under test depends only on the distribution of the random variable x under consideration. As before it is assumed that the distribution of x is known except for the values of a finite number of parameters $\theta_1, \ldots, \theta_k$ say. Thus, the distribution of x is given by a function $f(x, \theta_1, \ldots, \theta_k)$ where the functional form f is known, but the true values of the parameters $\theta_1, \ldots, \theta_k$ are unknown. Since the distribution of x is determined by the parameter point θ, the probability of

accepting H_0 will be function of θ. This function is denoted by $L(\theta)$ and will be called the operating characteristic (OC) function.

Life length is random and so we are led to a study of life distributions. Therefore we consider the notion of aging. Aging is conveniently studied in terms of the failure rate function. In the simple case where aging is not present, we obtain a constant failure rate, corresponding to the exponential distribution. The exponential distribution is in several senses the most fundamental distribution in reliability theory. The exponential life distribution provides a good description of the life length of a unit which does not age with time. A property of the exponential distribution which makes it especially important in reliability theory and applications is that the remaining life of a used component is independent of initial age. This property is referred to as "Memory less" property. Other important life distributions are Weibull distribution, Linear Failure Rate distribution and Makeham distribution.

The Weibull distribution is given by $F_x(t)=1-e^{-(\lambda t)^{\alpha}}$ for $t\geq 0$ where $\lambda,\alpha>0$. The Weibull distribution is an extreme value distribution in that it is the limiting distribution of the minimum of independent random variables. The failure rate $r(t)$ takes a particularly simple form $r(t)=\frac{f(t)}{\overline{F}(t)}=\alpha\lambda(\lambda t)^{\alpha-1}$ for $t>0$.

Thus the Weibull distribution F_α is IFR for $\alpha\geq 1$ and DFR for $0<\alpha\leq 1$ for $\alpha=1, F_\alpha(t)=1-e^{-\lambda t}$, the exponential distribution which is

both IFR and DFR. The parameter α is called the shape parameter as α increases, the failure rate function rises more steeply and the probability density becomes more peaked. The parameter λ is a scale parameter, that is the distribution depends on λ and t only through their product 'λt'.

The Weibull distribution has been used to describe fatigue failure (Weibull,1939), vacuum tube failure (Kao,1958) and ball bearing failure(Lieblein and Zelen,1956).It is perhaps the most popular parametric family of failure distributions at the present time.

1.1 ORGANISATION OF THE THESIS

This dissertation contains five chapters. Chapter I is an introductory chapter containing the basic ideas needed for further development.

Chapter II deals with a simple Sequential Non-Parametric test. Sequential analysis is a method of Statistical Inference whose characteristic feature is that the number of observations required by the procedure is not determined in advance of the experiment. This method follows a rule for making decisions which has been explained in the introduction section. After a particular Sequential test has been adopted, the probability that the process will terminate with the acceptance of the hypothesis H_0 under test

depends only on the distribution of the random variable X under consideration.

Bradley, Merchant and Wilcoxon(1966) proposed a non-parametric sequential test (Wald-open type) for the two sample problem. Their test was based on the likelihood ratio of the ranks of the observations. It's operating characteristic and Average Sample Number functions were obtained by simulation by Bradley at al (1966). Savage and Sethuraman(1966) showed that the test terminates with probability one and that its ASN is finite. Miller(1969,1972) gave two adhoc sequential tests of the closed type for one-sample problem. Phatarford and Subdury(1988) considered a Wald type test for the two-sample problem and they exploited the normal approximation to the Wilcoxon Mann-Whitney statistic. They first considered the Lehmann alternative and later on wider type of alternatives.

In our II chapter we consider a Wald type test for the two-sample problem. We take $H_0 : G(.) = F^{k_0}(.)$ and $H_1 : G(.) = F^{\frac{1}{k_0}}(.)(0 < k_0 < 1)$ and we wish to make a judgment between two new treatments. The test is based on a test statistic given by Shetty and Govindarajalu (1988).

$$U = \frac{1}{\binom{m}{3}\binom{n}{3}} \sum_{c} h\left(X_{i_1}, X_{i_2}, X_{i_3}; Y_{i_1}, Y_{i_2}, Y_{i_3}\right)$$

where $h(X_1, X_2, X_3; Y_1, Y_2, Y_3) = \begin{cases} 1, & if \quad M(X_1, X_2, X_3) < M(Y_1, Y_2, Y_3) \\ 0 & otherwise \end{cases}$

is a kernel of degree (3,3) which depends on the sub-sample medians. We find the mean and the variance for this U-statistic under the null hypothesis and the alternative hypothesis.

In the chapter III, we exploit the normal approximation to the U-statistic to construct the sequential probability ratio test. We also consider the Lehmann alternative as in the case of Phatarfod and Sudbury(1988).We find the operating characteristic function for the above, SPRT, We investigate the behaviour of the OC function for small values of n, namely, n=3,4,5,6 and 7 and for different values of k_0 say $k_0 = 0.5, 0.6$ and 0.7.

In the next chapter we take up testing exponentiality against Increasing Failure Rate Average alternatives. Many tests for exponentiality have been developed but the tests that have been developed specially for testing for increasing Failure Rate Average alternative are due to Deshpande(1983) and Bhadri(2005) among others. Deshpande's test is a special case of Bhadri(2005).

We propose a new class of statistics based on U-statistics whose kernel depends on sub-sample median. We obtain the mean and asymptotic variance of the U-statistic under the null hypothesis. The performance of the proposed test is evaluated in terms of Pitman asymptotic relative efficiency (PARE).

In the last chapter we study the small sample performance of the U-Statistic that we have developed in the previous chapter. A Monte Carlo study was carried out to estimate critical points, the exact significance level and the power for three specific alternatives corresponding to significance levels near to $\alpha = 0.05$ and $\alpha = 0.10$ of the three tests belonging to this class based on the U-Statistics for $b = 0.5$ and $b = 0.9$. The study is done for n=8 and 10. The three alternatives to the null hypothesis H_0 of exponentiality for which the power has been estimated are the Weibull distribution, Linear Failure Rate distribution and the Makeham distribution where all these distributions are increasing failure rate distributions and hence also increasing failure rate average.

CHAPTER II

A SIMPLE SEQUENTIAL NON-PARAMETRIC TEST

2.1. INTRODUCTION

Sequential analysis is a method of statistical inference whose characteristic feature is that the number of observations required by the procedure is not determined in advance of the experiment. The decision to terminate the experiment depends, at each stage, on the results of the observations previously made. A merit of the sequential method, as applied to testing statistical hypotheses, is that test procedures can be constructed which require, on the average, a smaller number of observations than equally reliable test procedures based on a predetermined number of observations.

The sequential method of testing a hypothesis H may be described as follows. A rule is given for making one of the following three decisions at any stage of the experiment (at the m^{th} trial for each integral value of m): (1) to accept the hypothesis H,(2) to reject the hypothesis H,(3) to continue the experiment by making an additional observation. Thus, such a test procedure is carried out sequentially. On the basis of the first observation one of the aforementioned three decisions is made. If the first or second decision is made, the process is terminated. If the third decision is made, a second trial is performed. Again, on the basis of the first two observations one of the three decisions is made. If the third decision is made, a third trial is performed, and so on. The process

is continued until either the first or the second decision is made. The number n of observations required by such a test procedure is a random variable, since the value of n depends on the outcome of the observations.

After a particular sequential test has been adopted, the probability that the process will terminate with the acceptance of the hypothesis H_0 under test depends only on the distribution of the random variable x under consideration. It is assumed that the distribution of x is known except for the values of a finite number of parameters, $\theta_1,...\theta_k$ say. Thus, the distribution of x is given by a function $f(x,\theta_1,...,\theta_k)$ where the functional form f is known, but the true values of the parameters $\theta_1,...\theta_k$ are unknown. Since the distribution of x is determined by the parameter θ, the probability of accepting H_0 will be a function of θ. This function will be denoted by $L(\theta)$ and will be called the operating characteristic (OC) function.

We shall denote by n the number of observations required by the sequential test. Then n is a random variable. Carrying out the same sequential test procedure repeatedly, we obtain, in general, different values of n(the average value of n in the long run, when the same test procedure is applied repeatedly). For any given test procedure the expected value of n depends only on the distribution of x. Since the distribution of x is determined by the parameter θ, the expected value of n depends only on θ. For any given

parameter θ, we shall denote the expected value of n by $E_\theta(n)$.We shall refer to the average sample number function $E_\theta(n)$ as the ASN function.

Bradley, Merchant and Wilcoxon(1966) proposed a nonparametric sequential test(Wald-open-type) for the two-sample problem. The null hypothesis is that the two populations are identical and the alternative hypothesis is the Lehmann alternative; that is, if we denote the c.d.f's of X and Y populations by F(.) and G(.) respectively, the null hypothesis is H_0:G(.)=F(.),and the alternative hypothesis is H_1:G(.)=F^k(.),where the value of k is specified. The test (called the MCSR-test) was based on the likelihood ratio of the ranks of the observations, its operating characteristic (O.C) and Average Sample Number (A.S.N) functions were obtained by simulation by Bradley et al(1966). Savage and Sethuraman(1966) showed that the test terminates with probability one, and that it's A.S.N. is finite. Miller(1969,1972) gave two adhoc sequential tests of the closed type for one-sample problem. The test is based on Wilcoxon signed rank statistic. Lai(1975) considered the analogous test for the two-sample problem. Phatarford and Subdury(1988) considered a Wald-type test for the two-sample problem. Unlike in Bradley, Merchant and Wilcoxon (1966) they exploited the normal approximation to the Wilcoxon Mann-Whitney statistic. They first considered the Lehmann alternative and later on wider type of alternatives.

In this paper we consider a Wald-type test for the two-sample problem. We take $H_0 : G(.) = F^{k_0}(.)$ and $H_1 : G(.) = F^{\frac{1}{k_0}}(.)$, $(0 < k_0 < 1)$. Here we wish to make a judgment between two new treatments and there is no prior performance involved. There would be a region of indifference $k_0 < k < k_0^{-1}$, where, if the true situation was $G(.) = F^k(.)$, it would not matter which of the two treatments was chosen. We propose the following Sequential test of strength (α, β). Observations are taken from the X and Y populations in pairs. At each stage, the U-statistic

$$U = \frac{1}{\binom{n}{3}\binom{n}{3}} \sum_c h(X_{i_1}, X_{i_2}, X_{i_3}; Y_{j_1}, Y_{j_2}, Y_{j_3})$$

where

$$h(X_1, X_2, X_3; Y_1, Y_2, Y_3) = \begin{cases} 1, & \text{if } M(X_1, X_2, X_3) < M(Y_1, Y_2, Y_3) \\ 0, & \text{otherwise} \end{cases}$$

$M(X_1, X_2, X_3)$=Median of X_1, X_2, X_3 and

$M(Y_1, Y_2, Y_3)$=Median of Y_1, Y_2, Y_3

is calculated. Sampling is continued as long as

$$B < \frac{f_{H_1}(U)}{f_{H_0}(U)} < A$$

where $A = \frac{1-\beta}{\alpha}$ and $B = \frac{\beta}{1-\alpha}$

and H_0 or H_1 is accepted according as the R.H.S or the L.H.S inequality is the first not satisfied. In section 2 we define our test

statistic. In section 3 we obtain the mean and variance for our test statistic.

2.2. THE PROPOSED TEST STATISTIC.

Let $X_1, X_2, ..., X_m$ and $Y_1, Y_2, ..., Y_n$ be random samples from continuous distributions with CDF's $F(x)$ and $G(x)$ respectively. Let U be the proposed test statistic proposed by Shetty and Govindarajalu(1988) for testing $H_0 : F(x) = G(x)$ Vs $H_1 : F(x) \geq G(x)$ with strict inequality for some x is given by

$$U = \frac{1}{\binom{m}{3}\binom{n}{3}} \sum_c h(X_{i_1}, X_{i_2}, X_{i_3}; Y_{j_1}, Y_{j_2}, Y_{j_3})$$

where

$$h(X_1, X_2, X_3; Y_1, Y_2, Y_3) = \begin{cases} 1, & if \quad M(X_1, X_2, X_3) < M(Y_1, Y_2, Y_3) \\ 0, & otherwise \end{cases}$$

is a kernel which depends on the sub-sample medians.

2.3. MEAN AND THE VARIANCE OF THE U-STATISTIC

2.3.1. MEAN

Now we have

$$\begin{aligned} E(U) &= E(h) \\ &= E(h(X_1, X_2, X_3; Y_1, Y_2, Y_3)) \\ &= P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3)] \end{aligned}$$

Now

under $H_0, F(X)=G(X)$

$$E_{H_0}(U)=\frac{1}{2}$$

2.3.2. VARIANCE of U when F(x)=G(x).

Now consider variance of U. Variance of U is given by

$$\text{Var(U)} = \frac{1}{\binom{n}{3}^2}\sum_{c=0}^{3}\sum_{d=0}^{3}\binom{3}{c}\binom{m-3}{3-c}\binom{3}{d}\binom{n-3}{3-d}\xi_{c,d} \qquad (2.3.2.1)$$

where

$\xi_{c,d}$ =Covariance between the two kernels wherein c x observations and d y observations are in common between the two kernels. (2.3.2.2)

Thus we have

$\xi_{0,0}=0$

$\xi_{1,0}=Cov[h(X_1,X_2,X_3;Y_1,Y_2,Y_3),h(X_1,X_4,X_5;Y_4,Y_5,Y_6)]$

$\xi_{0,1}=Cov[h(X_1,X_2,X_3;Y_1,Y_2,Y_3),h(X_4,X_5,X_6;Y_1,Y_4,Y_5)]$

$\xi_{2,0}=Cov[h(X_1,X_2,X_3;Y_1,Y_2,Y_3),h(X_1,X_2,X_4;Y_4,Y_5,Y_6)]$

$\xi_{0,2}=Cov[h(X_1,X_2,X_3;Y_1,Y_2,Y_3),h(X_4,X_5,X_6;Y_1,Y_2,Y_4)]$

$\xi_{3,0}=Cov[h(X_1,X_2,X_3;Y_1,Y_2,Y_3),h(X_1,X_2,X_3;Y_4,Y_5,Y_6)]$

$\xi_{0,3}=Cov[h(X_1,X_2,X_3;Y_1,Y_2,Y_3),h(X_4,X_5,X_6;Y_1,Y_2,Y_3)]$

$\xi_{1,2}=Cov[h(X_1,X_2,X_3;Y_1,Y_2,Y_3),h(X_1,X_4,X_5;Y_1,Y_2,Y_4)]$

$\xi_{2,1}=Cov[h(X_1,X_2,X_3;Y_1,Y_2,Y_3),h(X_1,X_2,X_4;Y_1,Y_4,Y_5)]$

$$\xi_{1,3} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_4, X_5; Y_1, Y_2, Y_3)]$$

$$\xi_{3,1} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_2, X_3; Y_1, Y_4, Y_5)]$$

$$\xi_{2,3} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_2, X_4; Y_1, Y_2, Y_3)]$$

$$\xi_{3,2} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_2, X_3; Y_1, Y_2, Y_4)]$$

$$\xi_{1,1} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_4, X_5; Y_1, Y_4, Y_5)]$$

$$\xi_{2,2} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_2, X_4; Y_1, Y_2, Y_4)]$$

$$\xi_{3,3} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_2, X_3; Y_1, Y_2, Y_3)]$$

Now consider

$$\xi_{1,0} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_4, X_5; Y_4, Y_5, Y_6)]$$

$$= E[h(X_1, X_2, X_3; Y_1, Y_2, Y_3)h(X_1, X_4, X_5; Y_4, Y_5, Y_6)] - \{E[h(X_1, X_2, X_3; Y_1, Y_2, Y_3)]\}^2$$

$$= P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_4, X_5) < Med(Y_4, Y_5, Y_6)] - \frac{1}{4}$$

Now

$$P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_4, X_5) < Med(Y_4, Y_5, Y_6)]$$
$$= \int_{-\infty}^{\infty} P^2[Med(x, X_2, X_3) < Med(Y_1, Y_2, Y_3)]dF(x)$$

Now

$$P[Med(x, X_2, X_3) < Med(Y_1, Y_2, Y_3)]$$

$$= 2P[x < Med(Y_1, Y_2, Y_3), X_2 < x < X_3] + 2P[X_2 < Med(Y_1, Y_2, Y_3), x < X_2 < X_3] +$$

$$2P[X_2 < Med(Y_1, Y_2, Y_3), X_3 < X_2 < x]$$

$$= 2P[X_2 < x < X_3 < Med(Y_1, Y_2, Y_3)] + 2P[X_2 < x < Med(Y_1, Y_2, Y_3) < X_3] +$$

$$2P[x < X_2 < X_3 < Med(Y_1, Y_2, Y_3)] + 2P[x < X_2 < Med(Y_1, Y_2, Y_3) < X_3] +$$

$$2P[X_3 < X_2 < x < Med(Y_1,Y_2,Y_3)]+2P[X_3 < X_2 < Med(Y_1,Y_2,Y_3) < x]$$

(2.3.2.2a).

Now consider the first term of (2.3.2.2a).

$$P[X_2 < x < X_3 < Med(Y_1,Y_2,Y_3)]$$

$$= P[X_2 < x]P[x < X_3 < Med(Y_1,Y_2,Y_3)]$$

$$= F(x)\left[\int_x^\infty P[x_3 < Med(Y_1,Y_2,Y_3)]dF(x_3)\right]$$

$$= F(x)\left[\int_x^\infty \sum_{i=0}^{1}\binom{3}{i}F^i(x_3)\overline{F}^{3-i}(x_3)dF(x_3)\right]$$

$$= F(x)\left[\int_{F(x)}^1 \sum_{i=0}^{1}\binom{3}{i}u^i(1-u)^{3-i}du\right] \qquad \text{letting } u = F(x_3)$$

$$= F(x)\left[\int_{F(x)}^1 \left[(1-u)^3+3u(1-u)^2\right]du\right]$$

$$= F(x)\left[\overline{F}^3(x)-\frac{1}{2}\overline{F}^4(x)\right].$$

Consider next the second term of (2.3.2.2a).

$$P[X_2 < x < Med(Y_1,Y_2,Y_3) < X_3]$$

$$= P[X_2 < x]P[x < Med(Y_1,Y_2,Y_3) < X_3]$$

$$= F(x)\left[\int_x^\infty P(z < X_3)6F(z)\overline{F}(z)dF(z)\right]$$

$$= F(x)\left[6\int_x^\infty \overline{F}(z)F(z)\overline{F}(z)dF(z)\right]$$

$$= 6F(x)\left[\int_{F(x)}^1 u(1-u)^2du\right] \qquad \text{letting } u = F(z)$$

$$= 6F(x)\left[\int_0^{\bar{F}(x)} v^2(1-v)dv\right] \qquad \text{letting } (1-u)=v$$

$$= 6F(x)\left[\frac{\bar{F}^3(x)}{3} - \frac{\bar{F}^4(x)}{4}\right]$$

$$= F(x)\left[2\bar{F}^3(x) - \frac{3}{2}\bar{F}^4(x)\right].$$

Now consider the third term of (2.3.2.2a)

$$P[x < X_2 < X_3 < Med(Y_1, Y_2, Y_3)]$$

$$= \int_x^{\infty} P(x_2 < X_3 < Med(Y_1, Y_2, Y_3))dF(x_2)$$

$$= \int_x^{\infty}\left[\int_{x_2}^{\infty} P(x_3 < Med(Y_1, Y_2, Y_3))dF(x_3)\right]dF(x_2)$$

$$= \int_x^{\infty}\left[\bar{F}^3(x_2) - \frac{1}{2}\bar{F}^4(x_2)\right]dF(x_2)$$

$$= \int_0^{\bar{F}(x)}\left(v^3 - \frac{1}{2}v^4\right)dv \qquad \text{letting } v = \bar{F}(x_2)$$

$$= \frac{\bar{F}^4(x)}{4} - \frac{\bar{F}^5(x)}{10}.$$

Further the fourth term of (2.3.2.2a)

$$P[x < X_2 < Med(Y_1, Y_2, Y_3) < X_3]$$

$$= \int_x^{\infty} P(x_2 < Med(Y_1, Y_2, Y_3) < X_3)dF(x_2)$$

$$= \int_x^{\infty}\left[\int_{x_2}^{\infty} P(z < X_3)6F(z)\bar{F}(z)dF(z)\right]dF(x_2)$$

$$= \int_x^{\infty}\left[2\bar{F}^3(x_2) - \frac{3}{2}\bar{F}^4(x_2)\right]dF(x_2)$$

$$= \frac{\overline{F}^4(x)}{2} - \frac{3\overline{F}^5(x)}{10}.$$

Similarly the fifth term of (2.3.2.2a)

$$P[X_3 < X_2 < x < Med(Y_1, Y_2, Y_3)]$$

$$= P[X_3 < X_2 < x]P[x < Med(Y_1, Y_2, Y_3)]$$

$$= \left[\int_{-\infty}^{x} P(X_3 < x_2)dF(x_2)\right]\left[\sum_{i=0}^{1}\binom{3}{i}F^i(x)\overline{F}^{3-i}(x)\right]$$

$$= \left[\int_{-\infty}^{x} F(x_2)dF(x_2)\right]\left[\overline{F}^3(x) + 3F(x)\overline{F}^2(x)\right]$$

$$= \int_0^{F(x)} udu\left[\overline{F}^3(x) + 3\overline{F}^2(x)(1 - \overline{F}(x))\right]$$

$$= \frac{F^2(x)}{2}\left[3\overline{F}^2(x) - 2\overline{F}^3(x)\right].$$

The sixth term of (2.3.2.2a)

$$P[X_3 < X_2 < Med(Y_1, Y_2, Y_3) < x]$$

$$= \int_{-\infty}^{x} P(X_3 < X_2 < z)6F(z)\overline{F}(z)dF(z)$$

$$= \int_{-\infty}^{x}\left[\int_{-\infty}^{z} P(X_3 < x_2)dF(x_2)6F(z)\overline{F}(z)dF(z)\right]$$

$$= 6\int_{-\infty}^{x}\left[\frac{F^3(z)}{2}\overline{F}(z)\right]dF(z)$$

$$= 3\int_0^{F(x)} u^3(1-u)du \quad \text{letting } F(z) = u$$

$$= 3\int_0^{F(x)}(u^3 - u^4)du$$

$$=3\left[\frac{F^4(x)}{4}-\frac{F^5(x)}{5}\right].$$

Now adding the six terms, we get

$$P[Med(x,X_2,X_3)<Med(Y_1,Y_2,Y_3)]$$

$$=2\left(F(x)\left[\bar{F}^3(x)-\frac{1}{2}\bar{F}^4(x)\right]\right)+2\left(F(x)\left[2\bar{F}^3(x)-\frac{3}{2}\bar{F}^4(x)\right]\right)$$
$$+2\left(\frac{\bar{F}^4(x)}{4}-\frac{\bar{F}^5(x)}{10}\right)+2\left(\frac{\bar{F}^4(x)}{2}-\frac{3\bar{F}^5(x)}{10}\right)$$
$$+2\left(\frac{F^2(x)}{2}\left[3\bar{F}^2(x)-2\bar{F}^3(x)\right]\right)+2\left(3\left[\frac{F^4(x)}{4}-\frac{F^5(x)}{5}\right]\right)$$

Further simplification yields

$$P[Med(x,X_2,X_3)<Med(Y_1,Y_2,Y_3)]=6u^4-\frac{12}{5}u^5-4u^3+\frac{7}{10}$$

Where $u=F(x)$.

Thus we get

$$P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_4,X_5)<Med(Y_4,Y_5,Y_6)]$$

$$=\int_{-\infty}^{\infty}P^2[Med(x,X_2,X_3)<Med(Y_1,Y_2,Y_3)]dF(x)$$

$$=\int_0^1\left(6u^4-\frac{12}{5}u^5-4u^3+\frac{7}{10}\right)^2du$$

$$=\frac{6299}{23100}.$$

Thus finally we have

$$\xi_{1,0}=P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_4,X_5)<Med(Y_4,Y_5,Y_6)]-\frac{1}{4}$$

$$=\frac{6299}{23100}-\frac{1}{4}$$

$$= \frac{131}{5775}.$$

Now consider

$$\xi_{0.1} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_4, X_5, X_6; Y_1, Y_4, Y_5)]$$

$$= E[h(X_1, X_2, X_3; Y_1, Y_2, Y_3) h(X_4, X_5, X_6; Y_1, Y_4, Y_5)] - \{E[h(X_1, X_2, X_3; Y_1, Y_2, Y_3)]\}^2$$

$$= P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_4, X_5, X_6) < Med(Y_1, Y_4, Y_5)] - \frac{1}{4}$$

Now

$$P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_4, X_5, X_6) < Med(Y_1, Y_4, Y_5)]$$

$$= \int_{-\infty}^{\infty} P^2[Med(X_1, X_2, X_3) < Med(y, Y_2, Y_3)] dF(y)$$

Further

$$P[Med(X_1, X_2, X_3) < Med(y, Y_2, Y_3)]$$

$$= 2P[Med(X_1, X_2, X_3) < y, Y_2 < y < Y_3] + 2P[Med(X_1, X_2, X_3) < Y_2, y < Y_2 < Y_3] +$$

$$2P[Med(X_1, X_2, X_3) < Y_2, Y_3 < Y_2 < y]$$

$$= 2P[Med(X_1, X_2, X_3) < Y_2 < y < Y_3] + 2P[Y_2 < Med(X_1, X_2, X_3) < y < Y_3] +$$

$$2P[Med(X_1, X_2, X_3) < y < Y_2 < Y_3] + 2P[y < Med(X_1, X_2, X_3) < Y_2 < Y_3] +$$

$$2P[Med(X_1, X_2, X_3) < Y_3 < Y_2 < y] + 2P[Y_3 < Med(X_1, X_2, X_3) < Y_2 < y]$$

(2.3.2.2b)

Now consider the first term of (2.3.2.2b)

$$P[Med(X_1, X_2, X_3) < Y_2 < y < Y_3]$$

$$= P[Med(X_1, X_2, X_3) < Y_2 < y] P[y < Y_3]$$

$$= \left[\int_{-\infty}^{y} P(Med(X_1, X_2, X_3) < y_2) dF(y_2)\right] [\overline{F}(y)]$$

$$= \left[\int_{-\infty}^{Y} \sum_{i=2}^{3} \binom{3}{i} F^{i}(y_2) \overline{F}^{3-i}(y_2) dF(y_2) \right] [\overline{F}(y)]$$

$$= \left[\int_{0}^{F(y)} \sum_{i=2}^{3} \binom{3}{i} u^{i} (1-u)^{3-i} du \right] [\overline{F}(y)] \qquad \text{letting } u = F(y_2)$$

$$= \left[F^{3}(y) - \frac{1}{2} F^{4}(y) \right] \overline{F}(y).$$

Now consider the second term of (2.3.2.2b).

$$P[Y_2 < Med(X_1, X_2, X_3) < y < Y_3]$$

$$= P[Y_2 < Med(X_1, X_2, X_3) < y] P[y < Y_3]$$

$$= \left[\int_{-\infty}^{y} P(Y_2 < z) 6F(z)\overline{F}(z) dF(z) \right] [\overline{F}(y)]$$

$$= \left[6 \int_{-\infty}^{y} F^{2}(z)\overline{F}(z) dF(z) \right] [\overline{F}(y)]$$

$$= 6 \left[\int_{0}^{F(y)} u^{2}(1-u) du \right] [\overline{F}(y)] \qquad \text{letting } u = F(z)$$

$$= \left[2F^{3}(y) - \frac{3}{2} F^{4}(y) \right] [\overline{F}(y)].$$

Now consider the third term of (2.3.2.2b).

$$P[Med(X_1, X_2, X_3) < y < Y_2 < Y_3]$$

$$= P[Med(X_1, X_2, X_3) < y] P[y < Y_2 < Y_3]$$

$$= \left[\sum_{i=2}^{3} \binom{3}{i} F^{i}(y) \overline{F}^{3-i}(y) \right] \left[\int_{y}^{\infty} P(y_2 < Y_3) dF(y_2) \right]$$

$$= \left[\binom{3}{2} F^{2}(y)\overline{F}(y) + \binom{3}{3} F^{3}(y) \right] \left[\int_{y}^{\infty} \overline{F}(y_2) dF(y_2) \right]$$

$$= [3F^2(y)\overline{F}(y) + F^3(y)]\left[\int_0^{\overline{F}(y)} u\,du\right] \qquad \text{letting } u = (1 - F(y_2))$$

$$= [3F^2(y)\overline{F}(y) + F^3(y)]\frac{\overline{F}^2(y)}{2}.$$

Now consider the fourth term of (2.3.2.2b).

$$P[y < Med(X_1, X_2, X_3) < Y_2 < Y_3]$$

$$= \int_y^\infty P(z < Y_2 < Y_3)\,6F(z)\overline{F}(z)\,dF(z)$$

$$= \int_y^\infty \left[\int_z^\infty P(y_2 < Y_3)\,dF(y_2)\right] 6F(z)\overline{F}(z)\,dF(z)$$

$$= \int_y^\infty \left[\frac{\overline{F}^2(z)}{2}\right] 6F(z)\overline{F}(z)\,dF(z)$$

$$= 3\int_y^\infty \overline{F}^3(z)F(z)\,dF(z)$$

$$= 3\int_0^{\overline{F}(y)} u^3(1-u)\,du \quad \text{letting } \overline{F}(z) = u$$

$$= \frac{3}{4}\overline{F}^4(y) - \frac{3}{5}\overline{F}^5(y).$$

Now consider the fifth term of (2.3.2.2b).

$$P[Med(X_1, X_2, X_3) < Y_3 < Y_2 < y]$$

$$= \int_{-\infty}^{y} P[Med(X_1, X_2, X_3) < Y_3 < y_2]\,dF(y_2)$$

$$= \int_{-\infty}^{y}\int_{-\infty}^{y_2} P[Med(X_1, X_2, X_3) < y_3]\,dF(y_3)\,dF(y_2)$$

$$= \int_{-\infty}^{y}\left[F^3(y_2) - \frac{1}{2}F^4(y_2)\right] dF(y_2)$$

$$= \int_0^{F(y)} \left(u^3 - \frac{1}{2}u^4 \right) du \qquad \text{letting } u = F(y_2)$$

$$= \frac{F^4(y)}{4} - \frac{F^5(y)}{10}.$$

Now consider the sixth term of (2.3.2.2b).

$$P[Y_3 < Med\,(X_1, X_2, X_3) < Y_2 < y]$$

$$= \int_{-\infty}^{y} P(Y_3 < Med(X_1, X_2, X_3) < y_2) dF(y_2)$$

$$= \int_{-\infty}^{y} \left[\int_{-\infty}^{y_2} P(Y_3 < z) 6F(z)\overline{F}(z) dF(z) dF(y_2) \right]$$

$$= \int_{-\infty}^{y} \left[2F^3(y_2) - \frac{3}{2}F^4(y_2) \right] dF(y_2)$$

$$= \int_0^{F(y)} \left(2u^3 - \frac{3}{2}u^4 \right) du \qquad \text{letting } u = F(y_2)$$

$$= \frac{1}{2}F^4(y) - \frac{3}{10}F^5(y).$$

Now adding the six terms we get

$$P[Med(X_1, X_2, X_3) < Med(y, Y_2, Y_3)]$$

$$= 2\left(\left[F^3(y) - \frac{1}{2}F^4(y) \right] \overline{F}(y) \right) + 2\left(\left[2F^3(y) - \frac{3}{2}F^4(y) \right] \left[\overline{F}(y) \right] \right)$$
$$+ 2\left(\left[3F^2(y)\overline{F}(y) + F^3(y) \right] \frac{\overline{F}^2(y)}{2} \right) + 2\left(\frac{3}{4}\overline{F}^4(y) - \frac{3}{5}\overline{F}^5(y) \right)$$
$$+ 2\left(\frac{F^4(y)}{4} - \frac{F^5(y)}{10} \right) + 2\left(\frac{1}{2}F^4(y) - \frac{3}{10}F^5(y) \right)$$

$$= 4v^3 - 6v^4 + \frac{12}{5}v^5 + \frac{3}{10} \quad \text{where } v = F(y).$$

Thus we get

$$P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_4,X_5,X_6)<Med(Y_1,Y_4,Y_5)]$$

$$=\int_{-\infty}^{\infty}P^2[Med(X_1,X_2,X_3)<Med(y,Y_2,Y_3)]dF(y)$$

$$=\int_0^1\left(4v^3-6v^4+\frac{12}{5}v^5+\frac{3}{10}\right)^2dv$$

$$=\frac{6299}{23100}.$$

Thus, finally we have

$$\xi_{0,1}=P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_4,X_5,X_6)<Med(Y_1,Y_4,Y_5)]-\frac{1}{4}$$

$$=\frac{6299}{23100}-\frac{1}{4}$$

$$=\frac{131}{5775}=\xi_{1,0}.$$

Now consider

$$\xi_{2,0}=Cov[h(X_1,X_2,X_3;Y_1,Y_2,Y_3),h(X_1,X_2,X_4;Y_4,Y_5,Y_6)]$$

$$=E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)h(X_1,X_2,X_4;Y_4,Y_5,Y_6)]-\{E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)]\}^2$$

$$=P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_2,X_4)<Med(Y_4,Y_5,Y_6)]-\frac{1}{4}$$

Now

$$P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_2,X_4)<Med(Y_4,Y_5,Y_6)]$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}P^2[Med(x_1,x_2,X_3)<Med(Y_1,Y_2,Y_3)]dF(x_1)dF(x_2)$$

$$=\iint_{x_1<x_2}P^2[Med(x_1,x_2,X_3)<Med(Y_1,Y_2,Y_3)]dF(x_1)dF(x_2)$$
$$+\iint_{x_2<x_1}P^2[Med(x_1,x_2,X_3)<Med(Y_1,Y_2,Y_3)]dF(x_1)dF(x_2) \qquad (2.3.2.2c)$$

Now consider case i) $x_1 < x_2$

$$\begin{aligned}
&P[Med(x_1,x_2,X_3) < Med(Y_1,Y_2,Y_3)] \\
&= P[x_1 < x_2 < X_3 < Med(Y_1,Y_2,Y_3)] + P[x_1 < x_2 < Med(Y_1,Y_2,Y_3) < X_3] \\
&+ P[X_3 < x_1 < x_2 < Med(Y_1,Y_2,Y_3)] + P[X_3 < x_1 < Med(Y_1,Y_2,Y_3) < x_2] \\
&+ P[x_1 < X_3 < x_2 < Med(Y_1,Y_2,Y_3)] + P[x_1 < X_3 < Med(Y_1,Y_2,Y_3) < x_2]
\end{aligned} \qquad (2.3.2.2d)$$

Now consider the first term of (2.3.2.2d).

$$P[x_1 < x_2 < X_3 < Med(Y_1,Y_2,Y_3)]$$

$$= \int_{x_2}^{\infty} P[Med(Y_1,Y_2,Y_3) > x_3] dF(x_3)$$

$$= \int_{x_2}^{\infty} \sum_{i=0}^{1} \binom{3}{i} F^i(x_3) \overline{F}^{3-i}(x_3) dF(x_3)$$

$$= \overline{F}^3(x_2) - \frac{1}{2}\overline{F}^4(x_2).$$

Now consider the second term of (2.3.2.2d).

$$P[x_1 < x_2 < Med(Y_1,Y_2,Y_3) < X_3]$$

$$= \int_{x_2}^{\infty} P(z < X_3) 6F(z)\overline{F}(z) dF(z)$$

$$= 2\overline{F}^3(x_2) - \frac{3}{2}\overline{F}^4(x_2).$$

Now consider the third term of (2.3.2.2d).

$$P[X_3 < x_1 < x_2 < Med(Y_1,Y_2,Y_3)]$$

$$= P[X_3 < x_1] P[x_2 < Med(Y_1,Y_2,Y_3)]$$

$$= F(x_1) \sum_{i=0}^{1} \binom{3}{i} F^i(x_2) \overline{F}^{3-i}(x_2)$$

$$= F(x_1)\left[3\overline{F}^2(x_2) - 2\overline{F}^3(x_2)\right].$$

Now consider the fourth term of (2.3.2.2d).

$$P[X_3 < x_1 < Med(Y_1,Y_2,Y_3) < x_2]$$

$$= P[X_3 < x_1]P[x_1 < Med(Y_1,Y_2,Y_3) < x_2]$$

$$= F(x_1)[P[Med(Y_1,Y_2,Y_3) < x_2] - P[Med(Y_1,Y_2,Y_3) < x_1]]$$

$$= F(x_1)[3F^2(x_2) - 2F^3(x_2) - [3F^2(x_1) - 2F^3(x_1)]]$$

$$= 3F^2(x_2)F(x_1) - 2F^3(x_2)F(x_1) - 3F^3(x_1) + 2F^4(x_1).$$

Now consider the fifth term of (2.3.2.2d).

$$P[x_1 < X_3 < x_2 < Med\,(Y_1,Y_2,Y_3)]$$

$$= P[x_1 < X_3 < x_2]P[x_2 < Med(Y_1,Y_2,Y_3)]$$

$$= [F(x_2) - F(x_1)][3\overline{F}^2(x_2) - 2\overline{F}^3(x_2)]$$

$$= 3F(x_2)\overline{F}^2(x_2) - 2F(x_2)\overline{F}^3(x_2) - 3F(x_1)\overline{F}^2(x_2) + 2F(x_1)\overline{F}^3(x_2).$$

Finally consider the sixth term of (2.3.2.2d).

$$P[x_1 < X_3 < Med(Y_1,Y_2,Y_3) < x_2]$$

$$= \iint_{x_1 < x_3 < z < x_2} 6F(z)\overline{F}(z)dF(x_3)dF(z)$$

$$= \int_{x_1}^{x_2}\int_{x_1}^{z} 6dF(x_3)F(z)\overline{F}(z)dF(z)$$

$$= 2F^3(x_2) - 2F^3(x_1) - \frac{3}{2}F^4(x_2) + \frac{3}{2}F^4(x_1) - 3F(x_1)F^2(x_2) + 3F^3(x_1) + 2F(x_1)F^3(x_2) - 2F^4(x_1).$$

Thus by adding the six terms, we have

$$P[Med(x_1,x_2,X_3) < Med(Y_1,Y_2,Y_3)]$$

$$\begin{aligned}
&= \left(\overline{F}^3(x_2) - \frac{1}{2}\overline{F}^4(x_2)\right) + \left(2\overline{F}^3(x_2) - \frac{3}{2}\overline{F}^4(x_2)\right) + \left(F(x_1)[3\overline{F}^2(x_2) - 2\overline{F}^3(x_2)]\right) \\
&+ \left(3F^2(x_2)F(x_1) - 2F^3(x_2)F(x_1) - 3F^3(x_1) + 2F^4(x_1)\right) \\
&+ \left(3F(x_2)\overline{F}^2(x_2) - 2F(x_2)\overline{F}^3(x_2) - 3F(x_1)\overline{F}^2(x_2) + 2F(x_1)\overline{F}^3(x_2)\right) \\
&+ \left(\begin{array}{l} 2F^3(x_2) - 2F^3(x_1) - \frac{3}{2}F^4(x_2) + \frac{3}{2}F^4(x_1) - 3F(x_1)F^2(x_2) + \\ 3F^3(x_1) + 2F(x_1)F^3(x_2) - 2F^4(x_1) \end{array}\right)
\end{aligned}$$

$$= 4F^3(x_2) - 3F^2(x_2) + 1 - \frac{3}{2}F^4(x_2) - 2F^3(x_1) + \frac{3}{2}F^4(x_1)$$

Thus we have the first term of (2.3.2.2c)

$$\int_{-\infty}^{\infty}\int_{-\infty}^{x_2} P^2[Med(x_1,x_2,X_3) < Med(Y_1,Y_2,Y_3)]dF(x_1)dF(x_2)$$

$$= \int_0^1 \int_0^{F(x_2)} \left[4F^3(x_2) - 3F^2(x_2) + 1 - \frac{3}{2}F^4(x_2) - 2F^3(x_1) + \frac{3}{2}F^4(x_1)\right]^2 dF(x_1)dF(x_2)$$

$$= \frac{311}{2100}.$$

Now consider case ii) $x_1 > x_2$

$$\begin{aligned} &P[Med(x_1,x_2,X_3) < Med(Y_1,Y_2,Y_3)] \\ &= P[x_2 < x_1 < X_3 < Med(Y_1,Y_2,Y_3)] + P[x_2 < x_1 < Med(Y_1,Y_2,Y_3) < X_3] \\ &+ P[X_3 < x_2 < x_1 < Med(Y_1,Y_2,Y_3)] + P[X_3 < x_2 < Med(Y_1,Y_2,Y_3) < x_1] \\ &+ P[x_2 < X_3 < x_1 < Med(Y_1,Y_2,Y_3)] + P[x_2 < X_3 < Med(Y_1,Y_2,Y_3) < x_1] \end{aligned} \quad (2.3.2.2e)$$

By symmetry, we have

$$P[x_2 < x_1 < X_3 < Med(Y_1,Y_2,Y_3)] = \overline{F}^3(x_1) - \frac{1}{2}\overline{F}^4(x_1).$$

$$P[x_2 < x_1 < Med(Y_1,Y_2,Y_3) < X_3] = 2\overline{F}^3(x_1) - \frac{3}{2}\overline{F}^4(x_1).$$

$$P[X_3 < x_2 < x_1 < Med(Y_1,Y_2,Y_3)] = F(x_2)[3\overline{F}^2(x_1) - 2\overline{F}^3(x_1)].$$

$$P[X_3 < x_2 < Med(Y_1,Y_2,Y_3) < x_1]$$

$$= F(x_2)[3F^2(x_1) - 2F^3(x_1) - [3F^2(x_2) - 2F^3(x_2)]]$$

$$= 3F^2(x_1)F(x_2) - 2F^3(x_1)F(x_2) - 3F^3(x_2) + 2F^4(x_2).$$

$$P[x_2 < X_3 < x_1 < Med(Y_1,Y_2,Y_3)]$$

$$= [F(x_1) - F(x_2)][3\overline{F}^2(x_1) - 2\overline{F}^3(x_1)]$$

$$= 3\overline{F}^2(x_1)F(x_1) - 3\overline{F}^2(x_1)F(x_2) - 2\overline{F}^3(x_1)F(x_1) + 2\overline{F}^3(x_1)F(x_2).$$

$$P[x_2 < X_3 < Med(Y_1,Y_2,Y_3) < x_1]$$

$$= 2F^3(x_1) - 2F^3(x_2) - \frac{3}{2}F^4(x_1) + \frac{3}{2}F^4(x_2) - 3F(x_2)F^2(x_1) + 3F^3(x_2) + 2F(x_2)F^3(x_1) - 2F^4(x_2).$$

Thus by adding the six terms, we have

$$P[Med(x_1,x_2,X_3) < Med(Y_1,Y_2,Y_3)]$$

$$= \begin{bmatrix} \left(\bar{F}^3(x_1) - \frac{1}{2}\bar{F}^4(x_1)\right) + \left(2\bar{F}^3(x_1) - \frac{3}{2}\bar{F}^4(x_1)\right) \\ + \left(F(x_2)\left[3\bar{F}^2(x_1) - 2\bar{F}^3(x_1)\right]\right) \\ + \left(3F^2(x_1)F(x_2) - 2F^3(x_1)F(x_2) - 3F^3(x_2) + 2F^4(x_2)\right) \\ + \left(3\bar{F}^2(x_1)F(x_1) - 3\bar{F}^2(x_1)F(x_2) - 2\bar{F}^3(x_1)F(x_1) + 2\bar{F}^3(x_1)F(x_2)\right) \\ + \left(2F^3(x_1) - 2F^3(x_2) - \frac{3}{2}F^4(x_1) + \frac{3}{2}F^4(x_2) - 3F(x_2)F^2(x_1) + 3F^3(x_2) + 2F(x_2)F^3(x_1) - 2F^4(x_2)\right) \end{bmatrix}$$

$$= \left[4F^3(x_1) - 3F^2(x_1) + 1 - \frac{3}{2}F^4(x_1) - 2F^3(x_2) + \frac{3}{2}F^4(x_2)\right].$$

Thus we have the second term of (2.3.2.2c)

$$\int_{-\infty}^{\infty}\int_{-\infty}^{x_1} P^2[Med(x_1,x_2,X_3) < Med(Y_1,Y_2,Y_3)]dF(x_2)dF(x_1)$$

$$= \int_0^1 \int_0^{F(x_1)} \left[4F^3(x_1) - 3F^2(x_1) + 1 - \frac{3}{2}F^4(x_1) - 2F^3(x_2) + \frac{3}{2}F^4(x_2)\right]^2 dF(x_2)dF(x_1)$$

$$= \frac{311}{2100}.$$

Now by adding the first and the second term of (2.3.2.2c),we have

$$P[Med(X_1,X_2,X_3) < Med(Y_1,Y_2,Y_3), Med(X_1,X_2,X_4) < Med(Y_1,Y_2,Y_3)]$$

$$= \left[\iint_{x_1<x_2} P^2[Med(x_1,x_2,X_3) < Med(Y_1,Y_2,Y_3)]dF(x_1)dF(x_2)\right] + \left[\iint_{x_2<x_1} P^2[Med(x_1,x_2,X_3) < Med(Y_1,Y_2,Y_3)]dF(x_1)dF(x_2)\right]$$

$$= \frac{311}{2100} + \frac{311}{2100}$$

$$= \frac{311}{1050}.$$

Thus finally we have

$$\xi_{2,0} = P[Med(X_1,X_2,X_3) < Med(Y_1,Y_2,Y_3), Med(X_1,X_2,X_4) < Med(Y_4,Y_5,Y_6)] - \frac{1}{4}$$

$$= \frac{311}{1050} - \frac{1}{4}$$

$$= \frac{97}{2100}.$$

Now consider

$$\xi_{0,2} = Cov[h(X_1,X_2,X_3;Y_1,Y_2,Y_3), h(X_4,X_5,X_6;Y_1,Y_2,Y_4)]$$

$$= E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)h(X_4,X_5,X_6;Y_1,Y_2,Y_4)] - \{E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)]\}^2$$

$$= P[Med(X_1,X_2,X_3) < Med(Y_1,Y_2,Y_3), Med(X_4,X_5,X_6) < Med(Y_1,Y_2,Y_4)] - \frac{1}{4}$$

As in the case of $\xi_{1,0}, \xi_{0,1}$ it follows from symmetry that

$$\xi_{0,2} = \xi_{2,0} = \frac{97}{2100}.$$

Now consider

$$\xi_{3,0} = Cov[h(X_1,X_2,X_3;Y_1,Y_2,Y_3), h(X_1,X_2,X_3;Y_4,Y_5,Y_6)]$$

$$= E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)h(X_1,X_2,X_3;Y_4,Y_5,Y_6)] - \{E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)]\}^2$$

$$= P[Med(X_1,X_2,X_3) < Med(Y_1,Y_2,Y_3), Med(X_1,X_2,X_3) < Med(Y_4,Y_5,Y_6)] - \frac{1}{4}.$$

Now

$$P[Med(X_1,X_2,X_3) < Med(Y_1,Y_2,Y_3), Med(X_1,X_2,X_3) < Med(Y_4,Y_5,Y_6)]$$

$$= \int P^2[x < Med(Y_1,Y_2,Y_3)] 6F(x)\overline{F}(x)dF(x)$$

$$= \int_{-\infty}^{\infty} (\overline{F}^3(x) + 3F(x)\overline{F}^2(x))^2 6F(x)\overline{F}(x)dF(x)$$

$$= \int_0^1 u^4(u+3(1-u))^2 6u(1-u)du \qquad \text{letting } u = F(x)$$

$$= \frac{1}{3}.$$

Thus

$$\xi_{3,0} = P[Med(X_1,X_2,X_3) < Med(Y_1,Y_2,Y_3), Med(X_1,X_2,X_3) < Med(Y_4,Y_5,Y_6)] - \frac{1}{4}$$

$$= \frac{1}{3} - \frac{1}{4}$$

$$= \frac{1}{12}.$$

Now consider

$$\xi_{0,3} = Cov[h(X_1,X_2,X_3;Y_1,Y_2,Y_3), h(X_4,X_5,X_6;Y_1,Y_2,Y_3)]$$

$$= E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)h(X_4,X_5,X_6;Y_1,Y_2,Y_3)] - \{E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)]\}^2$$

$$= P[Med(X_1,X_2,X_3) < Med(Y_1,Y_2,Y_3), Med(X_4,X_5,X_6) < Med(Y_1,Y_2,Y_3)] - \frac{1}{4}$$

$$= \xi_{3,0} = \frac{1}{12} \text{ by symmetry.}$$

Now consider

$$\xi_{3,3} = Cov[h(X_1,X_2,X_3;Y_1,Y_2,Y_3), h(X_1,X_2,X_3;Y_1,Y_2,Y_3)]$$

$$= Var[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)]$$

$$= E[h^2(X_1,X_2,X_3;Y_1,Y_2,Y_3)] - \frac{1}{4}$$

$$= \frac{1}{2} - \frac{1}{4}$$

$$= \frac{1}{4}.$$

Now consider

$$\xi_{1,2} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_4, X_5; Y_1, Y_2, Y_4)]$$

$$= E[h(X_1, X_2, X_3; Y_1, Y_2, Y_3)h(X_1, X_4, X_5; Y_1, Y_2, Y_4)] - \{E[h(X_1, X_2, X_3; Y_1, Y_2, Y_3)]\}^2$$

$$= P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_4, X_5) < Med(Y_1, Y_2, Y_4)] - \frac{1}{4}$$

Now

$$P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_4, X_5) < Med(Y_1, Y_2, Y_4)]$$

$$= \iiint P^2[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(y_1)dF(y_2)$$

$$= \iiint_{x_1 < y_1 < y_2} P^2[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(y_1)dF(y_2) +$$

$$\iiint_{x_1 < y_2 < y_1} P^2[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(y_1)dF(y_2) +$$

$$\iiint_{y_1 < x_1 < y_2} P^2[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(y_1)dF(y_2) +$$

$$\iiint_{y_2 < x_1 < y_1} P^2[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(y_1)dF(y_2) +$$

$$\iiint_{y_1 < y_2 < x_1} P^2[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(y_1)dF(y_2) +$$

$$\iiint_{y_2 < y_1 < x_1} P^2[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(y_1)dF(y_2)$$

(2.3.2.2f)

Now consider the first term of (2.3.2.2f).

$$\iiint_{x_1 < y_1 < y_2} P^2[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(y_1)dF(y_2).$$

we have

$$P[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)] = 2a_1 + 2a_2 + \ldots + 2a_{48}$$

where

$$a_1 = P[x_1 < y_1 < X_2 < y_2 < X_3 < Y_3]$$

$$= P[y_1 < X_2 < y_2]P[y_2 < X_3 < Y_3]$$

$$= [F(y_2) - F(y_1)]\left[\frac{\overline{F}^2(y_2)}{2}\right]$$

$$a_2 = P[x_1 < y_1 < X_2 < y_2 < Y_3 < X_3]$$

$$= P[y_1 < X_2 < y_2]P[y_2 < Y_3 < X_3]$$

$$= [F(y_2) - F(y_1)]\left[\frac{\overline{F}^2(y_2)}{2}\right]$$

$$a_3 = P[x_1 < y_1 < X_2 < X_3 < y_2 < Y_3]$$

$$= P[y_1 < X_2 < X_3 < y_2]P[y_2 < Y_3]$$

$$= \frac{[F(y_2) - F(y_1)]^2}{2}\overline{F}(y_2)$$

$$a_4 = P[x_1 < y_1 < X_2 < Y_3 < y_2 < X_3]$$

$$= P[y_1 < X_2 < Y_3 < y_2]P[y_2 < X_3]$$

$$= \frac{[F(y_2) - F(y_1)]^2}{2}\overline{F}(y_2)$$

$$a_5 = P[x_1 < y_1 < X_2 < X_3 < Y_3 < y_2]$$

$$= P[y_1 < X_2 < X_3 < Y_3 < y_2]$$

$$= \frac{[F(y_2) - F(y_1)]^3}{6}$$

$$a_6 = P[x_1 < y_1 < X_2 < Y_3 < X_3 < y_2]$$

$$= P[y_1 < X_2 < Y_3 < X_3 < y_2]$$

$$= \frac{[F(y_2) - F(y_1)]^3}{6}$$

$$a_7 = P[x_1 < X_2 < y_1 < y_2 < X_3 < Y_3]$$

$$= P[x_1 < X_2 < y_1]P[y_2 < X_3 < Y_3]$$

$$= [F(y_1) - F(x_1)]\left[\frac{\overline{F}^2(y_2)}{2}\right]$$

$$a_8 = P[x_1 < X_2 < y_1 < y_2 < Y_3 < X_3]$$

$$= P[x_1 < X_2 < y_1]P[y_2 < Y_3 < X_3]$$

$$= [F(y_1) - F(x_1)]\left[\frac{\overline{F}^2(y_2)}{2}\right]$$

$$a_9 = P[x_1 < X_2 < y_1 < X_3 < y_2 < Y_3]$$

$$= P[x_1 < X_2 < y_1]P[y_1 < X_3 < y_2]P[y_2 < Y_3]$$

$$= [F(y_1) - F(x_1)][F(y_2) - F(y_1)]\overline{F}(y_2)$$

$$a_{10} = P[x_1 < X_2 < y_1 < Y_3 < y_2 < X_3]$$

$$= P[x_1 < X_2 < y_1]P[y_1 < Y_3 < y_2]P[y_2 < X_3]$$

$$= [F(y_1) - F(x_1)][F(y_2) - F(y_1)]\overline{F}(y_2)$$

$$a_{11} = P[x_1 < X_2 < y_1 < X_3 < Y_3 < y_2]$$

$$= P[x_1 < X_2 < y_1]P[y_1 < X_3 < Y_3 < y_2]$$

$$= [F(y_1) - F(x_1)]\frac{[F(y_2) - F(y_1)]^2}{2}$$

$$a_{12} = P[x_1 < X_2 < y_1 < Y_3 < X_3 < y_2]$$

$$= P[x_1 < X_2 < y_1]P[y_1 < Y_3 < X_3 < y_2]$$

$$= [F(y_1) - F(x_1)]\frac{[F(y_2) - F(y_1)]^2}{2}$$

$$a_{13} = P[x_1 < X_2 < X_3 < y_1 < y_2 < Y_3]$$

$$= P[x_1 < X_2 < X_3 < y_1]P[y_2 < Y_3]$$

$$= \frac{[F(y_1) - F(x_1)]^2}{2} \overline{F}(y_2)$$

$$a_{14} = P[x_1 < X_2 < Y_3 < y_1 < y_2 < X_3]$$

$$= P[x_1 < X_2 < Y_3 < y_1]P[y_2 < X_3]$$

$$= \frac{[F(y_1) - F(x_1)]^2}{2} \overline{F}(y_2)$$

$$a_{15} = P[x_1 < Y_3 < X_2 < y_1 < y_2 < X_3]$$

$$= P[x_1 < Y_3 < X_2 < y_1]P[y_2 < X_3]$$

$$= \frac{[F(y_1) - F(x_1)]^2}{2} \overline{F}(y_2)$$

$$a_{16} = P[x_1 < X_2 < X_3 < y_1 < Y_3 < y_2]$$

$$= P[x_1 < X_2 < X_3 < y_1]P[y_1 < Y_3 < y_2]$$

$$= \frac{[F(y_1) - F(x_1)]^2}{2} [F(y_2) - F(y_1)]$$

$$a_{17} = P[x_1 < X_2 < Y_3 < y_1 < X_3 < y_2]$$

$$= P[x_1 < X_2 < Y_3 < y_1]P[y_1 < X_3 < y_2]$$

$$= \frac{[F(y_1) - F(x_1)]^2}{2} [F(y_2) - F(y_1)]$$

$$a_{18} = P[x_1 < Y_3 < X_2 < y_1 < X_3 < y_2]$$

$$= P[x_1 < Y_3 < X_2 < y_1]P[y_1 < X_3 < y_2]$$

$$= \frac{[F(y_1) - F(x_1)]^2}{2} [F(y_2) - F(y_1)]$$

$$a_{19} = P[x_1 < X_2 < X_3 < Y_3 < y_1 < y_2]$$

$$= P[x_1 < X_2 < X_3 < Y_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_1)]^3}{6}$$

$$a_{20} = P[x_1 < X_2 < Y_3 < X_3 < y_1 < y_2]$$

$$= P[x_1 < X_2 < Y_3 < X_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_1)]^3}{6}$$

$$a_{21} = P[x_1 < Y_3 < X_2 < X_3 < y_1 < y_2]$$

$$= P[x_1 < Y_3 < X_2 < X_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_1)]^3}{6}$$

$$a_{22} = P[X_2 < x_1 < y_1 < y_2 < X_3 < Y_3]$$

$$= P[X_2 < x_1]P[y_2 < X_3 < Y_3]$$

$$= [F(x_1)]\left[\frac{\overline{F}^2(y_2)}{2}\right]$$

$$a_{23} = P[X_2 < x_1 < y_1 < y_2 < Y_3 < X_3]$$

$$= P[X_2 < x_1]P[y_2 < Y_3 < X_3]$$

$$= [F(x_1)]\left[\frac{\overline{F}^2(y_2)}{2}\right]$$

$$a_{24} = P[X_2 < x_1 < y_1 < X_3 < y_2 < Y_3]$$

$$= P[X_2 < x_1]P[y_1 < X_3 < y_2]P[y_2 < Y_3]$$

$$= [F(x_1)][F(y_2) - F(y_1)]\overline{F}(y_2)$$

$$a_{25} = P[X_2 < x_1 < y_1 < Y_3 < y_2 < X_3]$$

$$= P[X_2 < x_1]P[y_1 < Y_3 < y_2]P[y_2 < X_3]$$

$$= [F(x_1)][F(y_2) - F(y_1)]\overline{F}(y_2)$$

$$a_{26} = P[X_2 < x_1 < y_1 < X_3 < Y_3 < y_2]$$

$$= P[X_2 < x_1]P[y_1 < X_3 < Y_3 < y_2]$$

$$= [F(x_1)]\frac{[F(y_2) - F(y_1)]^2}{2}$$

$$a_{27} = P[X_2 < x_1 < y_1 < Y_3 < X_3 < y_2]$$

$$= P[X_2 < x_1]P[y_1 < Y_3 < X_3 < y_2]$$

$$= [F(x_1)]\frac{[F(y_2) - F(y_1)]^2}{2}$$

$$a_{28} = P[X_2 < x_1 < X_3 < y_1 < y_2 < Y_3]$$

$$= P[X_2 < x_1]P[x_1 < X_3 < y_1]P[y_2 < Y_3]$$

$$= [F(x_1)][F(y_1) - F(x_1)]\overline{F}(y_2)$$

$$a_{29} = P[X_2 < x_1 < Y_3 < y_1 < y_2 < X_3]$$

$$= P[X_2 < x_1]P[x_1 < Y_3 < y_1]P[y_2 < X_3]$$

$$= [F(x_1)][F(y_1) - F(x_1)]\overline{F}(y_2)$$

$$a_{30} = P[Y_3 < x_1 < X_2 < y_1 < y_2 < X_3]$$

$$= P[Y_3 < x_1]P[x_1 < X_2 < y_1]P[y_2 < X_3]$$

$$= [F(x_1)][F(y_1) - F(x_1)]\overline{F}(y_2)$$

$$a_{31} = P[X_2 < x_1 < X_3 < y_1 < Y_3 < y_2]$$

$$= P[X_2 < x_1]P[x_1 < X_3 < y_1]P[y_1 < Y_3 < y_2]$$

$$= [F(x_1)][F(y_1) - F(x_1)][F(y_2) - F(y_1)]$$

$$a_{32} = P[X_2 < x_1 < Y_3 < y_1 < X_3 < y_2]$$

$$= P[X_2 < x_1]P[x_1 < Y_3 < y_1]P[y_1 < X_3 < y_2]$$

$$= [F(x_1)][F(y_1) - F(x_1)][F(y_2) - F(y_1)]$$

$$a_{33} = P[Y_3 < x_1 < X_2 < y_1 < X_3 < y_2]$$

$$= P[Y_3 < x_1]P[x_1 < X_2 < y_1]P[y_1 < X_3 < y_2]$$

$$= [F(x_1)][F(y_1) - F(x_1)][F(y_2) - F(y_1)]$$

$$a_{34} = P[X_2 < x_1 < X_3 < Y_3 < y_1 < y_2]$$

$$= P[X_2 < x_1]P[x_1 < X_3 < Y_3 < y_1]$$

$$= [F(x_1)]\frac{[F(y_1) - F(x_1)]^2}{2}$$

$$a_{35} = P[X_2 < x_1 < Y_3 < X_3 < y_1 < y_2]$$

$$= P[X_2 < x_1]P[x_1 < Y_3 < X_3 < y_1]$$

$$= [F(x_1)]\frac{[F(y_1) - F(x_1)]^2}{2}$$

$$a_{36} = P[Y_3 < x_1 < X_2 < X_3 < y_1 < y_2]$$

$$= P[Y_3 < x_1]P[x_1 < X_2 < X_3 < y_1]$$

$$= [F(x_1)]\frac{[F(y_1) - F(x_1)]^2}{2}$$

$$a_{37} = P[X_2 < X_3 < x_1 < y_1 < y_2 < Y_3]$$

$$= P[X_2 < X_3 < x_1]P[y_2 < Y_3]$$

$$= \frac{F^2(x_1)}{2}\overline{F}(y_2)$$

$$a_{38} = P[X_2 < Y_3 < x_1 < y_1 < y_2 < X_3]$$

$$= P[X_2 < Y_3 < x_1]P[y_2 < X_3]$$

$$= \frac{F^2(x_1)}{2}\overline{F}(y_2)$$

$$a_{39} = P[Y_3 < X_2 < x_1 < y_1 < y_2 < X_3]$$

$$= P[Y_3 < X_2 < x_1]P[y_2 < X_3]$$

$$= \frac{F^2(x_1)}{2}\overline{F}(y_2)$$

$$a_{40} = P[X_2 < X_3 < x_1 < y_1 < Y_3 < y_2]$$

$$= P[X_2 < X_3 < x_1]P[y_1 < Y_3 < y_2]$$

$$= \frac{F^2(x_1)}{2}[F(y_2) - F(y_1)]$$

$$a_{41} = P[X_2 < Y_3 < x_1 < y_1 < X_3 < y_2]$$

$$= P[X_2 < Y_3 < x_1]P[y_1 < X_3 < y_2]$$

$$= \frac{F^2(x_1)}{2}[F(y_2) - F(y_1)]$$

$$a_{42} = P[Y_3 < X_2 < x_1 < y_1 < X_3 < y_2]$$

$$= P[Y_3 < X_2 < x_1]P[y_1 < X_3 < y_2]$$

$$= \frac{F^2(x_1)}{2}[F(y_2) - F(y_1)]$$

$$a_{43} = P[X_2 < X_3 < x_1 < Y_3 < y_1 < y_2]$$

$$= P[X_2 < X_3 < x_1]P[x_1 < Y_3 < y_1]$$

$$= \frac{F^2(x_1)}{2}[F(y_1) - F(x_1)]$$

$$a_{44} = P[X_2 < Y_3 < x_1 < X_3 < y_1 < y_2]$$

$$= P[X_2 < Y_3 < x_1]P[x_1 < X_3 < y_1]$$

$$= \frac{F^2(x_1)}{2}[F(y_1) - F(x_1)]$$

$$a_{45} = P[Y_3 < X_2 < x_1 < X_3 < y_1 < y_2]$$

$$= P[Y_3 < X_2 < x_1]P[x_1 < X_3 < y_1]$$

$$= \frac{F^2(x_1)}{2}[F(y_1) - F(x_1)]$$

$$a_{46} = P[X_2 < X_3 < Y_3 < x_1 < y_1 < y_2]$$

$$= P[X_2 < X_3 < Y_3 < x_1]$$

$$= \frac{F^3(x_1)}{6}$$

$$a_{47} = P[X_2 < Y_3 < X_3 < x_1 < y_1 < y_2]$$

$$= P[X_2 < Y_3 < X_3 < x_1]$$

$$= \frac{F^3(x_1)}{6}$$

$$a_{48} = P[Y_3 < X_2 < X_3 < x_1 < y_1 < y_2]$$

$$= P[Y_3 < X_2 < X_3 < x_1]$$

$$= \frac{F^3(x_1)}{6}.$$

Now adding all the 48 terms we have

$$P[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)] = 2a_1 + 2a_2 + \ldots + 2a_{48}$$

$$= -\frac{2}{3}F^3(y_1) - 2F^2(y_2) + F^2(y_1) + \frac{2}{3}F^3(y_2) + 2F(y_1)$$

Thus integrating we have

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_2}\int_{-\infty}^{y_1} P^2[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(y_1)dF(y_2)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{y_2}\int_{-\infty}^{y_1}\left(-\frac{2}{3}F^3(y_1)-2F^2(y_2)+F^2(y_1)+\frac{2}{3}F^3(y_2)+2F(y_1)\right)^2 dF(x_1)dF(y_1)dF(y_2)$$

$$= \frac{277}{2520}.$$

Now consider the second term of (2.3.2.2f) that is case $x_1 < y_2 < y_1$

$$\iiint_{x_1<y_2<y_1} P^2[Med(x_1,X_2,X_3) < Med(y_1,y_2,Y_3)]dF(x_1)dF(y_1)dF(y_2).$$

We have

$$P[Med(x_1,X_2,X_3) < Med(y_1,y_2,Y_3)] = 2b_1 + 2b_2 + \ldots + 2b_{48}$$

Where

$$b_1 = P[x_1 < y_2 < X_2 < y_1 < X_3 < Y_3]$$

$$= P[y_2 < X_2 < y_1]P[y_1 < X_3 < Y_3]$$

$$= [F(y_1)-F(y_2)]\left[\frac{\overline{F}^2(y_1)}{2}\right]$$

$$b_2 = P[x_1 < y_2 < X_2 < y_1 < Y_3 < X_3]$$

$$= P[y_2 < X_2 < y_1]P[y_1 < Y_3 < X_3]$$

$$= [F(y_1)-F(y_2)]\left[\frac{\overline{F}^2(y_1)}{2}\right]$$

$$b_3 = P[x_1 < y_2 < X_2 < X_3 < y_1 < Y_3]$$

$$= P[y_2 < X_2 < X_3 < y_1]P[y_1 < Y_3]$$

$$= \frac{[F(y_1)-F(y_2)]^2}{2}\overline{F}(y_1)$$

$$b_4 = P[x_1 < y_2 < X_2 < Y_3 < y_1 < X_3]$$

$$= P[y_2 < X_2 < Y_3 < y_1]P[y_1 < X_3]$$

$$= \frac{[F(y_1) - F(y_2)]^2}{2} \overline{F}(y_1)$$

$$b_5 = P[x_1 < y_2 < X_2 < X_3 < Y_3 < y_1]$$

$$= P[y_2 < X_2 < X_3 < Y_3 < y_1]$$

$$= \frac{[F(y_1) - F(y_2)]^3}{6}$$

$$b_6 = P[x_1 < y_2 < X_2 < Y_3 < X_3 < y_1]$$

$$= P[y_2 < X_2 < Y_3 < X_3 < y_1]$$

$$= \frac{[F(y_1) - F(y_2)]^3}{6}$$

$$b_7 = P[x_1 < X_2 < y_2 < y_1 < X_3 < Y_3]$$

$$= P[x_1 < X_2 < y_2] P[y_1 < X_3 < Y_3]$$

$$= [F(y_2) - F(x_1)] \left[\frac{\overline{F}^2(y_1)}{2} \right]$$

$$b_8 = P[x_1 < X_2 < y_2 < y_1 < Y_3 < X_3]$$

$$= P[x_1 < X_2 < y_2] P[y_1 < Y_3 < X_3]$$

$$= [F(y_2) - F(x_1)] \left[\frac{\overline{F}^2(y_1)}{2} \right]$$

$$b_9 = P[x_1 < X_2 < y_2 < X_3 < y_1 < Y_3]$$

$$= P[x_1 < X_2 < y_2] P[y_2 < X_3 < y_1] P[y_1 < Y_3]$$

$$= [F(y_2) - F(x_1)][F(y_1) - F(y_2)] \overline{F}(y_1)$$

$$b_{10} = P[x_1 < X_2 < y_2 < Y_3 < y_1 < X_3]$$

$$= P[x_1 < X_2 < y_2] P[y_2 < Y_3 < y_1] P[y_1 < X_3]$$

$$= [F(y_2) - F(x_1)][F(y_1) - F(y_2)] \overline{F}(y_1)$$

$$b_{11} = P[x_1 < X_2 < y_2 < X_3 < Y_3 < y_1]$$

$$= P[x_1 < X_2 < y_2]P[y_2 < X_3 < Y_3 < y_1]$$

$$= [F(y_2) - F(x_1)]\frac{[F(y_1) - F(y_2)]^2}{2}$$

$$b_{12} = P[x_1 < X_2 < y_2 < Y_3 < X_3 < y_1]$$

$$= P[x_1 < X_2 < y_2]P[y_2 < Y_3 < X_3 < y_1]$$

$$= [F(y_2) - F(x_1)]\frac{[F(y_1) - F(y_2)]^2}{2}$$

$$b_{13} = P[x_1 < X_2 < X_3 < y_2 < y_1 < Y_3]$$

$$= P[x_1 < X_2 < X_3 < y_2]P[y_1 < Y_3]$$

$$= \frac{[F(y_2) - F(x_1)]^2}{2}\overline{F}(y_1)$$

$$b_{14} = P[x_1 < X_2 < Y_3 < y_2 < y_1 < X_3]$$

$$= P[x_1 < X_2 < Y_3 < y_2]P[y_1 < X_3]$$

$$= \frac{[F(y_2) - F(x_1)]^2}{2}\overline{F}(y_1)$$

$$b_{15} = P[x_1 < Y_3 < X_2 < y_2 < y_1 < X_3]$$

$$= P[x_1 < Y_3 < X_2 < y_2]P[y_1 < X_3]$$

$$= \frac{[F(y_2) - F(x_1)]^2}{2}\overline{F}(y_1)$$

$$b_{16} = P[x_1 < X_2 < X_3 < y_2 < Y_3 < y_1]$$

$$= P[x_1 < X_2 < X_3 < y_2]P[y_2 < Y_3 < y_1]$$

$$= \frac{[F(y_2) - F(x_1)]^2}{2}[F(y_1) - F(y_2)]$$

$$b_{17} = P[x_1 < X_2 < Y_3 < y_2 < X_3 < y_1]$$

$$= P[x_1 < X_2 < Y_3 < y_2]P[y_2 < X_3 < y_1]$$

$$= \frac{[F(y_2) - F(x_1)]^2}{2}[F(y_1) - F(y_2)]$$

$$b_{18} = P[x_1 < Y_3 < X_2 < y_2 < X_3 < y_1]$$

$$= P[x_1 < Y_3 < X_2 < y_2]P[y_2 < X_3 < y_1]$$

$$= \frac{[F(y_2) - F(x_1)]^2}{2}[F(y_1) - F(y_2)]$$

$$b_{19} = P[x_1 < X_2 < X_3 < Y_3 < y_2 < y_1]$$

$$= P[x_1 < X_2 < X_3 < Y_3 < y_2]$$

$$= \frac{[F(y_2) - F(x_1)]^3}{6}$$

$$b_{20} = P[x_1 < X_2 < Y_3 < X_3 < y_2 < y_1]$$

$$= P[x_1 < X_2 < Y_3 < X_3 < y_2]$$

$$= \frac{[F(y_2) - F(x_1)]^3}{6}$$

$$b_{21} = P[x_1 < Y_3 < X_2 < X_3 < y_2 < y_1]$$

$$= P[x_1 < Y_3 < X_2 < X_3 < y_2]$$

$$= \frac{[F(y_2) - F(x_1)]^3}{6}$$

$$b_{22} = P[X_2 < x_1 < y_2 < y_1 < X_3 < Y_3]$$

$$= P[X_2 < x_1]P[y_1 < X_3 < Y_3]$$

$$= [F(x_1)]\left[\frac{\overline{F}^2(y_1)}{2}\right]$$

$$b_{23} = P[X_2 < x_1 < y_2 < y_1 < Y_3 < X_3]$$

$$= P[X_2 < x_1]P[y_1 < Y_3 < X_3]$$

$$= [F(x_1)]\left[\frac{\overline{F}^2(y_1)}{2}\right]$$

$$b_{24} = P[X_2 < x_1 < y_2 < X_3 < y_1 < Y_3]$$

$$= P[X_2 < x_1]P[y_2 < X_3 < y_1]P[y_1 < Y_3]$$

$$= [F(x_1)][F(y_1) - F(y_2)]\overline{F}(y_1)$$

$$b_{25} = P[X_2 < x_1 < y_2 < Y_3 < y_1 < X_3]$$

$$= P[X_2 < x_1]P[y_2 < Y_3 < y_1]P[y_1 < X_3]$$

$$= [F(x_1)][F(y_1) - F(y_2)]\overline{F}(y_1)$$

$$b_{26} = P[X_2 < x_1 < y_2 < X_3 < Y_3 < y_1]$$

$$= P[X_2 < x_1]P[y_2 < X_3 < Y_3 < y_1]$$

$$= [F(x_1)]\frac{[F(y_1) - F(y_2)]^2}{2}$$

$$b_{27} = P[X_2 < x_1 < y_2 < Y_3 < X_3 < y_1]$$

$$= P[X_2 < x_1]P[y_2 < Y_3 < X_3 < y_1]$$

$$= [F(x_1)]\frac{[F(y_1) - F(y_2)]^2}{2}$$

$$b_{28} = P[X_2 < x_1 < X_3 < y_2 < y_1 < Y_3]$$

$$= P[X_2 < x_1]P[x_1 < X_3 < y_2]P[y_1 < Y_3]$$

$$= [F(x_1)][F(y_2) - F(x_1)]\overline{F}(y_1)$$

$$b_{29} = P[X_2 < x_1 < Y_3 < y_2 < y_1 < X_3]$$

$$= P[X_2 < x_1]P[x_1 < Y_3 < y_2]P[y_1 < X_3]$$

$$= [F(x_1)][F(y_2) - F(x_1)]\overline{F}(y_1)$$

$$b_{30} = P[Y_3 < x_1 < X_2 < y_2 < y_1 < X_3]$$

$$= P[Y_3 < x_1]P[x_1 < X_2 < y_2]P[y_1 < X_3]$$

$$= [F(x_1)][F(y_2) - F(x_1)]\overline{F}(y_1)$$

$$b_{31} = P[X_2 < x_1 < X_3 < y_2 < Y_3 < y_1]$$

$$= P[X_2 < x_1]P[x_1 < X_3 < y_2]P[y_2 < Y_3 < y_1]$$

$$= [F(x_1)][F(y_2) - F(x_1)][F(y_1) - F(y_2)]$$

$$b_{32} = P[X_2 < x_1 < Y_3 < y_2 < X_3 < y_1]$$

$$= P[X_2 < x_1]P[x_1 < Y_3 < y_2]P[y_2 < X_3 < y_1]$$

$$= [F(x_1)][F(y_2) - F(x_1)][F(y_1) - F(y_2)]$$

$$b_{33} = P[Y_3 < x_1 < X_2 < y_2 < X_3 < y_1]$$

$$= P[Y_3 < x_1]P[x_1 < X_2 < y_2]P[y_2 < X_3 < y_1]$$

$$= [F(x_1)][F(y_2) - F(x_1)][F(y_1) - F(y_2)]$$

$$b_{34} = P[X_2 < x_1 < X_3 < Y_3 < y_2 < y_1]$$

$$= P[X_2 < x_1]P[x_1 < X_3 < Y_3 < y_2]$$

$$= [F(x_1)]\frac{[F(y_2) - F(x_1)]^2}{2}$$

$$b_{35} = P[X_2 < x_1 < Y_3 < X_3 < y_2 < y_1]$$

$$= P[X_2 < x_1]P[x_1 < Y_3 < X_3 < y_2]$$

$$= [F(x_1)]\frac{[F(y_2) - F(x_1)]^2}{2}$$

$$b_{36} = P[Y_3 < x_1 < X_2 < X_3 < y_2 < y_1]$$

$$= P[Y_3 < x_1]P[x_1 < X_2 < X_3 < y_2]$$

$$= [F(x_1)]\frac{[F(y_2)-F(x_1)]^2}{2}$$

$$b_{37} = P[X_2 < X_3 < x_1 < y_2 < y_1 < Y_3]$$

$$= P[X_2 < X_3 < x_1]P[y_1 < Y_3]$$

$$= \frac{F^2(x_1)}{2}\overline{F}(y_1)$$

$$b_{38} = P[X_2 < Y_3 < x_1 < y_2 < y_1 < X_3]$$

$$= P[X_2 < Y_3 < x_1]P[y_1 < X_3]$$

$$= \frac{F^2(x_1)}{2}\overline{F}(y_1)$$

$$b_{39} = P[Y_3 < X_2 < x_1 < y_2 < y_1 < X_3]$$

$$= P[Y_3 < X_2 < x_1]P[y_1 < X_3]$$

$$= \frac{F^2(x_1)}{2}\overline{F}(y_1)$$

$$b_{40} = P[X_2 < X_3 < x_1 < y_2 < Y_3 < y_1]$$

$$= P[X_2 < X_3 < x_1]P[y_2 < Y_3 < y_1]$$

$$= \frac{F^2(x_1)}{2}[F(y_1)-F(y_2)]$$

$$b_{41} = P[X_2 < Y_3 < x_1 < y_2 < X_3 < y_1]$$

$$= P[X_2 < Y_3 < x_1]P[y_2 < X_3 < y_1]$$

$$= \frac{F^2(x_1)}{2}[F(y_1)-F(y_2)]$$

$$b_{42} = P[Y_3 < X_2 < x_1 < y_2 < X_3 < y_1]$$

$$= P[Y_3 < X_2 < x_1]P[y_2 < X_3 < y_1]$$

$$= \frac{F^2(x_1)}{2}[F(y_1) - F(y_2)]$$

$$b_{43} = P[X_2 < X_3 < x_1 < Y_3 < y_2 < y_1]$$

$$= P[X_2 < X_3 < x_1]P[x_1 < Y_3 < y_2]$$

$$= \frac{F^2(x_1)}{2}[F(y_2) - F(x_1)]$$

$$b_{44} = P[X_2 < Y_3 < x_1 < X_3 < y_2 < y_1]$$

$$= P[X_2 < Y_3 < x_1]P[x_1 < X_3 < y_2]$$

$$= \frac{F^2(x_1)}{2}[F(y_2) - F(x_1)]$$

$$b_{45} = P[Y_3 < X_2 < x_1 < X_3 < y_2 < y_1]$$

$$= P[Y_3 < X_2 < x_1]P[x_1 < X_3 < y_2]$$

$$= \frac{F^2(x_1)}{2}[F(y_2) - F(x_1)]$$

$$b_{46} = P[X_2 < X_3 < Y_3 < x_1 < y_2 < y_1]$$

$$= P[X_2 < X_3 < Y_3 < x_1]$$

$$= \frac{F^3(x_1)}{6}$$

$$b_{47} = P[X_2 < Y_3 < X_3 < x_1 < y_2 < y_1]$$

$$= P[X_2 < Y_3 < X_3 < x_1]$$

$$= \frac{F^3(x_1)}{6}$$

$$b_{48} = P[Y_3 < X_2 < X_3 < x_1 < y_2 < y_1]$$

$$= P[Y_3 < X_2 < X_3 < x_1]$$

$$= \frac{F^3(x_1)}{6}.$$

Now adding all the 48 terms we have

$$P[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)] = 2b_1 + 2b_2 + \ldots + 2b_{48}$$

$$= -\frac{2}{3}F^3(y_2) - 2F^2(y_1) + F^2(y_2) + \frac{2}{3}F^3(y_1) + 2F(y_2)$$

Thus integrating we have

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_1}\int_{-\infty}^{y_2} P^2[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)]\,dF(x_1)\,dF(y_2)\,dF(y_1)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{y_1}\int_{-\infty}^{y_2}\left(-\frac{2}{3}F^3(y_2) - 2F^2(y_1) + F^2(y_2) + \frac{2}{3}F^3(y_1) + 2F(y_2)\right)^2 dF(x_1)\,dF(y_2)\,dF(y_1)$$

$$= \frac{277}{2520}.$$

Now consider the third term of (2.3.2.2f)

$$\iiint_{y_1 < x_1 < y_2} P^2[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)]\,dF(x_1)\,dF(y_1)\,dF(y_2).$$

We have

$$P[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)] = 2c_1 + 2c_2 + \ldots + 2c_{30}$$

where

$$c_1 = P[y_1 < x_1 < X_2 < y_2 < X_3 < Y_3]$$

$$= P[x_1 < X_2 < y_2]P[y_2 < X_3 < Y_3]$$

$$= [F(y_2) - F(x_1)]\left[\frac{\overline{F}^2(y_2)}{2}\right]$$

$$c_2 = P[y_1 < x_1 < X_2 < y_2 < Y_3 < X_3]$$

$$= P[x_1 < X_2 < y_2]P[y_2 < Y_3 < X_3]$$

$$= [F(y_2) - F(x_1)]\left[\frac{\overline{F}^2(y_2)}{2}\right]$$

$$c_3 = P[y_1 < x_1 < X_2 < X_3 < y_2 < Y_3]$$

$$= P[x_1 < X_2 < X_3 < y_2]P[y_2 < Y_3]$$

$$= \frac{[F(y_2) - F(x_1)]^2}{2}\overline{F}(y_2)$$

$$c_4 = P[y_1 < x_1 < X_2 < Y_3 < y_2 < X_3]$$

$$= P[x_1 < X_2 < Y_3 < y_2]P[y_2 < X_3]$$

$$= \frac{[F(y_2) - F(x_1)]^2}{2}\overline{F}(y_2)$$

$$c_5 = P[y_1 < x_1 < X_2 < X_3 < Y_3 < y_2]$$

$$= P[x_1 < X_2 < X_3 < Y_3 < y_2]$$

$$= \frac{[F(y_2) - F(x_1)]^3}{6}$$

$$c_6 = P[y_1 < x_1 < X_2 < Y_3 < X_3 < y_2]$$

$$= P[x_1 < X_2 < Y_3 < X_3 < y_2]$$

$$= \frac{[F(y_2) - F(x_1)]^3}{6}$$

$$c_7 = P[y_1 < X_2 < x_1 < y_2 < X_3 < Y_3]$$

$$= P[y_1 < X_2 < x_1]P[y_2 < X_3 < Y_3]$$

$$= [F(x_1) - F(y_1)]\frac{\overline{F}^2(y_2)}{2}$$

$$c_8 = P[y_1 < X_2 < x_1 < y_2 < Y_3 < X_3]$$

$$= P[y_1 < X_2 < x_1]P[y_2 < Y_3 < X_3]$$

$$= [F(x_1) - F(y_1)]\frac{\overline{F}^2(y_2)}{2}$$

$$c_9 = P[y_1 < X_2 < x_1 < X_3 < y_2 < Y_3]$$

$$= P[y_1 < X_2 < x_1]P[x_1 < X_3 < y_2]P[y_2 < Y_3]$$

$$= [F(x_1) - F(y_1)][F(y_2) - F(x_1)]\overline{F}(y_2)$$

$$c_{10} = P[y_1 < X_2 < x_1 < Y_3 < y_2 < X_3]$$

$$= P[y_1 < \quad_2 < x_1]P[x_1 < Y_3 < y_2]P[y_2 < X_3]$$

$$= [F(x_1) - F(y_1)][F(y_2) - F(x_1)]\overline{F}(y_2)$$

$$c_{11} = P[y_1 < X_2 < x_1 < X_3 < Y_3 < y_2]$$

$$= P[y_1 < X_2 < x_1]P[x_1 < X_3 < Y_3 < y_2]$$

$$= [F(x_1) - F(y_1)]\frac{[F(y_2) - F(x_1)]^2}{2}$$

$$c_{12} = P[y_1 < X_2 < x_1 < Y_3 < X_3 < y_2]$$

$$= P[y_1 < X_2 < x_1]P[x_1 < Y_3 < X_3 < y_2]$$

$$= [F(x_1) - F(y_1)]\frac{[F(y_2) - F(x_1)]^2}{2}$$

$$c_{13} = P[y_1 < X_2 < X_3 < x_1 < y_2 < Y_3]$$

$$= P[y_1 < X_2 < X_3 < x_1]P[y_2 < Y_3]$$

$$= \frac{[F(x_1) - F(y_1)]^2}{2}\overline{F}(y_2)$$

$$c_{14} = P[y_1 < X_2 < X_3 < x_1 < Y_3 < y_2]$$

$$= P[y_1 < X_2 < X_3 < x_1]P[x_1 < Y_3 < y_2]$$

$$= \frac{[F(x_1) - F(y_1)]^2}{2}[F(y_2) - F(x_1)]$$

$$c_{15} = P[y_1 < X_2 < X_3 < Y_3 < x_1 < y_2]$$

$$= P[y_1 < X_2 < X_3 < Y_3 < x_1]$$

$$= \frac{[F(x_1) - F(y_1)]^3}{6}$$

$$c_{16} = P[X_2 < y_1 < x_1 < y_2 < X_3 < Y_3]$$

$$= P[X_2 < y_1]P[y_2 < X_3 < Y_3]$$

$$= [F(y_1)]\frac{\overline{F}^2(y_2)}{2}$$

$$c_{17} = P[X_2 < y_1 < x_1 < y_2 < Y_3 < X_3]$$

$$= P[X_2 < y_1]P[y_2 < Y_3 < X_3]$$

$$= [F(y_1)]\frac{\overline{F}^2(y_2)}{2}$$

$$c_{18} = P[X_2 < y_1 < x_1 < X_3 < y_2 < Y_3]$$

$$= P[X_2 < y_1]P[x_1 < X_3 < y_2]P[y_2 < Y_3]$$

$$= [F(y_1)][F(y_2) - F(x_1)]\overline{F}(y_2)$$

$$c_{19} = P[X_2 < y_1 < x_1 < Y_3 < y_2 < X_3]$$

$$= P[X_2 < y_1]P[x_1 < Y_3 < y_2]P[y_2 < X_3]$$

$$= [F(y_1)][F(y_2) - F(x_1)]\overline{F}(y_2)$$

$$c_{20} = P[X_2 < y_1 < x_1 < X_3 < Y_3 < y_2]$$

$$= P[X_2 < y_1]P[x_1 < X_3 < Y_3 < y_2]$$

$$= [F(y_1)]\frac{[F(y_2) - F(x_1)]^2}{2}$$

$$c_{21} = P[X_2 < y_1 < x_1 < Y_3 < X_3 < y_2]$$

$$= P[X_2 < y_1]P[x_1 < Y_3 < X_3 < y_2]$$

$$= [F(y_1)]\frac{[F(y_2)-F(x_1)]^2}{2}$$

$$c_{22} = P[X_2 < y_1 < X_3 < x_1 < y_2 < Y_3]$$

$$= P[X_2 < y_1]P[y_1 < X_3 < x_1]P[y_2 < Y_3]$$

$$= [F(y_1)][F(x_1)-F(y_1)]\overline{F}(y_2)$$

$$c_{23} = P[X_2 < y_1 < X_3 < x_1 < Y_3 < y_2]$$

$$= P[X_2 < y_1]P[y_1 < X_3 < x_1]P[x_1 < Y_3 < y_2]$$

$$= [F(y_1)][F(x_1)-F(y_1)][F(y_2)-F(x_1)]$$

$$c_{24} = P[X_2 < y_1 < X_3 < Y_3 < x_1 < y_2]$$

$$= P[X_2 < y_1]P[y_1 < X_3 < Y_3 < x_1]$$

$$= [F(y_1)]\frac{[F(x_1)-F(y_1)]^2}{2}$$

$$c_{25} = P[X_2 < X_3 < y_1 < x_1 < y_2 < Y_3]$$

$$= P[X_2 < X_3 < y_1]P[y_2 < Y_3]$$

$$= \frac{F^2(y_1)}{2}\overline{F}(y_2)$$

$$c_{26} = P[X_2 < X_3 < y_1 < x_1 < Y_3 < y_2]$$

$$= P[X_2 < X_3 < y_1]P[x_1 < Y_3 < y_2]$$

$$= \frac{F^2(y_1)}{2}[F(y_2)-F(x_1)]$$

$$c_{27} = P[X_2 < X_3 < y_1 < Y_3 < x_1 < y_2]$$

$$= P[X_2 < X_3 < y_1]P[y_1 < Y_3 < x_1]$$

$$= \frac{F^2(y_1)}{2}[F(x_1) - F(y_1)]$$

$$c_{28} = P[X_2 < X_3 < Y_3 < y_1 < x_1 < y_2]$$

$$= P[X_2 < X_3 < Y_3 < y_1]$$

$$= \frac{F^3(y_1)}{6}$$

$$c_{29} = P[X_2 < Y_3 < X_3 < y_1 < x_1 < y_2]$$

$$= P[X_2 < Y_3 < X_3 < y_1]$$

$$= \frac{F^3(y_1)}{6}$$

$$c_{30} = P[Y_3 < X_2 < X_3 < y_1 < x_1 < y_2]$$

$$= P[Y_3 < X_2 < X_3 < y_1]$$

$$= \frac{F^3(y_1)}{6}.$$

Now adding all the 30 terms we have

$$P[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)] = 2c_1 + 2c_2 + \ldots + 2c_{30}$$

$$= \frac{2}{3}F^3(y_2) - 2F^2(y_2) + \frac{2}{3}F^3(x_1) - F^2(x_1) + \frac{2}{3}F^3(y_1) + 2F(y_2).$$

Thus integrating we have

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_2}\int_{-\infty}^{x_1} P^2[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)]\,dF(y_1)\,dF(x_1)\,dF(y_2)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{y_2}\int_{-\infty}^{x_1}\left(\frac{2}{3}F^3(y_2) - 2F^2(y_2) + \frac{2}{3}F^3(x_1) - F^2(x_1) + \frac{2}{3}F^3(y_1) + 2F(y_2)\right)^2 dF(y_1)\,dF(x_1)\,dF(y_2)$$

$$= \frac{3}{70}.$$

Now consider the fourth term of (2.3.2.2f)

$$\iiint_{y_2<x_1<y_1} P^2[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(y_1)dF(y_2).$$

We have

$$P[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)] = 2d_1 + 2d_2 + ... + 2d_{30}$$

where

$$d_1 = P[y_2 < x_1 < X_2 < y_1 < X_3 < Y_3]$$

$$= P[x_1 < X_2 < y_1]P[y_1 < X_3 < Y_3]$$

$$= [F(y_1) - F(x_1)]\left[\frac{\overline{F}^2(y_1)}{2}\right]$$

$$d_2 = P[y_2 < x_1 < X_2 < y_1 < Y_3 < X_3]$$

$$= P[x_1 < X_2 < y_1]P[y_1 < Y_3 < X_3]$$

$$= [F(y_1) - F(x_1)]\left[\frac{\overline{F}^2(y_1)}{2}\right]$$

$$d_3 = P[y_2 < x_1 < X_2 < X_3 < y_1 < Y_3]$$

$$= P[x_1 < X_2 < X_3 < y_1]P[y_1 < Y_3]$$

$$= \frac{[F(y_1) - F(x_1)]^2}{2}\overline{F}(y_1)$$

$$d_4 = P[y_2 < x_1 < X_2 < Y_3 < y_1 < X_3]$$

$$= P[x_1 < X_2 < Y_3 < y_1]P[y_1 < X_3]$$

$$= \frac{[F(y_1) - F(x_1)]^2}{2}\overline{F}(y_1)$$

$$d_5 = P[y_2 < x_1 < X_2 < X_3 < Y_3 < y_1]$$

$$= P[x_1 < X_2 < X_3 < Y_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_1)]^3}{6}$$

$$d_6 = P[y_2 < x_1 < X_2 < Y_3 < X_3 < y_1]$$

$$= P[x_1 < X_2 < Y_3 < X_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_1)]^3}{6}$$

$$d_7 = P[y_2 < X_2 < x_1 < y_1 < X_3 < Y_3]$$

$$= P[y_2 < X_2 < x_1]P[y_1 < X_3 < Y_3]$$

$$= [F(x_1) - F(y_2)]\frac{\overline{F}^2(y_1)}{2}$$

$$d_8 = P[y_2 < X_2 < x_1 < y_1 < Y_3 < X_3]$$

$$= P[y_2 < X_2 < x_1]P[y_1 < Y_3 < X_3]$$

$$= [F(x_1) - F(y_2)]\frac{\overline{F}^2(y_1)}{2}$$

$$d_9 = P[y_2 < X_2 < x_1 < X_3 < y_1 < Y_3]$$

$$= P[y_2 < X_2 < x_1]P[x_1 < X_3 < y_1]P[y_1 < Y_3]$$

$$= [F(x_1) - F(y_2)][F(y_1) - F(x_1)]\overline{F}(y_1)$$

$$d_{10} = P[y_2 < X_2 < x_1 < Y_3 < y_1 < X_3]$$

$$= P[y_2 < X_2 < x_1]P[x_1 < Y_3 < y_1]P[y_1 < X_3]$$

$$= [F(x_1) - F(y_2)][F(y_1) - F(x_1)]\overline{F}(y_1)$$

$$d_{11} = P[y_2 < X_2 < x_1 < X_3 < Y_3 < y_1]$$

$$= P[y_2 < X_2 < x_1]P[x_1 < X_3 < Y_3 < y_1]$$

$$= [F(x_1) - F(y_2)]\frac{[F(y_1) - F(x_1)]^2}{2}$$

$$d_{12} = P[y_2 < X_2 < x_1 < Y_3 < X_3 < y_1]$$

$$= P[y_2 < X_2 < x_1]P[x_1 < Y_3 < X_3 < y_1]$$

$$= [F(x_1) - F(y_2)]\frac{[F(y_1) - F(x_1)]^2}{2}$$

$$d_{13} = P[y_2 < X_2 < X_3 < x_1 < y_1 < Y_3]$$

$$= P[y_2 < X_2 < X_3 < x_1]P[y_1 < Y_3]$$

$$= \frac{[F(x_1) - F(y_2)]^2}{2}\overline{F}(y_1)$$

$$d_{14} = P[y_2 < X_2 < X_3 < x_1 < Y_3 < y_1]$$

$$= P[y_2 < X_2 < X_3 < x_1]P[x_1 < Y_3 < y_1]$$

$$= \frac{[F(x_1) - F(y_2)]^2}{2}[F(y_1) - F(x_1)]$$

$$d_{15} = P[y_2 < X_2 < X_3 < Y_3 < x_1 < y_1]$$

$$= P[y_2 < X_2 < X_3 < Y_3 < x_1]$$

$$= \frac{[F(x_1) - F(y_2)]^3}{6}$$

$$d_{16} = P[X_2 < y_2 < x_1 < y_1 < X_3 < Y_3]$$

$$= P[X_2 < y_2]P[y_1 < X_3 < Y_3]$$

$$= [F(y_2)]\frac{\overline{F}^2(y_1)}{2}$$

$$d_{17} = P[X_2 < y_2 < x_1 < y_1 < Y_3 < X_3]$$

$$= P[X_2 < y_2]P[y_1 < Y_3 < X_3]$$

$$= [F(y_2)]\frac{\overline{F}^2(y_1)}{2}$$

$$d_{18} = P[X_2 < y_2 < x_1 < X_3 < y_1 < Y_3]$$

$$= P[X_2 < y_2]P[x_1 < X_3 < y_1]P[y_1 < Y_3]$$

$$= [F(y_2)][F(y_1) - F(x_1)]\overline{F}(y_1)$$

$$d_{19} = P[X_2 < y_2 < x_1 < Y_3 < y_1 < X_3]$$

$$= P[X_2 < y_2]P[x_1 < Y_3 < y_1]P[y_1 < X_3]$$

$$= [F(y_2)][F(y_1) - F(x_1)]\overline{F}(y_1)$$

$$d_{20} = P[X_2 < y_2 < x_1 < X_3 < Y_3 < y_1]$$

$$= P[X_2 < y_2]P[x_1 < X_3 < Y_3 < y_1]$$

$$= [F(y_2)]\frac{[F(y_1) - F(x_1)]^2}{2}$$

$$d_{21} = P[X_2 < y_2 < x_1 < Y_3 < X_3 < y_1]$$

$$= P[X_2 < y_2]P[x_1 < Y_3 < X_3 < y_1]$$

$$= [F(y_2)]\frac{[F(y_1) - F(x_1)]^2}{2}$$

$$d_{22} = P[X_2 < y_2 < X_3 < x_1 < y_1 < Y_3]$$

$$= P[X_2 < y_2]P[y_2 < X_3 < x_1]P[y_1 < Y_3]$$

$$= [F(y_2)][F(x_1) - F(y_2)]\overline{F}(y_1)$$

$$d_{23} = P[X_2 < y_2 < X_3 < x_1 < Y_3 < y_1]$$

$$= P[X_2 < y_2]P[y_2 < X_3 < x_1]P[x_1 < Y_3 < y_1]$$

$$= [F(y_2)][F(x_1) - F(y_2)][F(y_1) - F(x_1)]$$

$$d_{24} = P[X_2 < y_2 < X_3 < Y_3 < x_1 < y_1]$$

$$= P[X_2 < y_2]P[y_2 < X_3 < Y_3 < x_1]$$

$$= [F(y_2)]\frac{[F(x_1)-F(y_2)]^2}{2}$$

$$d_{25} = P[X_2 < X_3 < y_2 < x_1 < y_1 < Y_3]$$

$$= P[X_2 < X_3 < y_2]P[y_1 < Y_3]$$

$$= \frac{F^2(y_2)}{2}\overline{F}(y_1)$$

$$d_{26} = P[X_2 < X_3 < y_2 < x_1 < Y_3 < y_1]$$

$$= P[X_2 < X_3 < y_2]P[x_1 < Y_3 < y_1]$$

$$= \frac{F^2(y_2)}{2}[F(y_1)-F(x_1)]$$

$$d_{27} = P[X_2 < X_3 < y_2 < Y_3 < x_1 < y_1]$$

$$= P[X_2 < X_3 < y_2]P[y_2 < Y_3 < x_1]$$

$$= \frac{F^2(y_2)}{2}[F(x_1)-F(y_2)]$$

$$d_{28} = P[X_2 < X_3 < Y_3 < y_2 < x_1 < y_1]$$

$$= P[X_2 < X_3 < Y_3 < y_2]$$

$$= \frac{F^3(y_2)}{6}$$

$$d_{29} = P[X_2 < Y_3 < X_3 < y_2 < x_1 < y_1]$$

$$= P[X_2 < Y_3 < X_3 < y_2]$$

$$= \frac{F^3(y_2)}{6}$$

$$d_{30} = P[Y_3 < X_2 < X_3 < y_2 < x_1 < y_1]$$

$$= P[Y_3 < X_2 < X_3 < y_2]$$

$$= \frac{F^3(y_2)}{6}.$$

Now adding all the 30 terms we have

$$P[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)] = 2d_1 + 2d_2 + \ldots + 2d_{30}$$

$$= \frac{2}{3}F^3(y_1) - 2F^2(y_1) + \frac{2}{3}F^3(x_1) - F^2(x_1) + \frac{2}{3}F^3(y_2) + 2F(y_1).$$

Thus integrating we have

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_1}\int_{-\infty}^{x_1} P^2[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)]dF(y_2)dF(x_1)dF(y_1)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{y_1}\int_{-\infty}^{x_1}\left(\frac{2}{3}F^3(y_1) - 2F^2(y_1) + \frac{2}{3}F^3(x_1) - F^2(x_1) + \frac{2}{3}F^3(y_2) + 2F(y_1)\right)^2 dF(y_2)dF(x_1)dF(y_1)$$

$$= \frac{3}{70}.$$

Now consider the fifth term of (2.3.2.2f)

$$\iiint_{y_1 < y_2 < x_1} P^2[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(y_1)dF(y_2).$$

We have

$$P[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)] = 2e_1 + 2e_2 + \ldots + 2e_{12}$$

where

$$e_1 = P[y_1 < X_2 < X_3 < y_2 < x_1 < Y_3]$$

$$= P[y_1 < X_2 < X_3 < y_2]P[x_1 < Y_3]$$

$$= \left[\frac{[F(y_2) - F(y_1)]^2}{2}\right][\bar{F}(x_1)]$$

$$e_2 = P[y_1 < X_2 < X_3 < y_2 < Y_3 < x_1]$$

$$= P[y_1 < X_2 < X_3 < y_2]P[y_2 < Y_3 < x_1]$$

$$= \left[\frac{[F(y_2)-F(y_1)]^2}{2}\right][F(x_1)-F(y_2)]$$

$$e_3 = P[y_1 < X_2 < X_3 < Y_3 < y_2 < x_1]$$

$$= P[y_1 < X_2 < X_3 < Y_3 < y_2]$$

$$= \left[\frac{[F(y_2)-F(y_1)]^3}{6}\right]$$

$$e_4 = P[X_2 < y_1 < X_3 < y_2 < x_1 < Y_3]$$

$$= P[X_2 < y_1]P[y_1 < X_3 < y_2]P[x_1 < Y_3]$$

$$= [F(y_1)][F(y_2)-F(y_1)]\overline{F}(x_1)$$

$$e_5 = P[X_2 < y_1 < X_3 < y_2 < Y_3 < x_1]$$

$$= P[X_2 < y_1]P[y_1 < X_3 < y_2]P[y_2 < Y_3 < x_1]$$

$$= [F(y_1)][F(y_2)-F(y_1)][F(x_1)-F(y_2)]$$

$$e_6 = P[X_2 < y_1 < X_3 < Y_3 < y_2 < x_1]$$

$$= P[X_2 < y_1]P[y_1 < X_3 < Y_3 < y_2]$$

$$= F(y_1)\left[\frac{[F(y_2)-F(y_1)]^2}{2}\right]$$

$$e_7 = P[X_2 < X_3 < y_1 < y_2 < x_1 < Y_3]$$

$$= P[X_2 < X_3 < y_1]P[x_1 < Y_3]$$

$$= \left[\frac{[F^2(y_1)]}{2}\right]\overline{F}(x_1)$$

$$e_8 = P[X_2 < X_3 < y_1 < y_2 < Y_3 < x_1]$$

$$= P[X_2 < X_3 < y_1]P[y_2 < Y_3 < x_1]$$

$$= \left[\frac{[F^2(y_1)]}{2}\right][F(x_1) - F(y_2)]$$

$$e_9 = P[X_2 < X_3 < y_1 < Y_3 < y_2 < x_1]$$

$$= P[X_2 < X_3 < y_1]P[y_1 < Y_3 < y_2]$$

$$= \left[\frac{[F^2(y_1)]}{2}\right][F(y_2) - F(y_1)]$$

$$e_{10} = P[X_2 < X_3 < Y_3 < y_1 < y_2 < x_1]$$

$$= P[X_2 < X_3 < Y_3 < y_1]$$

$$= \left[\frac{[F^3(y_1)]}{6}\right]$$

$$e_{11} = P[X_2 < Y_3 < X_3 < y_1 < y_2 < x_1]$$

$$= P[X_2 < Y_3 < X_3 < y_1]$$

$$= \left[\frac{[F^3(y_1)]}{6}\right]$$

$$e_{12} = P[Y_3 < X_2 < X_3 < y_1 < y_2 < x_1]$$

$$= P[Y_3 < X_2 < X_3 < y_1]$$

$$= \left[\frac{[F^3(y_1)]}{6}\right].$$

Now adding all the 12 terms we have

$$P[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)] = 2e_1 + 2e_2 + \ldots + 2e_{12}$$

$$= -\frac{2}{3}F^3(y_2) + F^2(y_2) + \frac{2}{3}F^3(y_1).$$

Thus integrating we have

$$\int_{-\infty}^{\infty}\int_{-\infty}^{x_1}\int_{-\infty}^{y_2} P^2[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)]dF(y_1)dF(y_2)dF(x_1)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{x_1}\int_{-\infty}^{y_2}\left(-\frac{2}{3}F^3(y_2)+F^2(y_2)+\frac{2}{3}F^3(y_1)\right)^2 dF(y_1)dF(y_2)dF(x_1)$$

$$= \frac{5}{504}.$$

Now consider the sixth term of (2.3.2.2f)

$$\iiint_{y_2<y_1<x_1} P^2[Med(x_1,X_2,X_3)< Med(y_1,y_2,Y_3)]dF(x_1)dF(y_1)dF(y_2).$$

We have

$$P[Med(x_1,X_2,X_3)< Med(y_1,y_2,Y_3)]= 2f_1+2f_2+\ldots+2f_{12}$$

where

$$f_1 = P[y_2 < X_2 < X_3 < y_1 < x_1 < Y_3]$$

$$= P[y_2 < X_2 < X_3 < y_1]P[x_1 < Y_3]$$

$$= \left[\frac{[F(y_1)-F(y_2)]^2}{2}\right][\bar{F}(x_1)]$$

$$f_2 = P[y_2 < X_2 < X_3 < y_1 < Y_3 < x_1]$$

$$= P[y_2 < X_2 < X_3 < y_1]P[y_1 < Y_3 < x_1]$$

$$= \left[\frac{[F(y_1)-F(y_2)]^2}{2}\right][F(x_1)-F(y_1)]$$

$$f_3 = P[y_2 < X_2 < X_3 < Y_3 < y_1 < x_1]$$

$$= P[y_2 < X_2 < X_3 < Y_3 < y_1]$$

$$= \left[\frac{[F(y_1)-F(y_2)]^3}{6}\right]$$

$$f_4 = P[X_2 < y_2 < X_3 < y_1 < x_1 < Y_3]$$

$$= P[X_2 < y_2]P[y_2 < X_3 < y_1]P[x_1 < Y_3]$$

$$= [F(y_2)][F(y_1) - F(y_2)]\overline{F}(x_1)$$

$$f_5 = P[X_2 < y_2 < X_3 < y_1 < Y_3 < x_1]$$

$$= P[X_2 < y_2]P[y_2 < X_3 < y_1]P[y_1 < Y_3 < x_1]$$

$$= [F(y_2)][F(y_1) - F(y_2)][F(x_1) - F(y_1)]$$

$$f_6 = P[X_2 < y_2 < X_3 < Y_3 < y_1 < x_1]$$

$$= P[X_2 < y_2]P[y_2 < X_3 < Y_3 < y_1]$$

$$= F(y_2)\left[\frac{[F(y_1) - F(y_2)]^2}{2}\right]$$

$$f_7 = P[X_2 < X_3 < y_2 < y_1 < x_1 < Y_3]$$

$$= P[X_2 < X_3 < y_2]P[x_1 < Y_3]$$

$$= \left[\frac{[F^2(y_2)]}{2}\right]\overline{F}(x_1)$$

$$f_8 = P[X_2 < X_3 < y_2 < y_1 < Y_3 < x_1]$$

$$= P[X_2 < X_3 < y_2]P[y_1 < Y_3 < x_1]$$

$$= \left[\frac{[F^2(y_2)]}{2}\right][F(x_1) - F(y_1)]$$

$$f_9 = P[X_2 < X_3 < y_2 < Y_3 < y_1 < x_1]$$

$$= P[X_2 < X_3 < y_2]P[y_2 < Y_3 < y_1]$$

$$= \left[\frac{[F^2(y_2)]}{2}\right][F(y_1) - F(y_2)]$$

$$f_{10} = P[X_2 < X_3 < Y_3 < y_2 < y_1 < x_1]$$

$$= P[X_2 < X_3 < Y_3 < y_2]$$

$$= \left[\frac{[F^3(y_2)]}{6}\right]$$

$$f_{11} = P[X_2 < Y_3 < X_3 < y_2 < y_1 < x_1]$$

$$= P[X_2 < Y_3 < X_3 < y_2]$$

$$= \left[\frac{[F^3(y_2)]}{6}\right]$$

$$f_{12} = P[Y_3 < X_2 < X_3 < y_2 < y_1 < x_1]$$

$$= P[Y_3 < X_2 < X_3 < y_2]$$

$$= \left[\frac{[F^3(y_2)]}{6}\right].$$

Now adding all the 12 terms we have

$$P[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)] = 2f_1 + 2f_2 + \ldots + 2f_{12}$$

$$= -\frac{2}{3}F^3(y_1) + F^2(y_1) + \frac{2}{3}F^3(y_2).$$

Thus integrating we have

$$\int_{-\infty}^{\infty}\int_{-\infty}^{x_1}\int_{-\infty}^{y_1} P^2[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)]\,dF(y_2)\,dF(y_1)\,dF(x_1)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{x_1}\int_{-\infty}^{y_1}\left(-\frac{2}{3}F^3(y_1) + F^2(y_1) + \frac{2}{3}F^3(y_2)\right)^2 dF(y_2)\,dF(y_1)\,dF(x_1)$$

$$= \frac{5}{504}.$$

Thus finally adding the six terms we have (2.3.2.2f).

$$P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_4, X_5) < Med(Y_1, Y_2, Y_4)]$$

$$= \frac{41}{126}.$$

Thus finally we have

$$\xi_{1,2} = P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_4, X_5) < Med(Y_1, Y_2, Y_4)] - \frac{1}{4}$$

$$= \frac{41}{126} - \frac{1}{4}$$

$$= \frac{19}{252}.$$

Now consider

$$\xi_{2,1} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_2, X_4; Y_1, Y_4, Y_5)]$$

$$= E[h(X_1, X_2, X_3; Y_1, Y_2, Y_3)h(X_1, X_2, X_4; Y_1, Y_4, Y_5)] - \{E[h(X_1, X_2, X_3; Y_1, Y_2, Y_3)]\}^2$$

$$= P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_2, X_4) < Med(Y_1, Y_4, Y_5)] - \frac{1}{4}.$$

Now following the same steps as in the computation of $\xi_{1,2}$, it follows from symmetry that

$$\xi_{2,1} = \xi_{1,2} = \frac{19}{252}.$$

Now consider

$$\xi_{1,3} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_4, X_5; Y_1, Y_2, Y_3)]$$

$$= E[h(X_1, X_2, X_3; Y_1, Y_2, Y_3)h(X_1, X_4, X_5; Y_1, Y_2, Y_3)] - \{E[h(X_1, X_2, X_3; Y_1, Y_2, Y_3)]\}^2$$

$$= P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_4, X_5) < Med(Y_1, Y_2, Y_3)] - \frac{1}{4}$$

Now

$$P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_4, X_5) < Med(Y_1, Y_2, Y_3)]$$

$$= \iint_{-\infty < x < y < \infty} P^2[Med(x, X_2, X_3) < y] F(y)\overline{F}(y)dF(y)dF(x)$$

$$= \iint_{-\infty<x<y<\infty} P^2[Med(x, X_2, X_3) < y] f F(y) \bar{F}(y) dF(y) dF(x)$$
$$+ \iint_{-\infty<y<x<\infty} P^2[Med(x, X_2, X_3) < y] f F(y) \bar{F}(y) dF(y) dF(x) \qquad (2.3.2.2g)$$

Now consider the first term of (2.3.2.2g)

$$\iint_{-\infty<x<y<\infty} P^2[Med(x, X_2, X_3) < y] f F(y) \bar{F}(y) dF(y) dF(x).$$

We have

$$P[Med(x, X_2, X_3) < y] = 2g_1 + 2g_2 + \ldots + 2g_5$$

where,

$$g_1 = P[X_2 < x < X_3 < y]$$
$$= P[X_2 < x] P[x < X_3 < y]$$
$$= [F(x)][F(y) - F(x)]$$

$$g_2 = P[X_2 < x < y < X_3]$$
$$= P[X_2 < x] P[y < X_3]$$
$$= [F(x)][\bar{F}(y)]$$

$$g_3 = P[x < X_2 < X_3 < y]$$
$$= \frac{[F(y) - F(x)]^2}{2}$$

$$g_4 = P[x < X_2 < y < X_3]$$
$$= P[x < X_2 < y] P[y < X_3]$$
$$= [F(y) - F(x)][\bar{F}(y)]$$

$$g_5 = P[X_3 < X_2 < x < y]$$
$$= P[X_3 < X_2 < x]]$$

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$$= \frac{F^2(x)}{2}.$$

Now adding all the 5 terms we have

$$P[Med(x, X_2, X_3) < y] = 2g_1 + 2g_2 + ... + 2g_5$$

$$= -F^2(y) + 2F(y).$$

Thus integrating we have

$$\int_{-\infty}^{\infty}\int_{x}^{\infty} P^2[Med(x, X_2, X_3) < y]\,6F(y)\bar{F}(y)\,dF(y)\,dF(x)$$

$$= \int_{-\infty}^{\infty}\int_{x}^{\infty} \left(-F^2(y) + 2F(y)\right)^2 6F(y)\bar{F}(y)\,dF(y)\,dF(x)$$

$$= \frac{47}{140}.$$

Now consider the second term of (2.3.2.2g)

$$\iint_{-\infty<y<x<\infty} P^2[Med(x, X_2, X_3) < y]\,6F(y)\bar{F}(y)\,dF(y)\,dF(x).$$

We have

$$P[Med(x, X_2, X_3) < y]$$

$$= 2P[X_3 < X_2 < y < x]$$

$$= 2P[X_3 < X_2 < y]I_{(y<x)}$$

$$= 2\frac{F^2(y)}{2}.$$

Thus integrating we have

$$\int_{-\infty}^{\infty}\int_{-\infty}^{x} P^2[Med(x, X_2, X_3) < y]\,6F(y)\bar{F}(y)\,dF(y)\,dF(x)$$

$$= \frac{1}{28}.$$

Thus finally adding the two terms we have (2.3.2.2g)

$$P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_4,X_5)<Med(Y_1,Y_2,Y_3)]$$

$$=\frac{13}{35}.$$

Thus, finally we have

$$\xi_{1,3}=P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_4,X_5)<Med(Y_1,Y_2,Y_3)]-\frac{1}{4}$$

$$=\frac{13}{35}-\frac{1}{4}$$

$$=\frac{17}{140}.$$

Similarly,by symmetry it follows that

$$\xi_{3,1}=\xi_{1,3}=\frac{17}{140}$$

Now consider

$$\xi_{2,3}=Cov[h(X_1,X_2,X_3;Y_1,Y_2,Y_3),h(X_1,X_2,X_4;Y_1,Y_2,Y_3)]$$

$$=E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)h(X_1,X_2,X_4;Y_1,Y_2,Y_3)]-\{E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)]\}^2$$

$$=P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_2,X_4)<Med(Y_1,Y_2,Y_3)]-\frac{1}{4}$$

Now

$$P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_2,X_4)<Med(Y_1,Y_2,Y_3)]$$

$$=\iiint P^2[Med(x_1,x_2,X_3)<y]6F(y)\overline{F}(y)dF(y)dF(x_1)dF(x_2)$$

$$=\iiint_{-\infty<x_1<x_2<\infty} P^2[Med(x_1,x_2,X_3)<y]6F(y)\overline{F}(y)dF(y)dF(x_1)dF(x_2)$$
$$+\iiint_{-\infty<x_2<x_1<\infty} P^2[Med(x_1,x_2,X_3)<y]6F(y)\overline{F}(y)dF(y)dF(x_2)dF(x_1) \qquad (2.3.2.2h)$$

Now consider the first term of (2.3.2.2h)

$$\iiint_{-\infty<x_1<x_2<\infty} P^2[Med(x_1,x_2,X_3)<y]\,6F(y)\overline{F}(y)\,dF(y)\,dF(x_1)\,dF(x_2).$$

Evaluating $P[Med(x_1,x_2,X_3)<y]$ and substituting in the above we have

$$\int_{-\infty}^{\infty}\int_{x_1}^{\infty}\int_{x_2}^{\infty} P^2[Med(x_1,x_2,X_3)<y]\,6F(y)\overline{F}(y)\,dF(y)\,dF(x_1)\,dF(x_2)$$

$$=\int_{-\infty}^{\infty}\int_{x_1}^{\infty}\int_{x_2}^{\infty}(1+F(y))^2\,6F(y)\overline{F}(y)\,dF(y)\,dF(x_1)\,dF(x_2)$$

$$=\frac{59}{140}.$$

Similarly we have

$$\iiint_{-\infty<x_2<x_1<\infty} P^2[Med(x_1,x_2,X_3)<y]\,6F(y)\overline{F}(y)\,dF(y)\,dF(x_1)\,dF(x_2)$$

$$=\frac{59}{140}.$$

Thus finally adding the two terms we have (2.3.2.2h)

$$P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_2,X_4)<Med(Y_1,Y_2,Y_3)]$$

$$=\frac{59}{70}.$$

Thus finally we have

$$\xi_{2,3}=P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_2,X_4)<Med(Y_1,Y_2,Y_3)]-\frac{1}{4}$$

$$=\frac{59}{70}-\frac{1}{4}$$

$$=\frac{83}{140}.$$

Now consider

$$\xi_{3,2} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_2, X_3; Y_1, Y_2, Y_4)]$$

$$= E[h(X_1, X_2, X_3; Y_1, Y_2, Y_3)h(X_1, X_2, X_3; Y_1, Y_2, Y_4)] - \{E[h(X_1, X_2, X_3; Y_1, Y_2, Y_3)]\}^2$$

$$= P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_4)] - \frac{1}{4}$$

Again by symmetry,we have

$$\xi_{3,2} = \xi_{2,3} = \frac{83}{140}.$$

$$\xi_{1,1} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_4, X_5; Y_1, Y_4, Y_5)]$$

$$= E\left[h(X_1, X_2, X_3; Y_1, Y_2, Y_3)h(X_1, X_4, X_5; Y_1, Y_4, Y_5) - \{E[h(X_1, X_2, X_3; Y_1, Y_2, Y_3)]\}^2\right]$$

$$= P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_4, X_5) < Med(Y_1, Y_4, Y_5)] - \frac{1}{4}$$

Now,

$$P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_4, X_5) < Med(Y_1, Y_4, Y_5)]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P^2[Med(x, X_2, X_3) < Med(y, Y_2, Y_3)]dF(x)dF(y)$$

$$= h_1 + h_2$$

where

$$h_1 = \int_{-\infty}^{\infty}\int_{-\infty}^{y} P^2[Med(x, X_2, X_3) < Med(y, Y_2, Y_3)]dF(x)dF(y)$$

$$-\infty < x < y < \infty$$

$$h_2 = \int_{-\infty}^{\infty}\int_{y}^{\infty} P^2[Med(x, X_2, X_3) < Med(y, Y_2, Y_3)]dF(x)dF(y)$$

$$-\infty < y < x < \infty$$

For the case x<y,

$$P[Med(x, X_2, X_3) < Med(y, Y_2, Y_3)] = 4i_1 + 4i_2 + \ldots\ldots\ldots + 4i_{62}$$

where

$i_1 = P[x < y < X_2 < X_3 < Y_2 < Y_3]$ $i_2 = P[x < y < X_2 < Y_2 < X_3 < Y_3]$

$i_3 = P[x < y < X_2 < Y_2 < Y_3 < Y_3]$ $i_4 = P[x < X_2 < y < X_3 < Y_2 < Y_3]$

$i_5 = P[x < X_2 < y < Y_2 < X_3 < Y_3]$ $i_6 = P[x < X_2 < y < Y_2 < Y_3 < X_3]$

$i_7 = P[x < X_2 < X_3 < y < Y_2 < Y_3]$ $i_8 = P[x < X_2 < Y_2 < y < X_3 < Y_3]$

$i_9 = P[x < X_2 < Y_2 < y < Y_3 < X_3]$ $i_{10} = P[x < Y_2 < X_2 < y < X_3 < Y_3]$

$i_{11} = P[x < Y_2 < X_2 < y < Y_3 < X_3]$ $i_{12} = P[x < X_2 < X_3 < Y_2 < y < Y_3]$

$i_{13} = P[x < X_2 < Y_2 < X_3 < y < Y_3]$ $i_{14} = P[x < X_2 < Y_2 < Y_3 < y < X_3]$

$t_{15} = P[x < Y_2 < X_2 < X_3 < y < Y_3]$ $i_{16} = P[x < Y_2 < X_2 < Y_3 < y < X_3]$

$i_{17} = P[x < X_2 < X_3 < Y_2 < Y_3 < y]$ $i_{18} = P[x < X_2 < Y_2 < X_3 < Y_3 < y]$

$i_{19} = P[x < X_2 < Y_2 < Y_3 < X_3 < y]$ $i_{20} = P[x < Y_2 < X_2 < X_3 < Y_3 < y]$

$i_{21} = P[x < Y_2 < X_2 < Y_3 < X_3 < y]$ $i_{22} = P[X_2 < x < y < X_3 < Y_2 < Y_3]$

$i_{23} = P[X_2 < x < y < Y_2 < X_3 < Y_3]$ $i_{24} = P[X_2 < x < y < Y_2 < Y_3 < X_3]$

$i_{25} = P[X_2 < x < X_3 < y < Y_2 < Y_3]$ $i_{26} = P[X_2 < x < Y_2 < y < X_3 < Y_3]$

$i_{27} = P[X_2 < x < Y_2 < y < Y_3 < X_3]$ $i_{28} = P[Y_2 < x < X_2 < y < X_3 < Y_3]$

$i_{29} = P[Y_2 < x < X_2 < y < Y_3 < X_3]$ $i_{30} = P[X_2 < x < X_3 < Y_2 < y < Y_3]$

$i_{31} = P[X_2 < x < Y_2 < X_3 < y < Y_3]$ $i_{32} = P[X_2 < x < Y_2 < Y_3 < y < X_3]$

$i_{33} = P[Y_2 < x < X_2 < X_3 < y < Y_3]$ $i_{34} = P[Y_2 < x < X_2 < Y_3 < y < X_3]$

$i_{35} = P[X_2 < x < X_3 < Y_2 < Y_3 < y]$ $i_{36} = P[X_2 < x < Y_2 < X_3 < Y_3 < y]$

$i_{37} = P[X_2 < x < Y_2 < Y_3 < X_3 < y]$ $i_{38} = P[Y_2 < x < X_2 < X_3 < Y_3 < y]$

$i_{39} = P[Y_2 < x < X_2 < Y_3 < X_3 < y]$ $i_{40} = P[X_2 < X_3 < x < y < Y_2 < Y_3]$

$i_{41} = P[X_2 < Y_2 < x < y < X_3 < Y_3]$ $i_{42} = P[X_2 < Y_2 < x < y < Y_3 < X_3]$

$i_{43} = P[Y_2 < X_2 < x < y < X_3 < Y_3]$ $\quad$ $i_{44} = P[Y_2 < X_2 < x < y < Y_3 < X_3]$

$i_{45} = P[X_2 < X_3 < x < Y_2 < y < Y_3]$ $\quad$ $i_{46} = P[X_2 < Y_2 < x < X_3 < y < Y_3]$

$i_{47} = P[X_2 < Y_2 < x < Y_3 < y < X_3]$ $\quad$ $i_{48} = P[Y_2 < X_2 < x < X_3 < y < Y_3]$

$i_{49} = P[Y_2 < X_2 < x < Y_3 < y < X_3]$ $\quad$ $i_{50} = P[X_2 < X_3 < x < Y_2 < Y_3 < y]$

$i_{51} = P[X_2 < Y_2 < x < X_3 < Y_3 < y]$ $\quad$ $i_{52} = P[X_2 < Y_2 < x < Y_3 < X_3 < y]$

$i_{53} = P[Y_2 < X_2 < x < X_3 < Y_3 < y]$ $\quad$ $i_{54} = P[Y_2 < X_2 < x < Y_3 < X_3 < y]$

$i_{55} = P[X_2 < X_3 < Y_2 < x < y < Y_3]$ $\quad$ $i_{56} = P[X_2 < Y_2 < X_3 < x < y < Y_3]$

$i_{57} = P[Y_2 < X_2 < X_3 < x < y < Y_3]$ $\quad$ $i_{58} = P[X_2 < X_3 < Y_2 < x < Y_3 < y]$

$i_{59} = P[X_2 < Y_2 < X_3 < x < Y_3 < y]$ $\quad$ $i_{60} = P[X_2 < X_3 < Y_2 < Y_3 < x < y]$

$i_{61} = P[X_2 < Y_2 < X_3 < Y_3 < x < y]$ $\quad$ $i_{62} = P[Y_2 < X_2 < X_3 < Y_3 < x < y]$.

Now consider

$$i_1 = P[x < y < X_2 < X_3 < Y_2 < Y_3]$$

$$= P[y < X_2 < X_3 < Y_2 < Y_3]$$

$$= \int_y^\infty P[x_2 < X_3 < Y_2 < Y_3]dF(x_2)$$

$$= \int_y^\infty \int_{x_2}^\infty P[x_3 < Y_2 < Y_3]dF(x_3)dF(x_2)$$

$$= \int_y^\infty \int_{x_2}^\infty \int_{x_3}^\infty P[y_2 < Y_3]dF(y_2)dF(x_3)dF(x_2)$$

$$= \int_y^\infty \int_{x_2}^\infty \int_{x_3}^\infty \overline{F}(y_2)dF(y_2)dF(x_3)dF(x_2)$$

$$= \int_y^\infty \int_{x_2}^\infty \frac{\overline{F}^2(x_3)}{2}dF(x_3)dF(x_2)$$

$$= \int_y^\infty \frac{\overline{F}^3(x_2)}{6}dF(x_2)$$

$$= \frac{\overline{F}^4(y)}{24}$$

$$i_2 = P[x < y < X_2 < Y_2 < X_3 < Y_3]$$

$$= \frac{\overline{F}^4(y)}{24}$$

$$i_3 = P[x < y < X_2 < Y_2 < Y_3 < Y_3]$$

$$= \frac{\overline{F}^4(y)}{24}$$

$$i_4 = P[x < X_2 < y < X_3 < Y_2 < Y_3]$$

$$= P[x < X_2 < y]P[y < X_3 < Y_2 < Y_3]$$

$$= [F(y) - F(x)]\int_y^\infty P(x_3 < Y_2 < Y_3)dF(x_3)$$

$$= [F(y) - F(x)]\int_y^\infty \int_{x_3}^\infty P(y_2 < Y_3)dF((y_2)dF(x_3)$$

$$= [F(y) - F(x)]\int_y^\infty \int_{x_3}^\infty \overline{F}(y_2)dF(y_2)dF(x_3)$$

$$= [F(y) - F(x)]\int_y^\infty \frac{\overline{F}^2(x_3)}{2}dF(x_3)$$

$$= [F(y) - F(x)]\int_0^{\overline{F}(y)} \frac{u^2}{2}du$$

$$= [F(y) - F(x)]\frac{\overline{F}^3(y)}{6}$$

$$i_5 = P[x < X_2 < y < Y_2 < X_3 < Y_3]$$

$$= [F(y) - F(x)]\frac{\overline{F}^3(y)}{6}$$

$$i_6 = P[x < X_2 < y < Y_2 < Y_3 < X_3]$$

$$= [F(y) - F(x)]\frac{\overline{F}^3(y)}{6}$$

$$i_7 = P[x < X_2 < X_3 < y < Y_2 < Y_3]$$

$$= P[x < X_2 < X_3 < y]P[y < Y_2 < Y_3]$$

$$= \int_x^y P(x_2 < X_3 < y)dF(x_2)\int_y^\infty P(y_2 < Y_3)dF(y_2)$$

$$= \int_x^y [F(y) - F(x_2)]dF(x_2)\int_y^\infty \overline{F}(y_2)dF(y_2)$$

$$= \int_{F(x)}^{F(y)} [F(y) - u]du \int_0^{\overline{F}(y)} udu \qquad \text{Let } \overline{F}(y_2) = u$$

$$= \left\{ \int_{F(x)}^{F(y)} F(y)du - \int_{F(x)}^{F(y)} udu \right\} \frac{\overline{F}^2(y)}{2}$$

$$= \left\{ F(y)[F(y) - F(x)] - \frac{1}{2}[F^2(y) - F^2(x)] \right\} \frac{\overline{F}^2(y)}{2}$$

$$= \left\{ F^2(y) - F(y)F(x) - \frac{F^2(y)}{2} + \frac{F^2(x)}{2} \right\} \frac{\overline{F}^2(y)}{2}$$

$$= \left\{ \frac{F^2(y)}{2} + \frac{F^2(x)}{2} - F(y)F(x) \right\} \frac{\overline{F}^2(y)}{2}$$

$$= \left\{ \frac{1}{2}[F(y) - F(x)]^2 \right\} \frac{\overline{F}^2(y)}{2}$$

$$i_8 = P[x < X_2 < Y_2 < y < X_3 < Y_3]$$

$$= \left\{ \frac{1}{2}[F(y) - F(x)]^2 \right\} \frac{\overline{F}^2(y)}{2}$$

$$i_9 = P[x < X_2 < Y_2 < y < Y_3 < X_3]$$

$$= \left\{ \frac{1}{2}[F(y) - F(x)]^2 \right\} \frac{\overline{F}^2(y)}{2}$$

$$i_{10} = P[x < Y_2 < X_2 < y < X_3 < Y_3]$$

$$= \left\{ \frac{1}{2}[F(y) - F(x)]^2 \right\} \frac{\overline{F}^2(y)}{2}$$

$$i_{11} = P[x < Y_2 < X_2 < y < Y_3 < X_3]$$

$$= \left\{ \frac{1}{2}[F(y) - F(x)]^2 \right\} \frac{\overline{F}^2(y)}{2}$$

$$i_{12} = P[x < X_2 < X_3 < Y_2 < y < Y_3]$$

$$= P[x < X_2 < X_3 < Y_2 < y] P[y < Y_3]$$

$$= \int_x^y \int_x^{y_2} \int_x^{x_3} dF(x_2) dF(x_3) dF(y_2) (\overline{F}(y))$$

$$= \int_x^y \int_x^{y_2} [F(x_3) - F(x)] dF(x_3) dF(y_2) (\overline{F}(y))$$

$$= \int_x^y \frac{1}{2}[F(y_2) - F(x)]^2 dF(y_2) (\overline{F}(y))$$

$$= \frac{1}{6}[F(y) - F(x)]^3 . (\overline{F}(y))$$

$$i_{13} = P[x < X_2 < Y_2 < X_3 < y < Y_3]$$

$$= \frac{1}{6}[F(y) - F(x)]^3 . (\overline{F}(y))$$

$$i_{14} = P[x < X_2 < Y_2 < Y_3 < y < X_3]$$

$$= \frac{1}{6}[F(y) - F(x)]^3 . (\overline{F}(y))$$

$$i_{15} = P[x < Y_2 < X_2 < X_3 < y < Y_3]$$

$$= \frac{1}{6}[F(y) - F(x)]^3 . (\overline{F}(y))$$

$$i_{16} = P[x < Y_2 < X_2 < Y_3 < y < X_3]$$

$$= \frac{1}{6}[F(y) - F(x)]^3 . (\overline{F}(y))$$

$$i_{17} = P[x < X_2 < X_3 < Y_2 < Y_3 < y]$$

$$= \int_x^y \int_x^{y_3} \int_x^{y_2} \int_x^{x_3} dF(x_2) dF(x_3) dF(y_2) dF(y_3)$$

$$= \int_x^y \int_x^{y_3} \int_x^{y_2} [F(x_3) - F(x)] dF(x_3) dF(y_2) dF(y_3)$$

$$= \int_x^y \int_x^{y_3} \frac{1}{2} [F(y_2) - F(x)]^2 dF(y_2) dF(y_3)$$

$$= \int_x^y \frac{1}{6} [F(y_3) - F(x)]^3 dF(y_3)$$

$$= \frac{1}{24} [F(y) - F(x)]^4$$

$$i_{18} = P[x < X_2 < Y_2 < X_3 < Y_3 < y]$$

$$= \frac{1}{24} [F(y) - F(x)]^4$$

$$i_{19} = P[x < X_2 < Y_2 < Y_3 < X_3 < y]$$

$$= \frac{1}{24} [F(y) - F(x)]^4$$

$$i_{20} = P[x < Y_2 < X_2 < X_3 < Y_3 < y]$$

$$= \frac{1}{24} [F(y) - F(x)]^4$$

$$i_{21} = P[x < Y_2 < X_2 < Y_3 < X_3 < y]$$

$$= \frac{1}{24} [F(y) - F(x)]^4$$

$$i_{22} = P[X_2 < x < y < X_3 < Y_2 < Y_3]$$

$$= P[X_2 < x] P[y < X_3 < Y_2 < Y_3]$$

$$= F(x) \frac{\overline{F}^3(y)}{6}$$

$$i_{23} = P[X_2 < x < y < Y_2 < X_3 < Y_3]$$

$$= F(x) \frac{\overline{F}^3(y)}{6}$$

$$i_{24} = P[X_2 < x < y < Y_2 < Y_3 < X_3]$$

$$= F(x)\frac{\overline{F}^3(y)}{6}$$

$$i_{25} = P[X_2 < x < X_3 < y < Y_2 < Y_3]$$

$$= P[X_2 < x < X_3 < y]P[y < Y_2 < Y_3]$$

$$= P[X_2 < x]P[x < X_3 < y]P[y < Y_2 < Y_3]$$

$$= F(x)[F(y) - F(x)]\frac{\overline{F}^2(y)}{2}$$

$$i_{26} = P[X_2 < x < Y_2 < y < X_3 < Y_3]$$

$$= F(x)[F(y) - F(x)]\frac{\overline{F}^2(y)}{2}$$

$$i_{27} = P[X_2 < x < Y_2 < y < Y_3 < X_3]$$

$$= F(x)[F(y) - F(x)]\frac{\overline{F}^2(y)}{2}$$

$$i_{28} = P[Y_2 < x < X_2 < y < X_3 < Y_3]$$

$$= F(x)[F(y) - F(x)]\frac{\overline{F}^2(y)}{2}$$

$$i_{29} = P[Y_2 < x < X_2 < y < Y_3 < X_3]$$

$$= F(x)[F(y) - F(x)]\frac{\overline{F}^2(y)}{2}$$

$$i_{30} = P[X_2 < x < X_3 < Y_2 < y < Y_3]$$

$$= P[X_2 < x]P[x < X_3 < Y_2 < y]P[y < Y_3]$$

$$= F(x)\frac{[F(y) - F(x)]^2}{2}.\overline{F}(y)$$

$$i_{31} = P[X_2 < x < Y_2 < X_3 < y < Y_3]$$

$$= F(x)\frac{[F(y)-F(x)]^2}{2}.\bar{F}(y)$$

$$i_{32} = P[X_2 < x < Y_2 < Y_3 < y < X_3]$$

$$= F(x)\frac{[F(y)-F(x)]^2}{2}.\bar{F}(y)$$

$$i_{33} = P[Y_2 < x < X_2 < X_3 < y < Y_3]$$

$$= F(x)\frac{[F(y)-F(x)]^2}{2}.\bar{F}(y)$$

$$i_{34} = P[Y_2 < x < X_2 < Y_3 < y < X_3]$$

$$= F(x)\frac{[F(y)-F(x)]^2}{2}.\bar{F}(y)$$

$$i_{35} = P[X_2 < x < X_3 < Y_2 < Y_3 < y]$$

$$= P[X_2 < x]P[x < X_3 < Y_2 < Y_3 < y]$$

$$= F(x)\frac{[F(y)-F(x)]^3}{6}$$

$$i_{36} = P[X_2 < x < Y_2 < X_3 < Y_3 < y]$$

$$= F(x)\frac{[F(y)-F(x)]^3}{6}$$

$$i_{37} = P[X_2 < x < Y_2 < Y_3 < X_3 < y]$$

$$= F(x)\frac{[F(y)-F(x)]^3}{6}$$

$$i_{38} = P[Y_2 < x < X_2 < X_3 < Y_3 < y]$$

$$= F(x)\frac{[F(y)-F(x)]^3}{6}$$

$$i_{39} = P[Y_2 < x < X_2 < Y_3 < X_3 < y]$$

$$= F(x).\frac{[F(y)-F(x)]^3}{6}$$

$$i_{40} = P[X_2 < X_3 < x < y < Y_2 < Y_3]$$

$$= P[X_2 < X_3 < x]P[y < Y_2 < Y_3]$$

$$= \int_{-\infty}^{x} F(x_3)dF(x_3).\frac{\overline{F}^2(y)}{2}$$

$$= \int_0^{F(x)} udu.\frac{\overline{F}^2(y)}{2} \qquad \text{letting } u = F(x_3)$$

$$= \frac{F^2(x)}{2}.\frac{\overline{F}^2(y)}{2}$$

$$i_{41} = P[X_2 < Y_2 < x < y < X_3 < Y_3]$$

$$= \frac{F^2(x)}{2}.\frac{\overline{F}^2(y)}{2}$$

$$i_{42} = P[X_2 < Y_2 < x < y < Y_3 < X_3]$$

$$= \frac{F^2(x)}{2}.\frac{\overline{F}^2(y)}{2}$$

$$i_{43} = P[Y_2 < X_2 < x < y < X_3 < Y_3]$$

$$= \frac{F^2(x)}{2}.\frac{\overline{F}^2(y)}{2}$$

$$i_{44} = P[Y_2 < X_2 < x < y < Y_3 < X_3]$$

$$= \frac{F^2(x)}{2}.\frac{\overline{F}^2(y)}{2}$$

$$i_{45} = P[X_2 < X_3 < x < Y_2 < y < Y_3]$$

$$= P[X_2 < X_3 < x]P[x < Y_2 < y < Y_3]$$

$$= P[X_2 < X_3 < x]P[x < Y_2 < y]P[y < Y_3]$$

$$= \frac{F^2(x)}{2}.[F(y)-F(x)]\overline{F}(y)$$

$$i_{46} = P[X_2 < Y_2 < x < X_3 < y < Y_3]$$

$$= P[X_2 < Y_2 < x]P[x < X_3 < y < Y_3]$$

$$= \frac{F^2(x)}{2}.[F(y)-F(x)]\overline{F}(y)$$

$$i_{47} = P[X_2 < Y_2 < x < Y_3 < y < X_3]$$

$$= \frac{F^2(x)}{2}.[F(y)-F(x)]\overline{F}(y)$$

$$i_{48} = P[Y_2 < X_2 < x < X_3 < y < Y_3]$$

$$= \frac{F^2(x)}{2}.[F(y)-F(x)]\overline{F}(y)$$

$$i_{49} = P[Y_2 < X_2 < x < Y_3 < y < X_3]$$

$$= \frac{F^2(x)}{2}.[F(y)-F(x)]\overline{F}(y)$$

$$i_{50} = P[X_2 < X_3 < x < Y_2 < Y_3 < y]$$

$$= P[X_2 < X_3 < x]P[x < Y_2 < Y_3 < y]$$

$$= \frac{F^2(x)}{2}.\frac{[F(y)-F(x)]^2}{2}$$

$$i_{51} = P[X_2 < Y_2 < x < X_3 < Y_3 < y]$$

$$= \frac{F^2(x)}{2}.\frac{[F(y)-F(x)]^2}{2}$$

$$i_{52} = P[X_2 < Y_2 < x < Y_3 < X_3 < y]$$

$$= \frac{F^2(x)}{2}.\frac{[F(y)-F(x)]^2}{2}$$

$$i_{53} = P[Y_2 < X_2 < x < X_3 < Y_3 < y]$$

$$= \frac{F^2(x)}{2} \cdot \frac{[F(y)-F(x)]^2}{2}$$

$$i_{54} = P[Y_2 < X_2 < x < Y_3 < X_3 < y]$$

$$= \frac{F^2(x)}{2} \cdot \frac{[F(y)-F(x)]^2}{2}$$

$$i_{55} = P[X_2 < X_3 < Y_2 < x < y < Y_3]$$

$$= P[X_2 < X_3 < Y_2 < x]P[y < Y_3]$$

$$= \frac{F^3(x)}{6} \cdot \overline{F}(y)$$

$$i_{56} = P[X_2 < Y_2 < X_3 < x < y < Y_3]$$

$$= \frac{F^3(x)}{6} \cdot \overline{F}(y)$$

$$i_{57} = P[Y_2 < X_2 < X_3 < x < y < Y_3]$$

$$= \frac{F^3(x)}{6} \cdot \overline{F}(y)$$

$$i_{58} = P[X_2 < X_3 < Y_2 < x < Y_3 < y]$$

$$= P[X_2 < X_3 < Y_2 < x]P[x < Y_3 < y]$$

$$= \int_{-\infty}^{x} P[X_2 < X_3 < y_2]dF(y_2)[F(y)-F(x)]$$

$$= \int_{-\infty}^{x}\int_{\infty}^{y_2} P[X_2 < x_3]dF(x_3)dF(y_2)[F(y)-F(x)]$$

$$= \int_{-\infty}^{x}\int_{\infty}^{y_2} F(x_3)dF(x_3)dF(y_2)[F(y)-F(x)]$$

$$= \int_{-\infty}^{x} \frac{F^2(y_2)}{2} dF(y_2)[F(y)-F(x)]$$

$$= \int_{0}^{F(x)} \frac{u^2}{2} du.[F(y)-F(x)] \qquad \text{letting } u = F(y_2)$$

$$= \frac{F^3(x)}{6}[F(y) - F(x)]$$

$$i_{59} = P[X_2 < Y_2 < X_3 < x < Y_3 < y]$$

$$= \frac{F^3(x)}{6}[F(y) - F(x)]$$

$$i_{60} = P[X_2 < X_3 < Y_2 < Y_3 < x < y]$$

$$= P[X_2 < X_3 < Y_2 < Y_3 < x]$$

$$= \int_{-\infty}^{x}\int_{-\infty}^{y_3}\int_{-\infty}^{y_2} F(x_3)dF(x_3)dF(y_2)dF(y_3)$$

$$= \int_{-\infty}^{x}\int_{-\infty}^{y_3} \frac{F^2(y_2)}{2}dF(y_2)dF(y_3)$$

$$= \int_{-\infty}^{x} \frac{F^3(y_3)}{6}dF(y_3)$$

$$= \frac{F^4(x)}{24}$$

$$i_{61} = P[X_2 < Y_2 < X_3 < Y_3 < x < y]$$

$$= \frac{F^4(x)}{24}$$

$$i_{62} = P[Y_2 < X_2 < X_3 < Y_3 < x < y]$$

$$= \frac{F^4(x)}{24}.$$

Now adding the 62 terms we get case i) x<y

$$P[Med(x, X_2, X_3) < Med(y, Y_2, Y_3)] = 4t_1 + 4t_2 + \ldots\ldots\ldots + 4t_{62}$$

$$= \frac{1}{2} + F^4(y) + \frac{5}{3}F^4(x) + 2F^2(y) - \frac{8}{3}F^3(y) - \frac{2}{3}F^3(x)F(y) - \frac{4}{3}F^3(x).$$

Thus

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{y} P^2[Med(x,X_2,X_3) < Med(y,Y_2,Y_3)]dF(x)dF(y)$$

$$-\infty < x < y < \infty$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{y}\left(\frac{1}{2} + F^4(y) + \frac{5}{3}F^4(x) + 2F^2(y) - \frac{8}{3}F^3(y) - \frac{2}{3}F^3(x)F(y) - \frac{4}{3}F^3(x)\right)^2 dF(x)dF(y)$$

$$-\infty < x < y < \infty$$

$$= \frac{1097}{4536}.$$

Now consider for y<x

$$P[Med(x,X_2,X_3) < Med(y,Y_2,Y_3)] = 4j_1 + 4j_2 + \ldots\ldots + 4j_{27}$$

where

$$j_1 = P[y < x < X_2 < X_3 < Y_2 < Y_3] \qquad j_2 = P[y < x < X_2 < Y_2 < X_3 < Y_3]$$

$$j_3 = P[y < x < X_2 < Y_2 < Y_3 < X_3] \qquad j_4 = P[y < X_2 < x < X_3 < Y_2 < Y_3]$$

$$j_5 = P[y < X_2 < x < Y_2 < X_3 < Y_3] \qquad j_6 = P[y < X_2 < x < Y_2 < Y_3 < X_3]$$

$$j_7 = P[y < X_2 < X_3 < x < Y_2 < Y_3] \qquad j_8 = P[y < X_2 < X_3 < Y_2 < x < Y_3]$$

$$j_9 = P[y < X_2 < X_3 < Y_2 < Y_3 < x] \qquad j_{10} = P[X_2 < y < x < X_3 < Y_2 < Y_3]$$

$$j_{11} = P[X_2 < y < x < Y_2 < X_3 < Y_3] \qquad j_{12} = P[X_2 < y < x < Y_2 < Y_3 < X_3]$$

$$j_{13} = P[X_2 < y < X_3 < x < Y_2 < Y_3] \qquad j_{14} = P[X_2 < y < X_3 < Y_2 < x < Y_3]$$

$$j_{15} = P[X_2 < y < X_3 < Y_2 < Y_3 < x] \qquad j_{16} = P[X_2 < X_3 < y < x < Y_2 < Y_3]$$

$$j_{17} = P[X_2 < X_3 < y < Y_2 < x < Y_3] \qquad j_{18} = P[X_2 < X_3 < y < Y_2 < Y_3 < x]$$

$$j_{19} = P[X_2 < X_3 < Y_2 < y < x < Y_3] \qquad j_{20} = P[X_2 < Y_2 < X_3 < y < x < Y_3]$$

$$j_{21} = P[Y_2 < X_2 < X_3 < y < x < Y_3] \qquad j_{22} = P[X_2 < X_3 < Y_2 < y < Y_3 < x]$$

$$j_{23} = P[X_2 < Y_2 < X_3 < y < Y_3 < x] \qquad j_{24} = P[Y_2 < X_2 < X_3 < y < Y_3 < x]$$

$$j_{25} = P[X_2 < X_3 < Y_2 < Y_3 < y < x] \qquad j_{26} = P[X_2 < Y_2 < X_3 < Y_3 < y < x]$$

$$j_{27} = P[Y_2 < X_2 < X_3 < Y_3 < y < x]$$

Now

$$j_1 = P[y < x < X_2 < X_3 < Y_2 < Y_3]$$

$$= P[x < X_2 < X_3 < Y_2 < Y_3]$$

$$= \frac{\overline{F}^4(x)}{24}$$

$$j_2 = P[y < x < X_2 < Y_2 < X_3 < Y_3]$$

$$= \frac{\overline{F}^4(x)}{24}$$

$$j_3 = P[y < x < X_2 < Y_2 < Y_3 < X_3]$$

$$= \frac{\overline{F}^4(x)}{24}$$

$$j_4 = P[y < X_2 < x < X_3 < Y_2 < Y_3]$$

$$= P[y < X_2 < x]P[x < X_3 < Y_2 < Y_3]$$

$$= [F(x) - F(y)]\frac{\overline{F}^3(x)}{6}$$

$$j_5 = P[y < X_2 < x < Y_2 < X_3 < Y_3]$$

$$= [F(x) - F(y)]\frac{\overline{F}^3(x)}{6}$$

$$j_6 = P[y < X_2 < x < Y_2 < Y_3 < X_3]$$

$$= [F(x) - F(y)]\frac{\overline{F}^3(x)}{6}$$

$$j_7 = P[y < X_2 < X_3 < x < Y_2 < Y_3]$$

$$= P[y < X_2 < X_3 < x]P[x < Y_2 < Y_3]$$

$$= \int_y^x P(x_2 < X_3 < x) dF(x_2) \int_x^\infty P(y_2 < Y_3) dF(y_2)$$

$$= \frac{1}{2}[F(x) - F(y)]^2 \times \frac{\overline{F}^2(x)}{2}$$

$$j_8 = P[y < X_2 < X_3 < Y_2 < x < Y_3]$$

$$= P[y < X_2 < X_3 < Y_2 < x] P[x < Y_3]$$

$$= \frac{[F(x) - F(y)]^3}{6} . \overline{F}(x)$$

$$j_9 = P[y < X_2 < X_3 < Y_2 < Y_3 < x]$$

$$= \frac{[F(x) - F(y)]^4}{24}$$

$$j_{10} = P[X_2 < y < x < X_3 < Y_2 < Y_3]$$

$$= P[X_2 < y] P[x < X_3 < Y_2 < Y_3]$$

$$= F(y) \frac{\overline{F}^3(x)}{6}$$

$$j_{11} = P[X_2 < y < x < Y_2 < X_3 < Y_3]$$

$$= F(y) \frac{\overline{F}^3(x)}{6}$$

$$j_{12} = P[X_2 < y < x < Y_2 < Y_3 < X_3]$$

$$= F(y) \frac{\overline{F}^3(x)}{6}$$

$$j_{13} = P[X_2 < y < X_3 < x < Y_2 < Y_3]$$

$$= P[X_2 < y] P[y < X_3 < x] P[x < Y_2 < Y_3]$$

$$= F(y)[F(x) - F(y)] \times \frac{\overline{F}^2(x)}{2}$$

$$j_{14} = P[X_2 < y < X_3 < Y_2 < x < Y_3]$$

$$= P[X_2 < y]P[y < X_3 < Y_2 < x]P[x < Y_3]$$

$$= F(y)\frac{[F(x)-F(y)]^2}{2}.\overline{F}(x)$$

$$j_{15} = P[X_2 < y < X_3 < Y_2 < Y_3 < x]$$

$$= P[X_2 < y]P[y < X_3 < Y_2 < Y_3 < x]$$

$$= F(y)\frac{[F(x)-F(y)]^3}{6}$$

$$j_{16} = P[X_2 < X_3 < y < x < Y_2 < Y_3]$$

$$= P[X_2 < X_3 < y_3]P[x < Y_2 < Y_3]$$

$$= \frac{F^2(y)}{2}.\frac{\overline{F}^2(x)}{2}$$

$$j_{17} = P[X_2 < X_3 < y < Y_2 < x < Y_3]$$

$$= P[X_2 < X_3 < y]P[y < Y_2 < x]P[x < Y_3]$$

$$= \frac{F^2(y)}{2}[F(x)-F(y)]\overline{F}(x)$$

$$j_{18} = P[X_2 < X_3 < y < Y_2 < Y_3 < x]$$

$$= P[X_2 < X_3 < y]P[y < Y_2 < Y_3 < x]$$

$$= \frac{F^2(y)}{2}\frac{[F(x)-F(y)]^2}{2}$$

$$j_{19} = P[X_2 < X_3 < Y_2 < y < x < Y_3]$$

$$= P[X_2 < X_3 < Y_2 < y]P[x < Y_3]$$

$$= \frac{F^3(y)}{6}\overline{F}(x)$$

$$j_{20} = P[X_2 < Y_2 < X_3 < y < x < Y_3]\ .$$

$$= \frac{F^3(y)}{6}\overline{F}(x)$$

$$j_{21} = P[Y_2 < X_2 < X_3 < y < x < Y_3]$$

$$= \frac{F^3(y)}{6}\overline{F}(x)$$

$$j_{22} = P[X_2 < X_3 < Y_2 < y < Y_3 < x]$$

$$= P[X_2 < X_3 < Y_2 < y]P[y < Y_3 < x]$$

$$= \frac{F^3(y)}{6}[F(x) - F(y)]$$

$$j_{23} = P[X_2 < Y_2 < X_3 < y < Y_3 < x]$$

$$= \frac{F^3(y)}{6}[F(x) - F(y)]$$

$$j_{24} = P[Y_2 < X_2 < X_3 < y < Y_3 < x]$$

$$= \frac{F^3(y)}{6}[F(x) - F(y)]$$

$$j_{25} = P[X_2 < X_3 < Y_2 < Y_3 < y < x]$$

$$= P[X_2 < X_3 < Y_2 < Y_3 < y]$$

$$= \frac{F^4(y)}{24}$$

$$j_{26} = P[X_2 < Y_2 < X_3 < Y_3 < y < x]$$

$$= \frac{F^4(y)}{24}$$

$$j_{27} = P[Y_2 < X_2 < X_3 < Y_3 < y < x]$$

$$= \frac{F^4(y)}{24}.$$

Thus adding the 27 terms we have for y < x

$$P[Med(x,X_2,X_3)< Med(y,Y_2,Y_3)]= 4j_1+4j_2+.....+4j_{27}$$

$$=\frac{1}{2}-F^4(y)+\frac{8}{3}F^3(x)-F^4(x)+\frac{4}{3}F^3(y)-2F^2(x)\,.$$

Thus,

$$\int_{-\infty}^{\infty}\int_{y}^{\infty} P^2[Med(x,X_2,X_3)< Med(y,Y_2,Y_3)]dF(x)dF(y)$$
$$-\infty < y < x < \infty$$

$$=\int_{-\infty}^{\infty}\int_{y}^{\infty}\left(\frac{1}{2}-F^4(y)+\frac{8}{3}F^3(x)-F^4(x)+\frac{4}{3}F^3(y)-2F^2(x)\right)^2 dF(x)dF(y)$$
$$-\infty < y < x < \infty$$

$$=\frac{619}{12600}\,.$$

Thus adding the first and the second case we have,

$$P[Med(X_1,X_2,X_3)< Med(Y_1,Y_2,Y_3), Med(X_1,X_4,X_5)< Med(Y_1,Y_4,Y_5)]$$

$$=\frac{1097}{4536}+\frac{619}{12600}$$

$$=\frac{8249}{28350}\,.$$

Thus

$$\xi_{1,1} \quad = P[Med(X_1,X_2,X_3)< Med(Y_1,Y_2,Y_3), Med(X_1,X_4,X_5)< Med(Y_1,Y_4,Y_5)]-\frac{1}{4}$$

$$=\frac{8249}{28350}-\frac{1}{4}$$

$$=\frac{2323}{56700}\,.$$

Consider

$$\xi_{2,2} = Cov[h(X_1,X_2,X_3;Y_1,Y_2,Y_3), h(X_1,X_2X_4,;Y_1,Y_2,Y_4)]$$

$$= E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)h(X_1,X_2X_4,;Y_1,Y_2,Y_4)-\{E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)]\}^2]$$

$$= P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_2, X_4) < Med(Y_1, Y_2, Y_4)] - \frac{1}{4}.$$

Now

$$P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_2, X_4) < Med(Y_1, Y_2, Y_4)]$$

$$= \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dF(y_1)dF(y_2)$$

$$= k_1 + k_2 + ... + k_{20}$$

where

$$k_1 = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dF(y_1)dF(y_2)$$

$$x_1 < x_2 < y_1 < y_2$$

$$k_2 = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dF(y_1)dF(y_2)$$

$$x_1 < x_2 < y_2 < y_1$$

$$k_3 = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dF(y_1)dF(y_2)$$

$$x_1 < y_1 < x_2 < y_2$$

$$k_4 = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dF(y_1)dF(y_2)$$

$$x_1 < y_2 < x_2 < y_1$$

$$k_5 = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dF(y_1)dF(y_2)$$

$$x_1 < y_1 < y_2 < x_2$$

$$k_6 = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dF(y_1)dF(y_2)$$

$$x_1 < y_2 < y_1 < x_2$$

$$k_7 = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dF(y_1)dF(y_2)$$

$$x_2 < x_1 < y_1 < y_2$$

$$k_8 = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dF(y_1)dF(y_2)$$

$$x_2 < x_1 < y_2 < y_1$$

$$k_9 = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dF(y_1)dF(y_2)$$

$$x_2 < y_1 < x_1 < y_2$$

$$k_{10} = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dF(y_1)dF(y_2)$$

$$x_2 < y_2 < x_1 < y_1$$

$$k_{11} = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dF(y_1)dF(y_2)$$

$$x_2 < y_1 < y_2 < x_1$$

$$k_{12} = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dF(y_1)dF(y_2)$$

$$x_2 < y_2 < y_1 < x_1$$

$$k_{13} = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dF(y_1)dF(y_2)$$

$$y_1 < x_1 < y_2 < x_2$$

$$k_{14} = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dF(y_1)dF(y_2)$$

$$y_1 < x_2 < y_2 < x_1$$

$$k_{15} = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dF(y_1)dF(y_2)$$

$$y_1 < x_1 < x_2 < y_2$$

$$k_{16} = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dF(y_1)dF(y_2)$$

$$y_1 < x_2 < x_1 < y_2$$

$$k_{17} = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dF(y_1)dF(y_2)$$

$$y_2 < x_1 < y_1 < x_2$$

$$k_{18} = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dF(y_1)dF(y_2)$$

$$y_2 < x_2 < y_1 < x_1$$

$$k_{19} = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dF(y_1)dF(y_2)$$

$$y_2 < x_1 < x_2 < y_1$$

$$k_{20} = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dF(y_1)dF(y_2)$$

$$y_2 < x_2 < x_1 < y_1$$

Now consider

$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]$ when $x_1 < x_2 < y_1 < y_2$

$= l_1 + l_2 + ... + l_{30}$

where

$l_1 = P[x_1 < x_2 < X_3 < y_1 < y_2 < Y_3]$ $l_2 = P[x_1 < x_2 < y_1 < X_3 < y_2 < Y_3]$

$l_3 = P[x_1 < x_2 < y_1 < y_2 < X_3 < Y_3]$ $l_4 = P[x_1 < x_2 < y_1 < y_2 < Y_3 < X_3]$

$l_5 = P[x_1 < x_2 < Y_3 < y_1 < y_2 < X_3]$ $l_6 = P[x_1 < x_2 < y_1 < Y_3 < y_2 < X_3]$

$l_7 = P[x_1 < X_3 < x_2 < y_1 < y_2 < Y_3]$ $l_8 = P[x_1 < X_3 < x_2 < y_1 < Y_3 < y_2]$

$l_9 = P[x_1 < X_3 < x_2 < Y_3 < y_1 < y_2]$ $l_{10} = P[x_1 < Y_3 < x_2 < X_3 < y_1 < y_2]$

$l_{11} = P[x_1 < Y_3 < x_2 < y_1 < X_3 < y_2]$ $l_{12} = P[x_1 < Y_3 < x_2 < y_1 < y_2 < X_3]$

$l_{13} = P[x_1 < X_3 < Y_3 < x_2 < y_1 < y_2]$ $l_{14} = P[x_1 < Y_3 < X_3 < x_2 < y_1 < y_2]$

$l_{15} = P[X_3 < x_1 < x_2 < y_1 < y_2 < Y_3]$ $l_{16} = P[X_3 < x_1 < x_2 < y_1 < Y_3 < y_2]$

$l_{17} = P[Y_3 < x_1 < x_2 < y_1 < y_2 < X_3]$ $l_{18} = P[Y_3 < x_1 < x_2 < y_1 < X_3 < y_2]$

$l_{19} = P[X_3 < x_1 < x_2 < Y_3 < y_1 < y_2]$ $l_{20} = P[Y_3 < x_1 < x_2 < X_3 < y_1 < y_2]$

$l_{21} = P[X_3 < x_1 < Y_3 < x_2 < y_1 < y_2]$ $l_{22} = P[Y_3 < x_1 < X_3 < x_2 < y_1 < y_2]$

$$l_{23} = P[X_3 < Y_3 < x_1 < x_2 < y_1 < y_2] \qquad l_{24} = P[Y_3 < X_3 < x_1 < x_2 < y_1 < y_2]$$

$$l_{25} = P[x_1 < x_2 < Y_3 < y_1 < X_3 < y_2] \qquad l_{26} = P[x_1 < x_2 < X_3 < y_1 < Y_3 < y_2]$$

$$l_{27} = P[x_1 < x_2 < X_3 < Y_3 < y_1 < y_2] \qquad l_{28} = P[x_1 < x_2 < Y_3 < X_3 < y_1 < y_2]$$

$$l_{29} = P[x_1 < x_2 < y_1 < X_3 < Y_3 < y_2] \qquad l_{30} = P[x_1 < x_2 < y_1 < Y_3 < X_3 < y_2].$$

Now consider

$$l_1 = P[x_1 < x_2 < X_3 < y_1 < y_2 < Y_3]$$

$$= P[x_2 < X_3 < y_1]P[y_2 < Y_3]$$

$$= [F(y_1) - F(x_2)]\overline{F}(y_2)$$

$$l_2 = P[x_1 < x_2 < y_1 < X_3 < y_2 < Y_3]$$

$$= P[y_1 < X_3 < y_2 < Y_3]$$

$$= P[y_1 < X_3 < y_2]P[y_2 < Y_3]$$

$$= [F(y_2) - F(y_1)]\overline{F}(y_2)$$

$$l_3 = P[x_1 < x_2 < y_1 < y_2 < X_3 < Y_3]$$

$$= P[y_2 < X_3 < Y_3]$$

$$= \frac{\overline{F}^2(y_2)}{2}$$

$$l_4 = P[x_1 < x_2 < y_1 < y_2 < Y_3 < X_3]$$

$$= P[y_2 < Y_3 < X_3]$$

$$= \frac{\overline{F}^2(y_2)}{2}$$

$$l_5 = P[x_1 < x_2 < Y_3 < y_1 < y_2 < X_3]$$

$$= P[x_2 < Y_3 < y_1]P[y_2 < X_3]$$

$$= [F(y_1) - F(x_2)]\overline{F}(y_2)$$

$$l_6 = P[x_1 < x_2 < y_1 < Y_3 < y_2 < X_3]$$

$$= P[y_1 < Y_3 < y_2]P[y_2 < X_3]$$

$$= [F(y_2) - F(y_1)]\overline{F}(y_2)$$

$$l_7 = P[x_1 < X_3 < x_2 < y_1 < y_2 < Y_3]$$

$$= P[x_1 < X_3 < x_2]P[y_2 < Y_3]$$

$$= [F(x_2) - F(x_1)]\overline{F}(y_2)$$

$$l_8 = P[x_1 < X_3 < x_2 < y_1 < Y_3 < y_2]$$

$$= P[x_1 < X_3 < x_2]P[y_1 < Y_3 < y_2]$$

$$= [F(x_2) - F(x_1)][F(y_2) - F(y_1)]$$

$$l_9 = P[x_1 < X_3 < x_2 < Y_3 < y_1 < y_2]$$

$$= P[x_1 < X_3 < x_2 < Y_3 < y_1]$$

$$= P[x_1 < X_3 < x_2]P[x_2 < Y_3 < y_1]$$

$$= [F(x_2) - F(x_1)][F(y_1) - F(x_2)]$$

$$l_{10} = P[x_1 < Y_3 < x_2 < X_3 < y_1 < y_2]$$

$$= P[x_1 < Y_3 < x_2]P[x_2 < X_3 < y_1]$$

$$= [F(x_2) - F(x_1)][F(y_1) - F(x_2)]$$

$$l_{11} = P[x_1 < Y_3 < x_2 < y_1 < X_3 < y_2]$$

$$= P[x_1 < Y_3 < x_2]P[y_1 < X_3 < y_2]$$

$$= [F(x_2) - F(x_1)][F(y_2) - F(y_1)]$$

$$l_{12} = P[x_1 < Y_3 < x_2 < y_1 < y_2 < X_3]$$

$$= P[x_1 < Y_3 < x_2]P[y_2 < X_3]$$

$$= [F(x_2) - F(x_1)]\overline{F}(y_2)$$

$$l_{13} = P[x_1 < X_3 < Y_3 < x_2 < y_1 < y_2]$$

$$= P[x_1 < X_3 < Y_3 < x_2]$$

$$= \frac{[F(x_2) - F(x_1)]^2}{2}$$

$$l_{14} = P[x_1 < Y_3 < X_3 < x_2 < y_1 < y_2]$$

$$= P[x_1 < Y_3 < X_3 < x_2]$$

$$= \frac{[F(x_2) - F(x_1)]^2}{2}$$

$$l_{15} = P[X_3 < x_1 < x_2 < y_1 < y_2 < Y_3]$$

$$= P[X_3 < x_1 \;]P[y_2 < Y_3]$$

$$= [F(x_1)\overline{F}(y_2)]$$

$$l_{16} = P[X_3 < x_1 < x_2 < y_1 < Y_3 < y_2]$$

$$= P[X_3 < x_1]P[y_1 < Y_3 < y_2]$$

$$= F(x_1)[F(y_2) - F(y_1)]$$

$$l_{17} = P[Y_3 < x_1 < x_2 < y_1 < y_2 < X_3]$$

$$= P[Y_3 < x_1]P[y_2 < X_3]$$

$$= [F(x_1)\overline{F}(y_2)]$$

$$l_{18} = P[Y_3 < x_1 < x_2 < y_1 < X_3 < y_2]$$

$$= P[Y_3 < x_1]P[y_1 < X_3 < y_2]$$

$$= F(x_1)[F(y_2) - F(y_1)]$$

$$l_{19} = P[X_3 < x_1 < x_2 < Y_3 < y_1 < y_2]$$

$$= P[X_3 < x_1]P[x_2 < Y_3 < y_1]$$

$$= F(x_1)[F(y_1) - F(x_2)]$$

$$l_{20} = P[Y_3 < x_1 < x_2 < X_3 < y_1 < y_2]$$

$$= P[Y_3 < x_1]P[x_2 < X_3 < y_1]$$

$$= F(x_1)[F(y_1) - F(x_2)]$$

$$l_{21} = P[X_3 < x_1 < Y_3 < x_2 < y_1 < y_2]$$

$$= P[X_3 < x_1 < Y_3 < x_2]$$

$$= P[X_3 < x_1]P[x_1 < Y_3 < x_2]$$

$$= F(x_1)[F(x_2) - F(x_1)]$$

$$l_{22} = P[Y_3 < x_1 < X_3 < x_2 < y_1 < y_2]$$

$$= P[Y_3 < x_1]P[x_1 < X_3 < x_2]$$

$$= F(x_1)[F(x_2) - F(x_1)]$$

$$l_{23} = P[X_3 < Y_3 < x_1 < x_2 < y_1 < y_2]$$

$$= P[X_3 < Y_3 < x_1]$$

$$= \frac{F^2(x_1)}{2}$$

$$l_{24} = P[Y_3 < X_3 < x_1 < x_2 < y_1 < y_2]$$

$$= P[Y_3 < X_3 < x_1]$$

$$= \frac{F^2(x_1)}{2}$$

$$l_{25} = P[x_1 < x_2 < Y_3 < y_1 < X_3 < y_2]$$

$$= P[x_2 < Y_3 < y_1]P[y_1 < X_3 < y_2]$$

$$= [F(y_1) - F(x_2)][F(y_2) - F(y_1)]$$

$$l_{26} = P[x_1 < x_2 < X_3 < y_1 < Y_3 < y_2]$$

$$= P[x_2 < X_3 < y_1]P[y_1 < Y_3 < y_2]$$

$$= [F(y_1) - F(x_2)][F(y_2) - F(y_1)]$$

$$l_{27} = P[x_1 < x_2 < X_3 < Y_3 < y_1 < y_2]$$

$$= P[x_2 < X_3 < Y_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_2)]^2}{2}$$

$$l_{28} = P[x_1 < x_2 < Y_3 < X_3 < y_1 < y_2]$$

$$= P[x_2 < Y_3 < X_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_2)]^2}{2}$$

$$l_{29} = P[x_1 < x_2 < y_1 < X_3 < Y_3 < y_2]$$

$$= P[y_1 < X_3 < Y_3 < y_2]$$

$$= \frac{[F(y_2) - F(y_1)]^2}{2}$$

$$l_{30} = P[x_1 < x_2 < y_1 < Y_3 < X_3 < y_2]$$

$$= P[y_1 < Y_3 < X_3 < y_2]$$

$$= \frac{[F(y_2) - F(y_1)]^2}{2}$$

Adding the 30 terms we get for the case $x_1 < x_2 < y_1 < y_2$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = l_1 + l_2 + ... + l_{30}$$

$$= l^* \text{ say}$$

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_2}\int_{-\infty}^{y_1}\int_{-\infty}^{x_2} P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dF(y_1)dF(y_2)$$

$$-\infty < x_1 < x_2 < y_1 < y_2 < \infty$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{y_2}\int_{-\infty}^{y_1}\int_{-\infty}^{x_2} l^{*2}\, dF(x_1)dF(x_2)dF(y_1)dF(y_2)$$

$$-\infty < x_1 < x_2 < y_1 < y_2 < \infty$$

$$= \frac{1}{24}.$$

When $x_1 < x_2 < y_2 < y_1$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = m_1 + m_2 + ... + m_{30}$$

where

$$m_1 = P[x_1 < x_2 < X_3 < y_2 < y_1 < Y_3] \qquad m_2 = P[x_1 < x_2 < y_2 < X_3 < y_1 < Y_3]$$

$$m_3 = P[x_1 < x_2 < y_2 < y_1 < X_3 < Y_3] \qquad m_4 = P[x_1 < x_2 < y_2 < y_1 < Y_3 < X_3]$$

$$m_5 = P[x_1 < x_2 < Y_3 < y_2 < y_1 < X_3] \qquad m_6 = P[x_1 < x_2 < y_2 < Y_3 < y_1 < X_3]$$

$$m_7 = P[x_1 < X_3 < x_2 < y_2 < y_1 < Y_3] \qquad m_8 = P[x_1 < X_3 < x_2 < y_2 < Y_3 < y_1]$$

$$m_9 = P[x_1 < X_3 < x_2 < Y_3 < y_2 < y_1] \qquad m_{10} = P[x_1 < Y_3 < x_2 < X_3 < y_2 < y_1]$$

$$m_{11} = P[x_1 < Y_3 < x_2 < y_2 < X_3 < y_1] \qquad m_{12} = P[x_1 < Y_3 < x_2 < y_2 < y_1 < X_3]$$

$$m_{13} = P[x_1 < X_3 < Y_3 < x_2 < y_2 < y_1] \qquad m_{14} = P[x_1 < Y_3 < X_3 < x_2 < y_2 < y_1]$$

$$m_{15} = P[X_3 < x_1 < x_2 < y_2 < y_1 < Y_3] \qquad m_{16} = P[X_3 < x_1 < x_2 < y_2 < Y_3 < y_1]$$

$$m_{17} = P[Y_3 < x_1 < x_2 < y_2 < y_1 < X_3] \qquad m_{18} = P[Y_3 < x_1 < x_2 < y_2 < X_3 < y_1]$$

$$m_{19} = P[X_3 < x_1 < x_2 < Y_3 < y_2 < y_1] \qquad m_{20} = P[Y_3 < x_1 < x_2 < X_3 < y_2 < y_1]$$

$$m_{21} = P[X_3 < x_1 < Y_3 < x_2 < y_2 < y_1] \qquad m_{22} = P[Y_3 < x_1 < X_3 < x_2 < y_2 < y_1]$$

$$m_{23} = P[X_3 < Y_3 < x_1 < x_2 < y_2 < y_1] \qquad m_{24} = P[Y_3 < X_3 < x_1 < x_2 < y_2 < y_1]$$

$$m_{25} = P[x_1 < x_2 < Y_3 < y_2 < X_3 < y_1] \qquad m_{26} = P[x_1 < x_2 < X_3 < y_2 < Y_3 < y_1]$$

$$m_{27} = P[x_1 < x_2 < X_3 < Y_3 < y_2 < y_1] \qquad m_{28} = P[x_1 < x_2 < Y_3 < X_3 < y_2 < y_1]$$

$$m_{29} = P[x_1 < x_2 < y_2 < X_3 < Y_3 < y_1] \qquad m_{30} = P[x_1 < x_2 < y_2 < Y_3 < X_3 < y_1]$$

Now Consider

$$m_1 = P[x_1 < x_2 < X_3 < y_2 < y_1 < Y_3]$$

$$= P[x_2 < X_3 < y_2]P[y_1 < Y_3]$$

$$= [F(y_2) - F(x_2)]\overline{F}(y_1)$$

$$m_2 = P[x_1 < x_2 < y_2 < X_3 < y_1 < Y_3]$$

$$= P[y_2 < X_3 < y_1]P[y_1 < Y_3]$$

$$= [F(y_1) - F(y_2)]\overline{F}(y_1)$$

$$m_3 = P[x_1 < x_2 < y_2 < y_1 < X_3 < Y_3]$$

$$= P[y_1 < X_3 < Y_3]$$

$$= \frac{\overline{F}^2(y_1)}{2}$$

$$m_4 = P[x_1 < x_2 < y_2 < y_1 < Y_3 < X_3]$$

$$= P[y_1 < Y_3 < X_3]$$

$$= \frac{\overline{F}^2(y_1)}{2}$$

$$m_5 = P[x_1 < x_2 < Y_3 < y_2 < y_1 < X_3]$$

$$= P[x_2 < Y_3 < y_2]P[y_1 < X_3]$$

$$= [F(y_2) - F(x_2)]\overline{F}(y_1)$$

$$m_6 = P[x_1 < x_2 < y_2 < Y_3 < y_1 < X_3]$$

$$= P[y_2 < Y_3 < y_1]P[y_1 < X_3]$$

$$= [F(y_1) - F(y_2)]\overline{F}(y_1)$$

$$m_7 = P[x_1 < X_3 < x_2 < y_2 < y_1 < Y_3]$$

$$= P[x_1 < X_3 < x_2]P[y_1 < Y_3]$$

$$= [F(x_2) - F(x_1)]\overline{F}(y_1)$$

$$m_8 = P[x_1 < X_3 < x_2 < y_2 < Y_3 < y_1]$$

$$= P[x_1 < X_3 < x_2]P[y_2 < Y_3 < y_1]$$

$$= [F(x_2) - F(x_1)][F(y_1) - F(y_2)]$$

$$m_9 = P[x_1 < X_3 < x_2 < Y_3 < y_2 < y_1]$$

$$= P[x_1 < X_3 < x_2 < Y_3 < y_2]$$

$$= P[x_1 < X_3 < x_2]P[x_2 < Y_3 < y_2]$$

$$= [F(x_2) - F(x_1)][F(y_2) - F(x_2)]$$

$$m_{10} = P[x_1 < Y_3 < x_2 < X_3 < y_2 < y_1]$$

$$= P[x_1 < Y_3 < x_2]P[x_2 < X_3 < y_2]$$

$$= [F(x_2) - F(x_1)][F(y_2) - F(x_2)]$$

$$m_{11} = P[x_1 < Y_3 < x_2 < y_2 < X_3 < y_1]$$

$$= P[x_1 < Y_3 < x_2]P[y_2 < X_3 < y_1]$$

$$= [F(x_2) - F(x_1)][F(y_1) - F(y_2)]$$

$$m_{12} = P[x_1 < Y_3 < x_2 < y_2 < y_1 < X_3]$$

$$= P[x_1 < Y_3 < x_2]P[y_1 < X_3]$$

$$= [F(x_2) - F(x_1)]\overline{F}(y_1)$$

$$m_{13} = P[x_1 < X_3 < Y_3 < x_2 < y_2 < y_1]$$

$$= P[x_1 < X_3 < Y_3 < x_2]$$

$$= \frac{[F(x_2) - F(x_1)]^2}{2}$$

$$m_{14} = P[x_1 < Y_3 < X_3 < x_2 < y_2 < y_1]$$

$$= P[x_1 < Y_3 < X_3 < x_2]$$

$$= \frac{[F(x_2) - F(x_1)]^2}{2}$$

$$m_{15} = P[X_3 < x_1 < x_2 < y_2 < y_1 < Y_3]$$

$$= P[X_3 < x_1 \;]P[y_1 < Y_3]$$

$$= F(x_1)\overline{F}(y_1)$$

$$m_{16} = P[X_3 < x_1 < x_2 < y_2 < Y_3 < y_1]$$

$$= P[X_3 < x_1]P[y_2 < Y_3 < y_1]$$

$$= F(x_1)[F(y_1) - F(y_2)]$$

$$m_{17} = P[Y_3 < x_1 < x_2 < y_2 < y_1 < X_3]$$

$$= P[Y_3 < x_1]P[y_1 < X_3]$$

$$= F(x_1)\overline{F}(y_1)$$

$$m_{18} = P[Y_3 < x_1 < x_2 < y_2 < X_3 < y_1]$$

$$= P[Y_3 < x_1]P[y_2 < X_3 < y_1]$$

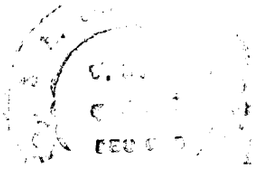
$$= F(x_1)[F(y_1) - F(y_2)]$$

$$m_{19} = P[X_3 < x_1 < x_2 < Y_3 < y_2 < y_1]$$

$$= P[X_3 < x_1]P[x_2 < Y_3 < y_2]$$

$$= F(x_1)[F(y_2) - F(x_2)]$$

$$m_{20} = P[Y_3 < x_1 < x_2 < X_3 < y_2 < y_1]$$



$$= P[Y_3 < x_1]P[x_2 < X_3 < y_2]$$

$$= F(x_1)[F(y_2) - F(x_2)]$$

$$m_{21} = P[X_3 < x_1 < Y_3 < x_2 < y_2 < y_1]$$

$$= P[X_3 < x_1 < Y_3 < x_2]$$

$$= P[X_3 < x_1]P[x_1 < Y_3 < x_2]$$

$$= F(x_1)[F(x_2) - F(x_1)]$$

$$m_{22} = P[Y_3 < x_1 < X_3 < x_2 < y_2 < y_1]$$

$$= P[Y_3 < x_1]P[x_1 < X_3 < x_2 \;]$$

$$= F(x_1)[F(x_2) - F(x_1)]$$

$$m_{23} = P[X_3 < Y_3 < x_1 < x_2 < y_2 < y_1]$$

$$= P[X_3 < Y_3 < x_1]$$

$$= \frac{F^2(x_1)}{2}$$

$$m_{24} = P[Y_3 < X_3 < x_1 < x_2 < y_2 < y_1]$$

$$= P[Y_3 < X_3 < x_1]$$

$$= \frac{F^2(x_1)}{2}$$

$$m_{25} = P[x_1 < x_2 < Y_3 < y_2 < X_3 < y_1]$$

$$= P[x_2 < Y_3 < y_2]P[y_2 < X_3 < y_1]$$

$$= [F(y_2) - F(x_2)][F(y_1) - F(y_2)]$$

$$m_{26} = P[x_1 < x_2 < X_3 < y_2 < Y_3 < y_1]$$

$$= P[x_2 < X_3 < y_2]P[y_2 < Y_3 < y_1]$$

$$= [F(y_2) - F(x_2)][F(y_1) - F(y_2)]$$

$$m_{27} = P[x_1 < x_2 < X_3 < Y_3 < y_2 < y_1]$$

$$= P[x_2 < X_3 < Y_3 < y_2]$$

$$= \frac{[F(y_2) - F(x_2)]^2}{2}$$

$$m_{28} = P[x_1 < x_2 < Y_3 < X_3 < y_2 < y_1]$$

$$= P[x_2 < Y_3 < X_3 < y_2]$$

$$= \frac{[F(y_2) - F(x_2)]^2}{2}$$

$$m_{29} = P[x_1 < x_2 < y_2 < X_3 < Y_3 < y_1]$$

$$= P[y_2 < X_3 < Y_3 < y_1]$$

$$= \frac{[F(y_1) - F(y_2)]^2}{2}$$

$$m_{30} = P[x_1 < x_2 < y_2 < Y_3 < X_3 < y_1]$$

$$= P[y_2 < Y_3 < X_3 < y_1]$$

$$= \frac{[F(y_1) - F(y_2)]^2}{2}.$$

Adding 30 terms we get for the case $x_1 < x_2 < y_2 < y_1$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = m_1 + m_2 + ... + m_{30}$$

$$= m^* \text{ say}$$

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_1}\int_{-\infty}^{y_2}\int_{-\infty}^{x_2} P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dF(y_2)dF(y_1)$$

$$-\infty < x_1 < x_2 < y_2 < y_1 < \infty$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{y_1}\int_{-\infty}^{y_2}\int_{-\infty}^{x_2} m^{*2}\, dF(x_1)dF(x_2)dF(y_2)dF(y_1)$$

$$-\infty < x_1 < x_2 < y_2 < y_1 < \infty$$

$$= \frac{1}{24}.$$

For the case $x_1 < y_1 < x_2 < y_2$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = n_1 + n_2 + ... + n_{30}$$

where

$n_1 = P[X_3 < x_1 < y_1 < x_2 < y_2 < Y_3]$ $\quad$ $n_2 = P[x_1 < X_3 < y_1 < x_2 < y_2 < Y_3]$

$n_3 = P[x_1 < y_1 < X_3 < x_2 < y_2 < Y_3]$ $\quad$ $n_4 = P[x_1 < y_1 < x_2 < X_3 < y_2 < Y_3]$

$n_5 = P[X_3 < x_1 < y_1 < x_2 < Y_3 < y_2]$ $\quad$ $n_6 = P[x_1 < X_3 < y_1 < x_2 < Y_3 < y_2]$

$n_7 = P[x_1 < y_1 < X_3 < x_2 < Y_3 < y_2]$ $\quad$ $n_8 = P[x_1 < y_1 < x_2 < Y_3 < y_2 < X_3]$

$n_9 = P[X_3 < x_1 < y_1 < Y_3 < x_2 < y_2]$ $\quad$ $n_{10} = P[x_1 < X_3 < y_1 < Y_3 < x_2 < y_2]$

$n_{11} = P[X_3 < x_1 < Y_3 < y_1 < x_2 < y_2]$ $\quad$ $n_{12} = P[Y_3 < x_1 < X_3 < y_1 < x_2 < y_2]$

$n_{13} = P[X_3 < Y_3 < x_1 < y_1 < x_2 < y_2]$ $\quad$ $n_{14} = P[Y_3 < X_3 < x_1 < y_1 < x_2 < y_2]$

$n_{15} = P[x_1 < X_3 < Y_3 < y_1 < x_2 < y_2]$ $\quad$ $n_{16} = P[x_1 < Y_3 < X_3 < y_1 < x_2 < y_2]$

$n_{17} = P[x_1 < y_1 < X_3 < Y_3 < x_2 < y_2]$ $\quad$ $n_{18} = P[x_1 < y_1 < x_2 < X_3 < Y_3 < y_2]$

$n_{19} = P[x_1 < y_1 < x_2 < Y_3 < X_3 < y_2]$ $\quad$ $n_{20} = P[x_1 < y_1 < x_2 < y_2 < X_3 < Y_3]$

$n_{21} = P[x_1 < y_1 < x_2 < y_2 < Y_3 < X_3]$.

Now Consider

$$n_1 = P[X_3 < x_1 < y_1 < x_2 < y_2 < Y_3]$$

$$= P[X_3 < x_1]P[y_2 < Y_3]$$

$$= F(x_1)\overline{F}(y_2)$$

$$n_2 = P[x_1 < X_3 < y_1 < x_2 < y_2 < Y_3]$$

$$= P[x_1 < X_3 < y_1]P[y_2 < Y_3]$$

$$= [F(y_1) - F(x_1)]\overline{F}(y_2)$$

$$n_3 = P[x_1 < y_1 < X_3 < x_2 < y_2 < Y_3]$$

$$= P[y_1 < X_3 < x_2]P[y_2 < Y_3]$$

$$= [F(x_2) - F(y_1)]\overline{F}(y_2)$$

$$n_4 = P[x_1 < y_1 < x_2 < X_3 < y_2 < Y_3]$$

$$= P[x_2 < X_3 < y_2 < Y_3]$$

$$= P[x_2 < X_3 < y_2]P[y_2 < Y_3]$$

$$= [F(y_2) - F(x_2)]\overline{F}(y_2)$$

$$n_5 = P[X_3 < x_1 < y_1 < x_2 < Y_3 < y_2]$$

$$= P[X_3 < x_1]P[x_2 < Y_3 < y_2]$$

$$= F(x_1)[F(y_2) - F(x_2)]$$

$$n_6 = P[x_1 < X_3 < y_1 < x_2 < Y_3 < y_2]$$

$$= P[x_1 < X_3 < y_1]P[x_2 < Y_3 < y_2]$$

$$= [F(y_1) - F(x_1)][F(y_2) - F(x_2)]$$

$$n_7 = P[x_1 < y_1 < X_3 < x_2 < Y_3 < y_2]$$

$$= P[y_1 < X_3 < x_2]P[x_2 < Y_3 < y_2]$$

$$= [F(x_2) - F(y_1)][F(y_2) - F(x_2)]$$

$$n_8 = P[x_1 < y_1 < x_2 < Y_3 < y_2 < X_3]$$

$$= P[x_2 < Y_3 < y_2 < X_3]$$

$$= P[x_2 < Y_3 < y_2]P[y_2 < X_3]$$

$$= [F(y_2) - F(x_2)]\overline{F}(y_2)$$

$$n_9 = P[X_3 < x_1 < y_1 < Y_3 < x_2 < y_2]$$

$$= P[X_3 < x_1]P[y_1 < Y_3 < x_2]$$

$$= F(x_1)[F(x_2) - F(y_1)]$$

$$n_{10} = P[x_1 < X_3 < y_1 < Y_3 < x_2 < y_2]$$

$$= P[x_1 < X_3 < y_1 < Y_3 < x_2]$$

$$= P[x_1 < X_3 < y_1]P[y_1 < Y_3 < x_2]$$

$$= [F(y_1) - F(x_1)][F(x_2) - F(y_1)]$$

$$n_{11} = P[X_3 < x_1 < Y_3 < y_1 < x_2 < y_2]$$

$$= P[X_3 < x_1]P[x_1 < Y_3 < y_1]$$

$$= F(x_1)[F(y_1) - F(x_1)]$$

$$n_{12} = P[Y_3 < x_1 < X_3 < y_1 < x_2 < y_2]$$

$$= P[Y_3 < x_1]P[x_1 < X_3 < y_1]$$

$$= F(x_1)[F(y_1) - F(x_1)]$$

$$n_{13} = P[X_3 < Y_3 < x_1 < y_1 < x_2 < y_2]$$

$$= P[X_3 < Y_3 < x_1]$$

$$= \frac{F^2(x_1)}{2}$$

$$n_{14} = P[Y_3 < X_3 < x_1 < y_1 < x_2 < y_2]$$

$$= P[Y_3 < X_3 < x_1]$$

$$= \frac{F^2(x_1)}{2}$$

$$n_{15} = P[x_1 < X_3 < Y_3 < y_1 < x_2 < y_2]$$

$$= P[x_1 < X_3 < Y_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_1)]^2}{2}$$

$$n_{16} = P[x_1 < Y_3 < X_3 < y_1 < x_2 < y_2]$$

$$= P[x_1 < Y_3 < X_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_1)]^2}{2}$$

$$n_{17} = P[x_1 < y_1 < X_3 < Y_3 < x_2 < y_2]$$

$$= P[y_1 < X_3 < Y_3 < x_2]$$

$$= \frac{[F(x_2) - F(y_1)]^2}{2}$$

$$n_{18} = P[x_1 < y_1 < x_2 < X_3 < Y_3 < y_2]$$

$$= P[x_2 < X_3 < Y_3 < y_2]$$

$$= \frac{[F(y_2) - F(x_2)]^2}{2}$$

$$n_{19} = P[x_1 < y_1 < x_2 < Y_3 < X_3 < y_2]$$

$$= P[x_2 < Y_3 < X_3 < y_2]$$

$$= \frac{[F(y_2) - F(x_2)]^2}{2}$$

$$n_{20} = P[x_1 < y_1 < x_2 < y_2 < X_3 < Y_3]$$

$$= P[y_2 < X_3 < Y_3]$$

$$= \frac{\overline{F}^2(y_2)}{2}$$

$$n_{21} = P[x_1 < y_1 < x_2 < y_2 < Y_3 < X_3]$$

$$= P[y_2 < Y_3 < X_3]$$

$$= \frac{\overline{F}^2(y_2)}{2}$$

Adding 21 terms we have for case $x_1 < y_1 < x_2 < y_2$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = n_1 + n_2 + \ldots\ldots + n_{21}$$

$$= 1 - F(x_2) + \frac{1}{2}F^2(y_1) + \frac{1}{2}F^2(x_2).$$

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_2}\int_{-\infty}^{x_2}\int_{-\infty}^{y_1} P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(y_1)dF(x_2)dF(y_2)$$

$$-\infty < x_1 < y_1 < x_2 < y_2 < \infty$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{y_2}\int_{-\infty}^{x_2}\int_{-\infty}^{y_1}\left(1 - F(x_2) + \frac{1}{2}F^2(y_1) + \frac{1}{2}F^2(x_2)\right)^2 dF(x_1)dF(y_1)dF(x_2)dF(y_2)$$

$$-\infty < x_1 < y_1 < x_2 < y_2 < \infty$$

$$= \frac{139}{6720}.$$

For the case $x_1 < y_2 < x_2 < y_1$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = o_1 + o_2 + \ldots\ldots\ldots\ldots + o_{21}$$

where

$o_1 = P[X_3 < x_1 < y_2 < x_2 < y_1 < Y_3]$ $\qquad$ $o_2 = P[x_1 < X_3 < y_2 < x_2 < y_1 < Y_3]$

$o_3 = P[x_1 < y_2 < X_3 < x_2 < y_1 < Y_3]$ $\qquad$ $o_4 = P[x_1 < y_2 < x_2 < X_3 < y_1 < Y_3]$

$o_5 = P[X_3 < x_1 < y_2 < x_2 < Y_3 < y_1]$ $\qquad$ $o_6 = P[x_1 < X_3 < y_2 < x_2 < Y_3 < y_1]$

$o_7 = P[x_1 < y_2 < X_3 < x_2 < Y_3 < y_1]$

$o_8 = P[x_1 < y_2 < x_2 < Y_3 < y_1 < X_3]$

$o_9 = P[X_3 < x_1 < y_2 < Y_3 < x_2 < y_1]$

$o_{10} = P[x_1 < X_3 < y_2 < Y_3 < x_2 < y_1]$

$o_{11} = P[X_3 < x_1 < Y_3 < y_2 < x_2 < y_1]$

$o_{12} = P[Y_3 < x_1 < X_3 < y_2 < x_2 < y_1]$

$o_{13} = P[X_3 < Y_3 < x_1 < y_2 < x_2 < y_1]$

$o_{14} = P[Y_3 < X_3 < x_1 < y_2 < x_2 < y_1]$

$o_{15} = P[x_1 < X_3 < Y_3 < y_2 < x_2 < y_1]$

$o_{16} = P[x_1 < Y_3 < X_3 < y_2 < x_2 < y_1]$

$o_{17} = P[x_1 < y_2 < X_3 < Y_3 < x_2 < y_1]$

$o_{18} = P[x_1 < y_2 < x_2 < X_3 < Y_3 < y_1]$

$o_{19} = P[x_1 < y_2 < x_2 < Y_3 < X_3 < y_1]$

$o_{20} = P[x_1 < y_2 < x_2 < y_1 < X_3 < Y_3]$

$o_{21} = P[x_1 < y_2 < x_2 < y_1 < Y_3 < X_3]$.

Now consider

$$o_1 = P[X_3 < x_1 < y_2 < x_2 < y_1 < Y_3]$$
$$= P[X_3 < x_1]P[y_1 < Y_3]$$
$$= F(x_1)\overline{F}(y_1)$$

$$o_2 = P[x_1 < X_3 < y_2 < x_2 < y_1 < Y_3]$$
$$= P[x_1 < X_3 < y_2]P[y_1 < Y_3]$$
$$= [F(y_2) - F(x_1)]\overline{F}(y_1)$$

$$o_3 = P[x_1 < y_2 < X_3 < x_2 < y_1 < Y_3]$$
$$= P[y_2 < X_3 < x_2]P[y_1 < Y_3]$$
$$= [F(x_2) - F(y_2)]\overline{F}(y_1)$$

$$o_4 = P[x_1 < y_2 < x_2 < X_3 < y_1 < Y_3]$$
$$= P[x_2 < X_3 < y_1 < Y_3]$$
$$= P[x_2 < X_3 < y_1]P[y_1 < Y_3]$$

$$= [F(y_1) - F(x_2)]\overline{F}(y_1)$$

$$o_5 = P[X_3 < x_1 < y_2 < x_2 < Y_3 < y_1]$$

$$= P[X_3 < x_1]P[x_2 < Y_3 < y_1]$$

$$= F(x_1)[F(y_1) - F(x_2)]$$

$$o_6 = P[x_1 < X_3 < y_2 < x_2 < Y_3 < y_1]$$

$$= P[x_1 < X_3 < y_2]P[x_2 < Y_3 < y_1]$$

$$= [F(y_2) - F(x_1)][F(y_1) - F(x_2)]$$

$$o_7 = P[x_1 < y_2 < X_3 < x_2 < Y_3 < y_1]$$

$$= P[y_2 < X_3 < x_2]P[x_2 < Y_3 < y_1]$$

$$= [F(x_2) - F(y_2)][F(y_1) - F(x_2)]$$

$$o_8 = P[x_1 < y_2 < x_2 < Y_3 < y_1 < X_3]$$

$$= P[x_2 < Y_3 < y_1 < X_3]$$

$$= P[x_2 < Y_3 < y_1]P[y_1 < X_3]$$

$$= [F(y_1) - F(x_2)]\overline{F}(y_1)$$

$$o_9 = P[X_3 < x_1 < y_2 < Y_3 < x_2 < y_1]$$

$$= P[X_3 < x_1]P[y_2 < Y_3 < x_2]$$

$$= F(x_1)[F(x_2) - F(y_2)]$$

$$o_{10} = P[x_1 < X_3 < y_2 < Y_3 < x_2 < y_1]$$

$$= P[x_1 < X_3 < y_2 < Y_3 < x_2]$$

$$= P[x_1 < X_3 < y_2]P[y_2 < Y_3 < x_2]$$

$$= [F(y_2) - F(x_1)][F(x_2) - F(y_2)]$$

$$o_{11} = P[X_3 < x_1 < Y_3 < y_2 < x_2 < y_1]$$

$$= P[X_3 < x_1]P[x_1 < Y_3 < y_2]$$

$$= F(x_1)[F(y_2) - F(x_1)]$$

$$o_{12} = P[Y_3 < x_1 < X_3 < y_2 < x_2 < y_1]$$

$$= P[Y_3 < x_1]P[x_1 < X_3 < y_2]$$

$$= F(x_1)[F(y_2) - F(x_1)]$$

$$o_{13} = P[X_3 < Y_3 < x_1 < y_2 < x_2 < y_1]$$

$$= P[X_3 < Y_3 < x_1]$$

$$= \frac{F^2(x_1)}{2}$$

$$o_{14} = P[Y_3 < X_3 < x_1 < y_2 < x_2 < y_1]$$

$$= P[Y_3 < X_3 < x_1]$$

$$= \frac{F^2(x_1)}{2}$$

$$o_{15} = P[x_1 < X_3 < Y_3 < y_2 < x_2 < y_1]$$

$$= P[x_1 < X_3 < Y_3 < y_2]$$

$$= \frac{[F(y_2) - F(x_1)]^2}{2}$$

$$o_{16} = P[x_1 < Y_3 < X_3 < y_2 < x_2 < y_1]$$

$$= P[x_1 < Y_3 < X_3 < y_2]$$

$$= \frac{[F(y_2) - F(x_1)]^2}{2}$$

$$o_{17} = P[x_1 < y_2 < X_3 < Y_3 < x_2 < y_1]$$

$$= P[y_2 < X_3 < Y_3 < x_2]$$

$$= \frac{[F(x_2) - F(y_2)]^2}{2}$$

$$o_{18} = P[x_1 < y_2 < x_2 < X_3 < Y_3 < y_1]$$

$$= P[x_2 < X_3 < Y_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_2)]^2}{2}$$

$$o_{19} = P[x_1 < y_2 < x_2 < Y_3 < X_3 < y_1]$$

$$= P[x_2 < Y_3 < X_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_2)]^2}{2}$$

$$o_{20} = P[x_1 < y_2 < x_2 < y_1 < X_3 < Y_3]$$

$$= P[y_1 < X_3 < Y_3]$$

$$= \frac{\overline{F}^2(y_1)}{2}$$

$$o_{21} = P[x_1 < y_2 < x_2 < y_1 < Y_3 < X_3]$$

$$= P[y_1 < Y_3 < X_3]$$

$$= \frac{\overline{F}^2(y_1)}{2}.$$

Adding 21 terms we have the case for $x_1 < y_2 < x_2 < y_1$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = o_1 + o_2 + \ldots\ldots\ldots\ldots + o_{21}$$

$$= 1 - F(x_2) + \frac{1}{2}F^2(y_2) + \frac{1}{2}F^2(x_2).$$

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_1}\int_{-\infty}^{x_2}\int_{-\infty}^{y_2} P^2[Med(x_1,x_2,X_3) < Med(y_1,y_2,Y_3)]dF(x_1)dF(y_2)dF(x_2)dF(y_1)$$

$$-\infty < x_1 < y_2 < x_2 < y_1 < \infty$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{y_1}\int_{-\infty}^{x_2}\int_{-\infty}^{y_2}\left(1 - F(x_2) + \frac{1}{2}F^2(y_2) + \frac{1}{2}F^2(x_2)\right)^2 dF(x_1)dF(y_2)dF(x_2)dF(y_1)$$

$$-\infty < x_1 < y_2 < x_2 < y_1 < \infty$$

$$= \frac{139}{6720}.$$

When $x_1 < y_1 < y_2 < x_2$, we have

$$P[Med(x_1,x_2,X_3) < Med(y_1,y_2,Y_3)] = p_1 + p_2 + \ldots\ldots + p_{15}$$

where

$$p_1 = P[X_3 < x_1 < y_1 < y_2 < x_2 < Y_3] \qquad p_2 = P[x_1 < X_3 < y_1 < y_2 < x_2 < Y_3]$$

$$p_3 = P[x_1 < y_1 < X_3 < y_2 < x_2 < Y_3] \qquad p_4 = P[X_3 < x_1 < y_1 < y_2 < Y_3 < x_2]$$

$$p_5 = P[x_1 < X_3 < y_1 < y_2 < Y_3 < x_2] \qquad p_6 = P[x_1 < y_1 < X_3 < y_2 < Y_3 < x_2]$$

$$p_7 = P[X_3 < x_1 < y_1 < Y_3 < y_2 < x_2] \qquad p_8 = P[x_1 < X_3 < y_1 < Y_3 < y_2 < x_2]$$

$$p_9 = P[x_1 < y_1 < X_3 < Y_3 < y_2 < x_2] \qquad p_{10} = P[X_3 < x_1 < Y_3 < y_1 < y_2 < x_2]$$

$$p_{11} = P[x_1 < X_3 < Y_3 < y_1 < y_2 < x_2] \qquad p_{12} = P[x_1 < Y_3 < X_3 < y_1 < y_2 < x_2]$$

$$p_{13} = P[Y_3 < X_3 < x_1 < y_1 < y_2 < x_2] \qquad p_{14} = P[Y_3 < x_1 < X_3 < y_1 < y_2 < x_2]$$

$$p_{15} = P[X_3 < Y_3 < x_1 < y_1 < y_2 < x_2].$$

Now evaluating the terms, we have

$$p_1 = P[X_3 < x_1 < y_1 < y_2 < x_2 < Y_3]$$

$$= P[X_3 < x_1]P[x_2 < Y_3]$$

$$= F(x_1)\overline{F}(x_2)$$

$$p_2 = P[x_1 < X_3 < y_1 < y_2 < x_2 < Y_3]$$

$$= P[x_1 < X_3 < y_1]P[x_2 < Y_3]$$

$$= [F(y_1) - F(x_1)]\overline{F}(x_2)$$

$$p_3 = P[x_1 < y_1 < X_3 < y_2 < x_2 < Y_3]$$

$$= P[y_1 < X_3 < y_2]P[x_2 < Y_3]$$

$$= [F(y_2) - F(y_1)]\overline{F}(x_2)$$

$$p_4 = P[X_3 < x_1 < y_1 < y_2 < Y_3 < x_2]$$

$$= P[X_3 < x_1]P[y_2 < Y_3 < x_2]$$

$$= F(x_1)[F(x_2) - F(y_2)]$$

$$p_5 = P[x_1 < X_3 < y_1 < y_2 < Y_3 < x_2]$$

$$= P[x_1 < X_3 < y_1]P[y_2 < Y_3 < x_2]$$

$$= [F(y_1) - F(x_1)][F(x_2) - F(y_2)]$$

$$p_6 = P[x_1 < y_1 < X_3 < y_2 < Y_3 < x_2]$$

$$= P[y_1 < X_3 < y_2]P[y_2 < Y_3 < x_2]$$

$$= [F(y_2) - F(y_1)][F(x_2) - F(y_2)]$$

$$p_7 = P[X_3 < x_1 < y_1 < Y_3 < y_2 < x_2]$$

$$= P[X_3 < x_1]P[y_1 < Y_3 < y_2]$$

$$= F(x_1)[F(y_2) - F(y_1)]$$

$$p_8 = P[x_1 < X_3 < y_1 < Y_3 < y_2 < x_2]$$

$$= P[x_1 < X_3 < y_1]P[y_1 < Y_3 < y_2]$$

$$= [F(y_1) - F(x_1)][F(y_2) - F(y_1)]$$

$$p_9 = P[x_1 < y_1 < X_3 < Y_3 < y_2 < x_2]$$

$$= P[y_1 < X_3 < Y_3 < y_2]$$

$$= \frac{[F(y_2) - F(y_1)]^2}{2}$$

$$p_{10} = P[X_3 < x_1 < Y_3 < y_1 < y_2 < x_2]$$

$$= P[X_3 < x_1 < Y_3 < y_1]$$

$$= P[X_3 < x_1]P[x_1 < Y_3 < y_1]$$

$$= F(x_1)[F(y_1) - F(x_1)]$$

$$p_{11} = P[x_1 < X_3 < Y_3 < y_1 < y_2 < x_2]$$

$$= P[x_1 < X_3 < Y_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_1)]^2}{2}$$

$$p_{12} = P[x_1 < Y_3 < X_3 < y_1 < y_2 < x_2]$$

$$= P[x_1 < Y_3 < X_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_1)]^2}{2}$$

$$p_{13} = P[Y_3 < X_3 < x_1 < y_1 < y_2 < x_2]$$

$$= P[Y_3 < X_3 < x_1]$$

$$= \frac{F^2(x_1)}{2}$$

$$p_{14} = P[Y_3 < x_1 < X_3 < y_1 < y_2 < x_2]$$

$$= P[Y_3 < x_1 < X_3 < y_1]$$

$$= P[Y_3 < x_1]P[x_1 < X_3 < y_1]$$

$$= F(x_1)[F(y_1) - F(x_1)]$$

$$p_{15} = P[X_3 < Y_3 < x_1 < y_1 < y_2 < x_2]$$

$$= P[X_3 < Y_3 < x_1]$$

$$= \frac{F^2(x_1)}{2}.$$

Adding 15 terms we have for $x_1 < y_1 < y_2 < x_2$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = p_1 + p_2 + ... + p_{15}$$

$$= F(y_2) + \frac{1}{2}F^2(y_1) - \frac{1}{2}F^2(y_2).$$

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{x_2}\int_{-\infty}^{y_2}\int_{-\infty}^{y_1} P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(y_1)dF(y_2)dF(x_2)$$

$$-\infty < x_1 < y_1 < y_2 < x_2 < \infty$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{x_2}\int_{-\infty}^{y_2}\int_{-\infty}^{y_1}\left(F(y_2) + \frac{1}{2}F^2(y_1) - \frac{1}{2}F^2(y_2)\right)^2 dF(x_1)dF(y_1)dF(y_2)dF(x_2)$$

$$-\infty < x_1 < y_1 < y_2 < x_2 < \infty$$

$$= \frac{11}{960}.$$

For the case $x_1 < y_2 < y_1 < x_2$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = q_1 + q_2 + ... + q_{15}$$

where

$q_1 = P[X_3 < x_1 < y_2 < y_1 < x_2 < Y_3]$ $\quad$ $q_2 = P[x_1 < X_3 < y_2 < y_1 < x_2 < Y_3]$

$q_3 = P[x_1 < y_2 < X_3 < y_1 < x_2 < Y_3]$ $\quad$ $q_4 = P[X_3 < x_1 < y_2 < y_1 < Y_3 < x_2]$

$q_5 = P[x_1 < X_3 < y_2 < y_1 < Y_3 < x_2]$ $\quad$ $q_6 = P[x_1 < y_2 < X_3 < y_1 < Y_3 < x_2]$

$q_7 = P[X_3 < x_1 < y_2 < Y_3 < y_1 < x_2]$ $\quad$ $q_8 = P[x_1 < X_3 < y_2 < Y_3 < y_1 < x_2]$

$q_9 = P[x_1 < y_2 < X_3 < Y_3 < y_1 < x_2]$ $\quad$ $q_{10} = P[X_3 < x_1 < Y_3 < y_2 < y_1 < x_2]$

$q_{11} = P[x_1 < X_3 < Y_3 < y_2 < y_1 < x_2]$ $\quad$ $q_{12} = P[x_1 < Y_3 < X_3 < y_2 < y_1 < x_2]$

$q_{13} = P[Y_3 < X_3 < x_1 < y_2 < y_1 < x_2]$ $\quad$ $q_{14} = P[Y_3 < x_1 < X_3 < y_2 < y_1 < x_2]$

$q_{15} = P[X_3 < Y_3 < x_1 < y_2 < y_1 < x_2]$.

Further

$$q_1 = P[X_3 < x_1 < y_2 < y_1 < x_2 < Y_3]$$
$$= P[X_3 < x_1]P[x_2 < Y_3]$$
$$= F(x_1)\overline{F}(x_2)$$

$$q_2 = P[x_1 < X_3 < y_2 < y_1 < x_2 < Y_3]$$
$$= P[x_1 < X_3 < y_2]P[x_2 < Y_3]$$
$$= [F(y_2) - F(x_1)]\overline{F}(x_2)$$

$$q_3 = P[x_1 < y_2 < X_3 < y_1 < x_2 < Y_3]$$
$$= P[y_2 < X_3 < y_1]P[x_2 < Y_3]$$
$$= [F(y_1) - F(y_2)]\overline{F}(x_2)$$

$$q_4 = P[X_3 < x_1 < y_2 < y_1 < Y_3 < x_2]$$
$$= P[X_3 < x_1]P[y_1 < Y_3 < x_2]$$
$$= F(x_1)[F(x_2) - F(y_1)]$$

$$q_5 = P[x_1 < X_3 < y_2 < y_1 < Y_3 < x_2]$$
$$= P[x_1 < X_3 < y_2]P[y_1 < Y_3 < x_2]$$
$$= [F(y_2) - F(x_1)][F(x_2) - F(y_1)]$$

$$q_6 = P[x_1 < y_2 < X_3 < y_1 < Y_3 < x_2]$$
$$= P[y_2 < X_3 < y_1]P[y_1 < Y_3 < x_2]$$
$$= [F(y_1) - F(y_2)][F(x_2) - F(y_1)]$$

$$q_7 = P[X_3 < x_1 < y_2 < Y_3 < y_1 < x_2]$$

$$= P[X_3 < x_1]P[y_2 < Y_3 < y_1]$$

$$= F(x_1)[F(y_1) - F(y_2)]$$

$$q_8 = P[x_1 < X_3 < y_2 < Y_3 < y_1 < x_2]$$

$$= P[x_1 < X_3 < y_2]P[y_2 < Y_3 < y_1]$$

$$= [F(y_2) - F(x_1)][F(y_1) - F(y_2)]$$

$$q_9 = P[x_1 < y_2 < X_3 < Y_3 < y_1 < x_2]$$

$$= P[y_2 < X_3 < Y_3 < y_1]$$

$$= \frac{[F(y_1) - F(y_2)]^2}{2}$$

$$q_{10} = P[X_3 < x_1 < Y_3 < y_2 < y_1 < x_2]$$

$$= P[X_3 < x_1 < Y_3 < y_2]$$

$$= P[X_3 < x_1]P[x_1 < Y_3 < y_2]$$

$$= F(x_1)[F(y_2) - F(x_1)]$$

$$q_{11} = P[x_1 < X_3 < Y_3 < y_2 < y_1 < x_2]$$

$$= P[x_1 < X_3 < Y_3 < y_2]$$

$$= \frac{[F(y_2) - F(x_1)]^2}{2}$$

$$q_{12} = P[x_1 < Y_3 < X_3 < y_2 < y_1 < x_2]$$

$$= P[x_1 < Y_3 < X_3 < y_2]$$

$$= \frac{[F(y_1) - F(x_1)]^2}{2}$$

$$q_{13} = P[Y_3 < X_3 < x_1 < y_2 < y_1 < x_2]$$

$$= P[Y_3 < X_3 < x_1]$$

$$= \frac{F^2(x_1)}{2}$$

$$q_{14} = P[Y_3 < x_1 < X_3 < y_2 < y_1 < x_2]$$

$$= P[Y_3 < x_1 < X_3 < y_2]$$

$$= P[Y_3 < x_1]P[x_1 < X_3 < y_2]$$

$$= F(x_1)[F(y_2) - F(x_1)]$$

$$q_{15} = P[X_3 < Y_3 < x_1 < y_2 < y_1 < x_2]$$

$$= P[X_3 < Y_3 < x_1]$$

$$= \frac{F^2(x_1)}{2}$$

Adding 15 terms we have for $x_1 < y_2 < y_1 < x_2$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = q_1 + q_2 + ... + q_{15}$$

$$= F(y_1) + \frac{1}{2}F^2(y_2) - \frac{1}{2}F^2(y_1).$$

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{x_2}\int_{-\infty}^{y_1}\int_{-\infty}^{y_2} P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(y_2)dF(y_1)dF(x_2)$$

$$-\infty < x_1 < y_2 < y_1 < x_2 < \infty$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{x_2}\int_{-\infty}^{y_1}\int_{-\infty}^{y_2}\left(F(y_1) + \frac{1}{2}F^2(y_2) - \frac{1}{2}F^2(y_1)\right)^2 dF(x_1)dF(y_2)dF(y_1)dF(x_2)$$

$$-\infty < x_1 < y_2 < y_1 < x_2 < \infty$$

$$= \frac{11}{960}.$$

Now for $x_2 < x_1 < y_1 < y_2$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = r_1 + r_2 + ... + r_{30}$$

where

$r_1 = P[X_3 < x_2 < x_1 < y_1 < y_2 < Y_3]$ $r_2 = P[x_2 < X_3 < x_1 < y_1 < y_2 < Y_3]$

$r_3 = P[x_2 < x_1 < X_3 < y_1 < y_2 < Y_3]$ $r_4 = P[x_2 < x_1 < y_1 < X_3 < y_2 < Y_3]$

$r_5 = P[x_2 < x_1 < y_1 < y_2 < X_3 < Y_3]$ $r_6 = P[x_2 < x_1 < y_1 < y_2 < Y_3 < X_3]$

$r_7 = P[X_3 < x_2 < x_1 < y_1 < Y_3 < y_2]$ $r_8 = P[x_2 < X_3 < x_1 < y_1 < Y_3 < y_2]$

$r_9 = P[x_2 < x_1 < X_3 < y_1 < Y_3 < y_2]$ $r_{10} = P[x_2 < x_1 < y_1 < X_3 < Y_3 < y_2]$

$r_{11} = P[x_2 < x_1 < y_1 < Y_3 < X_3 < y_2]$ $r_{12} = P[x_2 < x_1 < y_1 < Y_3 < y_2 < X_3]$

$r_{13} = P[X_3 < x_2 < x_1 < Y_3 < y_1 < y_2]$ $r_{14} = P[x_2 < X_3 < x_1 < Y_3 < y_1 < y_2]$

$r_{15} = P[x_2 < x_1 < X_3 < Y_3 < y_1 < y_2]$ $r_{16} = P[x_2 < x_1 < Y_3 < X_3 < y_1 < y_2]$

$r_{17} = P[x_2 < x_1 < Y_3 < y_1 < X_3 < y_2]$ $r_{18} = P[x_2 < x_1 < Y_3 < y_1 < y_2 < X_3]$

$r_{19} = P[X_3 < x_2 < Y_3 < x_1 < y_1 < y_2]$ $r_{20} = P[x_2 < X_3 < Y_3 < x_1 < y_1 < y_2]$

$r_{21} = P[x_2 < Y_3 < X_3 < x_1 < y_1 < y_2]$ $r_{22} = P[x_2 < Y_3 < x_1 < X_3 < y_1 < y_2]$

$r_{23} = P[x_2 < Y_3 < x_1 < y_1 < X_3 < y_2]$ $r_{24} = P[x_2 < Y_3 < x_1 < y_1 < y_2 < X_3]$

$r_{25} = P[X_3 < Y_3 < x_2 < x_1 < y_1 < y_2]$ $r_{26} = P[Y_3 < X_3 < x_2 < x_1 < y_1 < y_2]$

$r_{27} = P[Y_3 < x_2 < X_3 < x_1 < y_1 < y_2]$ $r_{28} = P[Y_3 < x_2 < x_1 < X_3 < y_1 < y_2]$

$r_{29} = P[Y_3 < x_2 < x_1 < y_1 < X_3 < y_2]$ $r_{30} = P[Y_3 < x_2 < x_1 < y_1 < y_2 < X_3]$

Further

$$r_1 = P[X_3 < x_2 < x_1 < y_1 < y_2 < Y_3]$$

$$= P[X_3 < x_2]P[y_2 < Y_3]$$

$$= F(x_2)\overline{F}(y_2)$$

$$r_2 = P[x_2 < X_3 < x_1 < y_1 < y_2 < Y_3]$$

$$= P[x_2 < X_3 < x_1]P[y_2 < Y_3]$$

$$= [F(x_1) - F(x_2)]\overline{F}(y_2)$$

$$r_3 = P[x_2 < x_1 < X_3 < y_1 < y_2 < Y_3]$$

$$= P[x_1 < X_3 < y_1]P[y_2 < Y_3]$$

$$= [F(y_1) - F(x_1)]\overline{F}(y_2)$$

$$r_4 = P[x_2 < x_1 < y_1 < X_3 < y_2 < Y_3]$$

$$= P[y_1 < X_3 < y_2]P[y_2 < Y_3]$$

$$= [F(y_2) - F(y_1)]\overline{F}(y_2)$$

$$r_5 = P[x_2 < x_1 < y_1 < y_2 < X_3 < Y_3]$$

$$= P[y_2 < X_3 < Y_3]$$

$$= \frac{\overline{F}^2(y_2)}{2}$$

$$r_6 = P[x_2 < x_1 < y_1 < y_2 < Y_3 < X_3]$$

$$= P[y_2 < Y_3 < X_3]$$

$$= \frac{\overline{F}^2(y_2)}{2}$$

$$r_7 = P[X_3 < x_2 < x_1 < y_1 < Y_3 < y_2]$$

$$= P[X_3 < x_2]P[y_1 < Y_3 < y_2]$$

$$= F(x_2)[F(y_2) - F(y_1)]$$

$$r_8 = P[x_2 < X_3 < x_1 < y_1 < Y_3 < y_2]$$

$$= P[x_2 < X_3 < x_1]P[y_1 < Y_3 < y_2]$$

$$= [F(x_1) - F(x_2)][F(y_2) - F(y_1)]$$

$$r_9 = P[x_2 < x_1 < X_3 < y_1 < Y_3 < y_2]$$

$$= P[x_1 < X_3 < y_1]P[y_1 < Y_3 < y_2]$$

$$= [F(y_1) - F(x_1)][F(y_2) - F(y_1)]$$

$$r_{10} = P[x_2 < x_1 < y_1 < X_3 < Y_3 < y_2]$$

$$= P[y_1 < X_3 < Y_3 < y_2]$$

$$= \frac{[F(y_2) - F(y_1)]^2}{2}$$

$$r_{11} = P[x_2 < x_1 < y_1 < Y_3 < X_3 < y_2]$$

$$= P[y_1 < Y_3 < X_3 < y_2]$$

$$= \frac{[F(y_2) - F(y_1)]^2}{2}$$

$$r_{12} = P[x_2 < x_1 < y_1 < Y_3 < y_2 < X_3]$$

$$= P[y_1 < Y_3 < y_2]P[y_2 < X_3]$$

$$= [F(y_2) - F(y_1)]\overline{F}(y_2)$$

$$r_{13} = P[X_3 < x_2 < x_1 < Y_3 < y_1 < y_2]$$

$$= P[X_3 < x_2]P[x_1 < Y_3 < y_1]$$

$$= F(x_2)[F(y_1) - F(x_1)]$$

$$r_{14} = P[x_2 < X_3 < x_1 < Y_3 < y_1 < y_2]$$

$$= P[x_2 < X_3 < x_1]P[x_1 < Y_3 < y_1]$$

$$= [F(x_1) - F(x_2)][F(y_1) - F(x_1)]$$

$$r_{15} = P[x_2 < x_1 < X_3 < Y_3 < y_1 < y_2]$$

$$= P[x_1 < X_3 < Y_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_1)]^2}{2}$$

$$r_{16} = P[x_2 < x_1 < Y_3 < X_3 < y_1 < y_2]$$

$$= P[x_1 < Y_3 < X_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_1)]^2}{2}$$

$$r_{17} = P[x_2 < x_1 < Y_3 < y_1 < X_3 < y_2]$$

$$= P[x_1 < Y_3 < y_1]P[y_1 < X_3 < y_2]$$

$$= [F(y_1) - F(x_1)][F(y_2) - F(y_1)]$$

$$r_{18} = P[x_2 < x_1 < Y_3 < y_1 < y_2 < X_3]$$

$$= P[x_1 < Y_3 < y_1]P[y_2 < X_3]$$

$$= [F(y_1) - F(x_1)]\overline{F}(y_2)$$

$$r_{19} = P[X_3 < x_2 < Y_3 < x_1 < y_1 < y_2]$$

$$= P[X_3 < x_2]P[x_2 < Y_3 < x_1]$$

$$= F(x_2)[F(x_1) - F(x_2)]$$

$$r_{20} = P[x_2 < X_3 < Y_3 < x_1 < y_1 < y_2]$$

$$= P[x_2 < X_3 < Y_3 < x_1]$$

$$= \frac{[F(x_1) - F(x_2)]^2}{2}$$

$$F_{21} = P[x_2 < Y_3 < X_3 < x_1 < y_1 < y_2]$$

$$= P[x_2 < Y_3 < X_3 < x_1]$$

$$= \frac{[F(x_1) - F(x_2)]^2}{2}$$

$$r_{22} = P[x_2 < Y_3 < x_1 < X_3 < y_1 < y_2]$$

$$= P[x_2 < Y_3 < x_1]P[x_1 < X_3 < y_1]$$

$$= [F(x_1) - F(x_2)][F(y_1) - F(x_1)]$$

$$r_{23} = P[x_2 < Y_3 < x_1 < y_1 < X_3 < y_2]$$

$$= P[x_2 < Y_3 < x_1]P[y_1 < X_3 < y_2]$$

$$= [F(x_1) - F(x_2)][F(y_2) - F(y_1)]$$

$$r_{24} = P[x_2 < Y_3 < x_1 < y_1 < y_2 < X_3]$$

$$= P[x_2 < Y_3 < x_1]P[y_2 < X_3]$$

$$= [F(x_1) - F(x_2)]\overline{F}(y_2)$$

$$r_{25} = P[X_3 < Y_3 < x_2 < x_1 < y_1 < y_2]$$

$$= P[X_3 < Y_3 < x_2]$$

$$= \frac{F^2(x_2)}{2}$$

$$r_{26} = P[Y_3 < X_3 < x_2 < x_1 < y_1 < y_2]$$

$$= P[Y_3 < X_3 < x_2]$$

$$= \frac{F^2(x_2)}{2}$$

$$r_{27} = P[Y_3 < x_2 < X_3 < x_1 < y_1 < y_2]$$

$$= P[Y_3 < x_2 < X_3 < x_1]$$

$$= P[Y_3 < x_2]P[x_2 < X_3 < x_1]$$

$$= F(x_2)[F(x_1) - F(x_2)]$$

$$r_{28} = P[Y_3 < x_2 < x_1 < X_3 < y_1 < y_2]$$

$$= P[Y_3 < x_2]P[x_1 < X_3 < y_1]$$

$$= F(x_2)[F(y_1) - F(x_1)]$$

$$r_{29} = P[Y_3 < x_2 < x_1 < y_1 < X_3 < y_2]$$

$$= P[Y_3 < x_2]P[y_1 < X_3 < y_2]$$

$$= F(x_2)[F(y_2) - F(y_1)]$$

$$r_{30} = P[Y_3 < x_2 < x_1 < y_1 < y_2 < X_3]$$

$$= P[Y_3 < x_2]P[y_2 < X_3]$$

$$= F(x_2)\bar{F}(y_2).$$

Adding the 30 terms we have for

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = r_1 + r_2 + ... + r_{30}$$

$$= r^* \text{ say} \qquad \text{When } x_2 < x_1 < y_1 < y_2.$$

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_2}\int_{-\infty}^{y_1}\int_{-\infty}^{x_1} P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_2)dF(x_1)dF(y_1)dF(y_2)$$

$$-\infty < x_2 < x_1 < y_1 < y_2 < \infty$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{y_2}\int_{-\infty}^{y_1}\int_{-\infty}^{x_1} r^{*2} dF(x_2)dF(x_1)dF(y_1)dF(y_2)$$

$$-\infty < x_2 < x_1 < y_1 < y_2 < \infty$$

$$= \frac{1}{24}.$$

In the eighth case i.e. when $x_2 < x_1 < y_2 < y_1$

$$P[Med(x_1,x_2,X_3)< Med(y_1,y_2,Y_3)]= s_1 + s_2 + ... + s_{30}$$

$$s_1 = P[X_3 < x_2 < x_1 < y_2 < y_1 < Y_3] \qquad s_2 = P[x_2 < X_3 < x_1 < y_2 < y_1 < Y_3]$$

$$s_3 = P[x_2 < x_1 < X_3 < y_2 < y_1 < Y_3] \qquad s_4 = P[x_2 < x_1 < y_2 < X_3 < y_1 < Y_3]$$

$$s_5 = P[x_2 < x_1 < y_2 < y_1 < X_3 < Y_3] \qquad s_6 = P[x_2 < x_1 < y_2 < y_1 < Y_3 < X_3]$$

$$s_7 = P[X_3 < x_2 < x_1 < y_2 < Y_3 < y_1] \qquad s_8 = P[x_2 < X_3 < x_1 < y_2 < Y_3 < y_1]$$

$$s_9 = P[x_2 < x_1 < X_3 < y_2 < Y_3 < y_1] \qquad s_{10} = P[x_2 < x_1 < y_2 < X_3 < Y_3 < y_1]$$

$$s_{11} = P[x_2 < x_1 < y_2 < Y_3 < X_3 < y_1] \qquad s_{12} = P[x_2 < x_1 < y_2 < Y_3 < y_1 < X_3]$$

$$s_{13} = P[X_3 < x_2 < x_1 < Y_3 < y_2 < y_1] \qquad s_{14} = P[x_2 < X_3 < x_1 < Y_3 < y_2 < y_1]$$

$$s_{15} = P[x_2 < x_1 < X_3 < Y_3 < y_2 < y_1] \qquad s_{16} = P[x_2 < x_1 < Y_3 < X_3 < y_2 < y_1]$$

$$s_{17} = P[x_2 < x_1 < Y_3 < y_2 < X_3 < y_1] \qquad s_{18} = P[x_2 < x_1 < Y_3 < y_2 < y_1 < X_3]$$

$$s_{19} = P[X_3 < x_2 < Y_3 < x_1 < y_2 < y_1] \qquad s_{20} = P[x_2 < X_3 < Y_3 < x_1 < y_2 < y_1]$$

$$s_{21} = P[x_2 < Y_3 < X_3 < x_1 < y_2 < y_1] \qquad s_{22} = P[x_2 < Y_3 < x_1 < X_3 < y_2 < y_1]$$

$$s_{23} = P[x_2 < Y_3 < x_1 < y_2 < X_3 < y_1] \qquad s_{24} = P[x_2 < Y_3 < x_1 < y_2 < y_1 < X_3]$$

$$s_{25} = P[X_3 < Y_3 < x_2 < x_1 < y_2 < y_1] \qquad s_{26} = P[Y_3 < X_3 < x_2 < x_1 < y_2 < y_1]$$

$$s_{27} = P[Y_3 < x_2 < X_3 < x_1 < y_2 < y_1] \qquad s_{28} = P[Y_3 < x_2 < x_1 < X_3 < y_2 < y_1]$$

$$s_{29} = P[Y_3 < x_2 < x_1 < y_2 < X_3 < y_1] \qquad s_{30} = P[Y_3 < x_2 < x_1 < y_2 < y_1 < X_3]$$

Further

$$s_1 = P[X_3 < x_2 < x_1 < y_2 < y_1 < Y_3]$$

$$= P[X_3 < x_2]P[y_1 < Y_3]$$

$$= F(x_2)\overline{F}(y_1)$$

$$s_2 = P[x_2 < X_3 < x_1 < y_2 < y_1 < Y_3]$$

$$= P[x_2 < X_3 < x_1]P[y_1 < Y_3]$$

$$= [F(x_1) - F(x_2)]\overline{F}(y_1)$$

$$s_3 = P[x_2 < x_1 < X_3 < y_2 < y_1 < Y_3]$$

$$= P[x_1 < X_3 < y_2]P[y_1 < Y_3]$$

$$= [F(y_2) - F(x_1)]\overline{F}(y_1)$$

$$s_4 = P[x_2 < x_1 < y_2 < X_3 < y_1 < Y_3]$$

$$= P[y_2 < X_3 < y_1]P[y_1 < Y_3]$$

$$= [F(y_1) - F(y_2)]\overline{F}(y_1)$$

$$s_5 = P[x_2 < x_1 < y_2 < y_1 < X_3 < Y_3]$$

$$= P[y_1 < X_3 < Y_3]$$

$$= \frac{\overline{F}^2(y_1)}{2}$$

$$s_6 = P[x_2 < x_1 < y_2 < y_1 < Y_3 < X_3]$$

$$= P[y_1 < Y_3 < X_3]$$

$$= \frac{\overline{F}^2(y_1)}{2}$$

$$s_7 = P[X_3 < x_2 < x_1 < y_2 < Y_3 < y_1]$$

$$= P[X_3 < x_2]P[y_2 < Y_3 < y_1]$$

$$= F(x_2)[F(y_1) - F(y_2)]$$

$$s_8 = P[x_2 < X_3 < x_1 < y_2 < Y_3 < y_1]$$

$$= P[x_2 < X_3 < x_1]P[y_2 < Y_3 < y_1]$$

$$= [F(x_1) - F(x_2)][F(y_1) - F(y_2)]$$

$$s_9 = P[x_2 < x_1 < X_3 < y_2 < Y_3 < y_1]$$

$$= P[x_1 < X_3 < y_2]P[y_2 < Y_3 < y_1]$$

$$= [F(y_2) - F(x_1)][F(y_1) - F(y_2)]$$

$$s_{10} = P[x_2 < x_1 < y_2 < X_3 < Y_3 < y_1]$$

$$= P[y_2 < X_3 < Y_3 < y_1]$$

$$= \frac{[F(y_1) - F(y_2)]^2}{2}$$

$$s_{11} = P[x_2 < x_1 < y_2 < Y_3 < X_3 < y_1]$$

$$= P[y_2 < Y_3 < X_3 < y_1]$$

$$= \frac{[F(y_1) - F(y_2)]^2}{2}$$

$$s_{12} = P[x_2 < x_1 < y_2 < Y_3 < y_1 < X_3]$$

$$= P[y_2 < Y_3 < y_1]P[y_1 < X_3]$$

$$= [F(y_1) - F(y_2)]\overline{F}(y_1)$$

$$s_{13} = P[X_3 < x_2 < x_1 < Y_3 < y_2 < y_1]$$

$$= P[X_3 < x_2]P[x_1 < Y_3 < y_2]$$

$$= F(x_2)[F(y_2) - F(x_1)]$$

$$s_{14} = P[x_2 < X_3 < x_1 < Y_3 < y_2 < y_1]$$

$$= P[x_2 < X_3 < x_1]P[x_1 < Y_3 < y_2]$$

$$= [F(x_1) - F(x_2)][F(y_2) - F(x_1)]$$

$$s_{15} = P[x_2 < x_1 < X_3 < Y_3 < y_2 < y_1]$$

$$= P[x_1 < X_3 < Y_3 < y_2]$$

$$= \frac{[F(y_2) - F(x_1)]^2}{2}$$

$$s_{16} = P[x_2 < x_1 < Y_3 < X_3 < y_2 < y_1]$$

$$= P[x_1 < Y_3 < X_3 < y_2]$$

$$= \frac{[F(y_2) - F(x_1)]^2}{2}$$

$$s_{17} = P[x_2 < x_1 < Y_3 < y_2 < X_3 < y_1]$$

$$= P[x_1 < Y_3 < y_2]P[y_2 < X_3 < y_1]$$

$$= [F(y_2) - F(x_1)][F(y_1) - F(y_2)]$$

$$s_{18} = P[x_2 < x_1 < Y_3 < y_2 < y_1 < X_3]$$

$$= P[x_1 < Y_3 < y_2]P[y_1 < X_3]$$

$$= [F(y_2) - F(x_1)]\overline{F}(y_1)$$

$$s_{19} = P[X_3 < x_2 < Y_3 < x_1 < y_2 < y_1]$$

$$= P[X_3 < x_2]P[x_2 < Y_3 < x_1]$$

$$= F(x_2)[F(x_1) - F(x_2)]$$

$$s_{20} = P[x_2 < X_3 < Y_3 < x_1 < y_2 < y_1]$$

$$= P[x_2 < X_3 < Y_3 < x_1]$$

$$= \frac{[F(x_1) - F(x_2)]^2}{2}$$

$$s_{21} = P[x_2 < Y_3 < X_3 < x_1 < y_2 < y_1]$$

$$= P[x_2 < Y_3 < X_3 < x_1]$$

$$= \frac{[F(x_1) - F(x_2)]^2}{2}$$

$$s_{22} = P[x_2 < Y_3 < x_1 < X_3 < y_2 < y_1]$$

$$= P[x_2 < Y_3 < x_1]P[x_1 < X_3 < y_2]$$

$$= [F(x_1) - F(x_2)][F(y_2) - F(x_1)]$$

$$s_{23} = P[x_2 < Y_3 < x_1 < y_2 < X_3 < y_1]$$

$$= P[x_2 < Y_3 < x_1]P[y_2 < X_3 < y_1]$$

$$= [F(x_1) - F(x_2)][F(y_1) - F(y_2)]$$

$$s_{24} = P[x_2 < Y_3 < x_1 < y_2 < y_1 < X_3]$$

$$= P[x_2 < Y_3 < x_1]P[y_1 < X_3]$$

$$= [F(x_1) - F(x_2)]\overline{F}(y_1)$$

$$s_{25} = P[X_3 < Y_3 < x_2 < x_1 < y_2 < y_1]$$

$$= P[X_3 < Y_3 < x_2]$$

$$= \frac{F^2(x_2)}{2}$$

$$s_{26} = P[Y_3 < X_3 < x_2 < x_1 < y_2 < y_1]$$

$$= P[Y_3 < X_3 < x_2]$$

$$= \frac{F^2(x_2)}{2}$$

$$s_{27} = P[Y_3 < x_2 < X_3 < x_1 < y_2 < y_1]$$

$$= P[Y_3 < x_2 < X_3 < x_1]$$

$$= P[Y_3 < x_2]P[x_2 < X_3 < x_1]$$

$$= F(x_2)[F(x_1) - F(x_2)]$$

$$s_{28} = P[Y_3 < x_2 < x_1 < X_3 < y_2 < y_1]$$

$$= P[Y_3 < x_2]P[x_1 < X_3 < y_2]$$

$$= F(x_2)[F(y_2) - F(x_1)]$$

$$s_{29} = P[Y_3 < x_2 < x_1 < y_2 < X_3 < y_1]$$

$$= P[Y_3 < x_2]P[y_2 < X_3 < y_1]$$

$$= F(x_2)[F(y_1) - F(y_2)]$$

$$s_{30} = P[Y_3 < x_2 < x_1 < y_2 < y_1 < X_3]$$

$$= P[Y_3 < x_2]P[y_1 < X_3]$$

$$= F(x_2)\overline{F}(y_1).$$

Adding the 30 terms we have for case

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = s_1 + s_2 + ... + s_{30}$$

$$= s^* \text{ say when } x_2 < x_1 < y_2 < y_1$$

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_1}\int_{-\infty}^{y_2}\int_{-\infty}^{x_1} P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_2)dF(x_1)dF(y_2)dF(y_1)$$

$$-\infty < x_2 < x_1 < y_2 < y_1 < \infty$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{y_1}\int_{-\infty}^{y_2}\int_{-\infty}^{x_1} s^{*2} dF(x_2)dF(x_1)dF(y_2)dF(y_1)$$

$$-\infty < x_2 < x_1 < y_2 < y_1 < \infty$$

$$= \frac{1}{24}.$$

When $x_2 < y_1 < x_1 < y_2$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = t_1 + t_2 + ... + t_{21}$$

where

$$t_1 = P[X_3 < x_2 < y_1 < x_1 < y_2 < Y_3] \qquad t_2 = P[x_2 < X_3 < y_1 < x_1 < y_2 < Y_3]$$

$t_3 = P[x_2 < y_1 < X_3 < x_1 < y_2 < Y_3]$

$t_4 = P[x_2 < y_1 < x_1 < X_3 < y_2 < Y_3]$

$t_5 = P[x_2 < y_1 < x_1 < y_2 < X_3 < Y_3]$

$t_6 = P[x_2 < y_1 < x_1 < y_2 < Y_3 < X_3]$

$t_7 = P[X_3 < x_2 < y_1 < x_1 < Y_3 < y_2]$

$t_8 = P[x_2 < X_3 < y_1 < x_1 < Y_3 < y_2]$

$t_9 = P[x_2 < y_1 < X_3 < x_1 < Y_3 < y_2]$

$t_{10} = P[x_2 < y_1 < x_1 < X_3 < Y_3 < y_2]$

$t_{11} = P[x_2 < y_1 < x_1 < Y_3 < X_3 < y_2]$

$t_{12} = P[x_2 < y_1 < x_1 < Y_3 < y_2 < X_3]$

$t_{13} = P[X_3 < x_2 < y_1 < Y_3 < x_1 < y_2]$

$t_{14} = P[x_2 < X_3 < y_1 < Y_3 < x_1 < y_2]$

$t_{15} = P[x_2 < y_1 < X_3 < Y_3 < x_1 < y_2]$

$t_{16} = P[X_3 < x_2 < Y_3 < y_1 < x_1 < y_2]$

$t_{17} = P[x_2 < X_3 < Y_3 < y_1 < x_1 < y_2]$

$t_{18} = P[x_2 < Y_3 < X_3 < y_1 < x_1 < y_2]$

$t_{19} = P[X_3 < Y_3 < x_2 < y_1 < x_1 < y_2]$

$t_{20} = P[Y_3 < X_3 < x_2 < y_1 < x_1 < y_2]$

$t_{21} = P[Y_3 < x_2 < X_3 < y_1 < x_1 < y_2]$.

Further

$t_1 = P[X_3 < x_2 < y_1 < x_1 < y_2 < Y_3]$

$= P[X_3 < x_2]P[y_2 < Y_3]$

$= F(x_2)\overline{F}(y_2)$

$t_2 = P[x_2 < X_3 < y_1 < x_1 < y_2 < Y_3]$

$= P[x_2 < X_3 < y_1]P[y_2 < Y_3]$

$= [F(y_1) - F(x_2)]\overline{F}(y_2)$

$t_3 = P[x_2 < y_1 < X_3 < x_1 < y_2 < Y_3]$

$= P[y_1 < X_3 < x_1]P[y_2 < Y_3]$

$= [F(x_1) - F(y_1)]\overline{F}(y_2)$

$t_4 = P[x_2 < y_1 < x_1 < X_3 < y_2 < Y_3]$

$= P[x_1 < X_3 < y_2 < Y_3]$

$$= P[x_1 < X_3 < y_2]P[y_2 < Y_3]$$

$$= [F(y_2) - F(x_1)]\overline{F}(y_2)$$

$$t_5 = P[x_2 < y_1 < x_1 < y_2 < X_3 < Y_3]$$

$$= P[y_2 < X_3 < Y_3]$$

$$= \frac{\overline{F}^2(y_2)}{2}$$

$$t_6 = P[x_2 < y_1 < x_1 < y_2 < Y_3 < X_3]$$

$$= P[y_2 < Y_3 < X_3]$$

$$= \frac{\overline{F}^2(y_2)}{2}$$

$$t_7 = P[X_3 < x_2 < y_1 < x_1 < Y_3 < y_2]$$

$$= P[X_3 < x_2]P[x_1 < Y_3 < y_2]$$

$$= F(x_2)[F(y_2) - F(x_1)]$$

$$t_8 = P[x_2 < X_3 < y_1 < x_1 < Y_3 < y_2]$$

$$= P[x_2 < X_3 < y_1]P[x_1 < Y_3 < y_2]$$

$$= [F(y_1) - F(x_2)][F(y_2) - F(x_1)]$$

$$t_9 = P[x_2 < y_1 < X_3 < x_1 < Y_3 < y_2]$$

$$= P[y_1 < X_3 < x_1]P[x_1 < Y_3 < y_2]$$

$$= [F(x_1) - F(y_1)][F(y_2) - F(x_1)]$$

$$t_{10} = P[x_2 < y_1 < x_1 < X_3 < Y_3 < y_2]$$

$$= P[x_1 < X_3 < Y_3 < y_2]$$

$$= \frac{[F(y_2) - F(x_1)]^2}{2}$$

$$t_{11} = P[x_2 < y_1 < x_1 < Y_3 < X_3 < y_2]$$

$$= P[x_1 < Y_3 < X_3 < y_2]$$

$$= \frac{[F(y_2) - F(x_1)]^2}{2}$$

$$t_{12} = P[x_2 < y_1 < x_1 < Y_3 < y_2 < X_3]$$

$$= P[x_1 < Y_3 < y_2]P[y_2 < X_3]$$

$$= [F(y_2) - F(x_1)]\overline{F}(y_2)$$

$$t_{13} = P[X_3 < x_2 < y_1 < Y_3 < x_1 < y_2]$$

$$= P[X_3 < x_2]P[y_1 < Y_3 < x_1]$$

$$= F(x_2)[F(x_1) - F(y_1)]$$

$$t_{14} = P[x_2 < X_3 < y_1 < Y_3 < x_1 < y_2]$$

$$= P[x_2 < X_3 < y_1]P[y_1 < Y_3 < x_1]$$

$$= [F(y_1) - F(x_2)][F(x_1) - F(y_1)]$$

$$t_{15} = P[x_2 < y_1 < X_3 < Y_3 < x_1 < y_2]$$

$$= P[y_1 < X_3 < Y_3 < x_1]$$

$$= \frac{[F(x_1) - F(y_1)]^2}{2}$$

$$t_{16} = P[X_3 < x_2 < Y_3 < y_1 < x_1 < y_2]$$

$$= P[X_3 < x_2]P[x_2 < Y_3 < y_1]$$

$$= F(x_2)[F(y_1) - F(x_2)]$$

$$t_{17} = P[x_2 < X_3 < Y_3 < y_1 < x_1 < y_2]$$

$$= P[x_2 < X_3 < Y_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_2)]^2}{2}$$

$$t_{18} = P[x_2 < Y_3 < X_3 < y_1 < x_1 < y_2]$$

$$= P[x_2 < Y_3 < X_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_2)]^2}{2}$$

$$t_{19} = P[X_3 < Y_3 < x_2 < y_1 < x_1 < y_2]$$

$$= P[X_3 < Y_3 < x_2]$$

$$= \frac{F^2(x_2)}{2}$$

$$t_{20} = P[Y_3 < X_3 < x_2 < y_1 < x_1 < y_2]$$

$$= P[Y_3 < X_3 < x_2]$$

$$= \frac{F^2(x_2)}{2}$$

$$t_{21} = P[Y_3 < x_2 < X_3 < y_1 < x_1 < y_2]$$

$$= P[Y_3 < x_2]P[x_2 < X_3 < y_1]$$

$$= F(x_2)[F(y_1) - F(x_2)].$$

Adding the 21 terms we have

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = t_1 + t_2 + ... + t_{21}$$

$$= 1 - F(x_1) + \frac{1}{2}F^2(y_1) + \frac{1}{2}F^2(x_1) \quad \text{when} \quad x_2 < y_1 < x_1 < y_2,$$

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_2}\int_{-\infty}^{x_1}\int_{-\infty}^{y_1} P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_2)dF(y_1)dF(x_1)dF(y_2)$$

$$-\infty < x_2 < y_1 < x_1 < y_2 < \infty$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{y_2}\int_{-\infty}^{x_1}\left(1-F(x_1)+\frac{1}{2}F^2(y_1)+\frac{1}{2}F^2(x_1)\right)^2 dF(x_2)dF(y_1)dF(x_1)dF(y_2)$$

$$-\infty < x_2 < y_1 < x_1 < y_2 < \infty$$

$$= \frac{139}{6720}.$$

Further when $x_2 < y_2 < x_1 < y_1$, we have

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = u_1 + u_2 + ... + u_{21}$$

where

$u_1 = P[X_3 < x_2 < y_2 < x_1 < y_1 < Y_3]$ $\quad$ $u_2 = P[x_2 < X_3 < y_2 < x_1 < y_1 < Y_3]$

$u_3 = P[x_2 < y_2 < X_3 < x_1 < y_1 < Y_3]$ $\quad$ $u_4 = P[x_2 < y_2 < x_1 < X_3 < y_1 < Y_3]$

$u_5 = P[x_2 < y_2 < x_1 < y_1 < X_3 < Y_3]$ $\quad$ $u_6 = P[x_2 < y_2 < x_1 < y_1 < Y_3 < X_3]$

$u_7 = P[X_3 < x_2 < y_2 < x_1 < Y_3 < y_1]$ $\quad$ $u_8 = P[x_2 < X_3 < y_2 < x_1 < Y_3 < y_1]$

$u_9 = P[x_2 < y_2 < X_3 < x_1 < Y_3 < y_1]$ $\quad$ $u_{10} = P[x_2 < y_2 < x_1 < X_3 < Y_3 < y_1]$

$u_{11} = P[x_2 < y_2 < x_1 < Y_3 < X_3 < y_1]$ $\quad$ $u_{12} = P[x_2 < y_2 < x_1 < Y_3 < y_1 < X_3]$

$u_{13} = P[X_3 < x_2 < y_2 < Y_3 < x_1 < y_1]$ $\quad$ $u_{14} = P[x_2 < X_3 < y_2 < Y_3 < x_1 < y_1]$

$u_{15} = P[x_2 < y_2 < X_3 < Y_3 < x_1 < y_1]$ $\quad$ $u_{16} = P[X_3 < x_2 < Y_3 < y_2 < x_1 < y_1]$

$u_{17} = P[x_2 < X_3 < Y_3 < y_2 < x_1 < y_1]$ $\quad$ $u_{18} = P[x_2 < Y_3 < X_3 < y_2 < x_1 < y_1]$

$u_{19} = P[X_3 < Y_3 < x_2 < y_2 < x_1 < y_1]$ $\quad$ $u_{20} = P[Y_3 < X_3 < x_2 < y_2 < x_1 < y_1]$

$u_{21} = P[Y_3 < x_2 < X_3 < y_2 < x_1 < y_1]$.

Now consider

$$u_1 = P[X_3 < x_2 < y_2 < x_1 < y_1 < Y_3]$$

$$= P[X_3 < x_2]P[y_1 < Y_3]$$

$$= F(x_2)\overline{F}(y_1)$$

$$u_2 = P[x_2 < X_3 < y_2 < x_1 < y_1 < Y_3]$$

$$= P[x_2 < X_3 < y_2]P[y_1 < Y_3]$$

$$= [F(y_2) - F(x_2)]\overline{F}(y_1)$$

$$u_3 = P[x_2 < y_2 < X_3 < x_1 < y_1 < Y_3]$$

$$= P[y_2 < X_3 < x_1]P[y_1 < Y_3]$$

$$= [F(x_1) - F(y_2)]\overline{F}(y_1)$$

$$u_4 = P[x_2 < y_2 < x_1 < X_3 < y_1 < Y_3]$$

$$= P[x_1 < X_3 < y_1 < Y_3]$$

$$= P[x_1 < X_3 < y_1]P[y_1 < Y_3]$$

$$= [F(y_1) - F(x_1)]\overline{F}(y_1)$$

$$u_5 = P[x_2 < y_2 < x_1 < y_1 < X_3 < Y_3]$$

$$= P[y_1 < X_3 < Y_3]$$

$$= \frac{\overline{F}^2(y_1)}{2}$$

$$u_6 = P[x_2 < y_2 < x_1 < y_1 < Y_3 < X_3]$$

$$= P[y_1 < Y_3 < X_3]$$

$$= \frac{\overline{F}^2(y_1)}{2}$$

$$u_7 = P[X_3 < x_2 < y_2 < x_1 < Y_3 < y_1]$$

$$= P[X_3 < x_2]P[x_1 < Y_3 < y_1]$$

$$= F(x_2)[F(y_1) - F(x_1)]$$

$$u_8 = P[x_2 < X_3 < y_2 < x_1 < Y_3 < y_1]$$

$$= P[x_2 < X_3 < y_2]P[x_1 < Y_3 < y_1]$$

$$= [F(y_2) - F(x_2)][F(y_1) - F(x_1)]$$

$$u_9 = P[x_2 < y_2 < X_3 < x_1 < Y_3 < y_1]$$

$$= P[y_2 < X_3 < x_1]P[x_1 < Y_3 < y_1]$$

$$= [F(x_1) - F(y_2)][F(y_1) - F(x_1)]$$

$$u_{10} = P[x_2 < y_2 < x_1 < X_3 < Y_3 < y_1]$$

$$= P[x_1 < X_3 < Y_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_1)]^2}{2}$$

$$u_{11} = P[x_2 < y_2 < x_1 < Y_3 < X_3 < y_1]$$

$$= P[x_1 < Y_3 < X_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_1)]^2}{2}$$

$$u_{12} = P[x_2 < y_2 < x_1 < Y_3 < y_1 < X_3]$$

$$= P[x_1 < Y_3 < y_1]P[y_1 < X_3]$$

$$= [F(y_1) - F(x_1)]\overline{F}(y_1)$$

$$u_{13} = P[X_3 < x_2 < y_2 < Y_3 < x_1 < y_1]$$

$$= P[X_3 < x_2]P[y_2 < Y_3 < x_1]$$

$$= F(x_2)[F(x_1) - F(y_2)]$$

$$u_{14} = P[x_2 < X_3 < y_2 < Y_3 < x_1 < y_1]$$

$$= P[x_2 < X_3 < y_2]P[y_2 < Y_3 < x_1]$$

$$= [F(y_2) - F(x_2)][F(x_1) - F(y_2)]$$

$$u_{15} = P[x_2 < y_2 < X_3 < Y_3 < x_1 < y_1]$$

$$= P[y_2 < X_3 < Y_3 < x_1]$$

$$= \frac{[F(x_1) - F(y_2)]^2}{2}$$

$$u_{16} = P[X_3 < x_2 < Y_3 < y_2 < x_1 < y_1]$$

$$= P[X_3 < x_2]P[x_2 < Y_3 < y_2]$$

$$= F(x_2)[F(y_2) - F(x_2)]$$

$$u_{17} = P[x_2 < X_3 < Y_3 < y_2 < x_1 < y_1]$$

$$= P[x_2 < X_3 < Y_3 < y_2]$$

$$= \frac{[F(y_2) - F(x_2)]^2}{2}$$

$$u_{18} = P[x_2 < Y_3 < X_3 < y_2 < x_1 < y_1]$$

$$= P[x_2 < Y_3 < X_3 < y_2]$$

$$= \frac{[F(y_2) - F(x_2)]^2}{2}$$

$$u_{19} = P[X_3 < Y_3 < x_2 < y_2 < x_1 < y_1]$$

$$= P[X_3 < Y_3 < x_2]$$

$$= \frac{F^2(x_2)}{2}$$

$$u_{20} = P[Y_3 < X_3 < x_2 < y_2 < x_1 < y_1]$$

$$= P[Y_3 < X_3 < x_2]$$

$$= \frac{F^2(x_2)}{2}$$

$$u_{21} = P[Y_3 < x_2 < X_3 < y_2 < x_1 < y_1]$$

$$= P[Y_3 < x_2]P[x_2 < X_3 < y_2]$$

$$= F(x_2)[F(y_2) - F(x_2)].$$

Adding the 21 terms we have for $x_2 < y_2 < x_1 < y_1$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = u_1 + u_2 + ... + u_{21}$$

$$= 1 - F(x_1) + \frac{1}{2}F^2(y_2) + \frac{1}{2}F^2(x_1)$$

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_1}\int_{-\infty}^{x_1}\int_{-\infty}^{y_2} P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_2)dF(y_2)dF(x_1)dF(y_1)$$

$$-\infty < x_2 < y_2 < x_1 < y_1 < \infty$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{y_1}\int_{-\infty}^{x_1}\int_{-\infty}^{y_2}\left(1 - F(x_1) + \frac{1}{2}F^2(y_2) + \frac{1}{2}F^2(x_1)\right)^2 dF(x_2)dF(y_2)dF(x_1)dF(y_1)$$

$$-\infty < x_2 < y_2 < x_1 < y_1 < \infty$$

$$= \frac{139}{6720}.$$

Consider case 11 i.e. when $x_2 < y_1 < y_2 < x_1$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = v_1 + v_2 + ... + v_{15}$$

where

$v_1 = P[X_3 < x_2 < y_1 < y_2 < x_1 < Y_3]$ $\quad$ $v_2 = P[x_2 < X_3 < y_1 < y_2 < x_1 < Y_3]$

$v_3 = P[x_2 < y_1 < X_3 < y_2 < x_1 < Y_3]$ $\quad$ $v_4 = P[X_3 < x_2 < y_1 < y_2 < Y_3 < x_1]$

$v_5 = P[x_2 < X_3 < y_1 < y_2 < Y_3 < x_1]$ $\quad$ $v_6 = P[x_2 < y_1 < X_3 < y_2 < Y_3 < x_1]$

$v_7 = P[X_3 < x_2 < y_1 < Y_3 < y_2 < x_1]$ $\quad$ $v_8 = P[x_2 < X_3 < y_1 < Y_3 < y_2 < x_1]$

$v_9 = P[x_2 < y_1 < X_3 < Y_3 < y_2 < x_1]$ $\quad$ $v_{10} = P[X_3 < x_2 < Y_3 < y_1 < y_2 < x_1]$

$v_{11} = P[x_2 < X_3 < Y_3 < y_1 < y_2 < x_1]$ $\quad$ $v_{12} = P[x_2 < Y_3 < X_3 < y_1 < y_2 < x_1]$

$$v_{13} = P[X_3 < Y_3 < x_2 < y_1 < y_2 < x_1] \qquad v_{14} = P[Y_3 < X_3 < x_2 < y_1 < y_2 < x_1]$$

$$v_{15} = P[Y_3 < x_2 < X_3 < y_1 < y_2 < x_1]$$

Now Consider

$$v_1 = P[X_3 < x_2 < y_1 < y_2 < x_1 < Y_3]$$

$$= P[X_3 < x_2]P[x_1 < Y_3]$$

$$= F(x_2)\overline{F}(x_1)$$

$$v_2 = P[x_2 < X_3 < y_1 < y_2 < x_1 < Y_3]$$

$$= P[x_2 < X_3 < y_1]P[x_1 < Y_3]$$

$$= [F(y_1) - F(x_2)]\overline{F}(x_1)$$

$$v_3 = P[x_2 < y_1 < X_3 < y_2 < x_1 < Y_3]$$

$$= P[y_1 < X_3 < y_2]P[x_1 < Y_3]$$

$$= [F(y_2) - F(y_1)]\overline{F}(x_1)$$

$$v_4 = P[X_3 < x_2 < y_1 < y_2 < Y_3 < x_1]$$

$$= P[X_3 < x_2]P[y_2 < Y_3 < x_1]$$

$$= F(x_2)[F(x_1) - F(y_2)]$$

$$v_5 = P[x_2 < X_3 < y_1 < y_2 < Y_3 < x_1]$$

$$= P[x_2 < X_3 < y_1]P[y_2 < Y_3 < x_1]$$

$$= [F(y_1) - F(x_2)][F(x_1) - F(y_2)]$$

$$v_6 = P[x_2 < y_1 < X_3 < y_2 < Y_3 < x_1]$$

$$= P[y_1 < X_3 < y_2]P[y_2 < Y_3 < x_1]$$

$$= [F(y_2) - F(y_1)][F(x_1) - F(y_2)]$$

$$v_7 = P[X_3 < x_2 < y_1 < Y_3 < y_2 < x_1]$$

$$= P[X_3 < x_2]P[y_1 < Y_3 < y_2]$$

$$= F(x_2)[F(y_2) - F(y_1)]$$

$$v_8 = P[x_2 < X_3 < y_1 < Y_3 < y_2 < x_1]$$

$$= P[x_2 < X_3 < y_1]P[y_1 < Y_3 < y_2]$$

$$= [F(y_1) - F(x_2)][F(y_2) - F(y_1)]$$

$$v_9 = P[x_2 < y_1 < X_3 < Y_3 < y_2 < x_1]$$

$$= P[y_1 < X_3 < Y_3 < y_2]$$

$$= \frac{[F(y_2) - F(y_1)]^2}{2}$$

$$v_{10} = P[X_3 < x_2 < Y_3 < y_1 < y_2 < x_1]$$

$$= P[X_3 < x_2 < Y_3 < y_1]$$

$$= P[X_3 < x_2]P[x_2 < Y_3 < y_1]$$

$$= F(x_2)[F(y_1) - F(x_2)]$$

$$v_{11} = P[x_2 < X_3 < Y_3 < y_1 < y_2 < x_1]$$

$$= P[x_2 < X_3 < Y_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_2)]^2}{2}$$

$$v_{12} = P[x_2 < Y_3 < X_3 < y_1 < y_2 < x_1]$$

$$= P[x_2 < Y_3 < X_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_2)]^2}{2}$$

$$v_{13} = P[X_3 < Y_3 < x_2 < y_1 < y_2 < x_1]$$

$$= P[X_3 < Y_3 < x_2]$$

$$= \frac{F^2(x_2)}{2}$$

$$v_{14} = P[Y_3 < X_3 < x_2 < y_1 < y_2 < x_1]$$

$$= P[Y_3 < X_3 < x_2]$$

$$= \frac{F^2(x_2)}{2}$$

$$v_{15} = P[Y_3 < x_2 < X_3 < y_1 < y_2 < x_1]$$

$$= P[Y_3 < x_2]P[x_2 < X_3 < y_1]$$

$$= F(x_2)[F(y_1) - F(x_2)]$$

Thus, adding 15 terms we have

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = v_1 + v_2 + ... + v_{15}$$

$$= F(y_2) + \frac{1}{2}F^2(y_1) - \frac{1}{2}F^2(y_2) \quad \text{for the case } x_2 < y_1 < y_2 < x_1 .$$

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{x_1}\int_{-\infty}^{y_2}\int_{-\infty}^{y_1} P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_2)dF(y_1)dF(y_2)dF(x_1)$$

$$-\infty < x_2 < y_1 < y_2 < x_1 < \infty$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{x_1}\int_{-\infty}^{y_2}\int_{-\infty}^{y_1}\left(F(y_2) + \frac{1}{2}F^2(y_1) - \frac{1}{2}F^2(y_2)\right)^2 dF(x_2)dF(y_1)dF(y_2)dF(x_1)$$

$$-\infty < x_2 < y_1 < y_2 < x_1 < \infty$$

$$= \frac{11}{960}.$$

Further consider the case 12: $x_2 < y_2 < y_1 < x_1$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = w_1 + w_2 + \ldots\ldots + w_{15}$$

where

$$w_1 = P[X_3 < x_2 < y_2 < y_1 < x_1 < Y_3] \qquad w_2 = P[x_2 < X_3 < y_2 < y_1 < x_1 < Y_3]$$

$$w_3 = P[x_2 < y_2 < X_3 < y_1 < x_1 < Y_3] \qquad w_4 = P[X_3 < x_2 < y_2 < y_1 < Y_3 < x_1]$$

$$w_5 = P[x_2 < X_3 < y_2 < y_1 < Y_3 < x_1] \qquad w_6 = P[x_2 < y_2 < X_3 < y_1 < Y_3 < x_1]$$

$$w_7 = P[X_3 < x_2 < y_2 < Y_3 < y_1 < x_1] \qquad w_8 = P[x_2 < X_3 < y_2 < Y_3 < y_1 < x_1]$$

$$w_9 = P[x_2 < y_2 < X_3 < Y_3 < y_1 < x_1] \qquad w_{10} = P[X_3 < x_2 < Y_3 < y_2 < y_1 < x_1]$$

$$w_{11} = P[x_2 < X_3 < Y_3 < y_2 < y_1 < x_1] \qquad w_{12} = P[x_2 < Y_3 < X_3 < y_2 < y_1 < x_1]$$

$$w_{13} = P[X_3 < Y_3 < x_2 < y_2 < y_1 < x_1] \qquad w_{14} = P[Y_3 < X_3 < x_2 < y_2 < y_1 < x_1]$$

$$w_{15} = P[Y_3 < x_2 < X_3 < y_2 < y_1 < x_1].$$

Now Consider

$$w_1 = P[X_3 < x_2 < y_2 < y_1 < x_1 < Y_3]$$

$$= P[X_3 < x_2]P[x_1 < Y_3]$$

$$= F(x_2)\overline{F}(x_1)$$

$$w_2 = P[x_2 < X_3 < y_2 < y_1 < x_1 < Y_3]$$

$$= P[x_2 < X_3 < y_2]P[x_1 < Y_3]$$

$$= [F(y_2) - F(x_2)]\overline{F}(x_1)$$

$$w_3 = P[x_2 < y_2 < X_3 < y_1 < x_1 < Y_3]$$

$$= P[y_2 < X_3 < y_1]P[x_1 < Y_3]$$

$$= [F(y_1) - F(y_2)]\overline{F}(x_1)$$

$$w_4 = P[X_3 < x_2 < y_2 < y_1 < Y_3 < x_1]$$

$$= P[X_3 < x_2]P[y_1 < Y_3 < x_1]$$

$$= F(x_2)[F(x_1) - F(y_1)]$$

$$w_5 = P[x_2 < X_3 < y_2 < y_1 < Y_3 < x_1]$$

$$= P[x_2 < X_3 < y_2]P[y_1 < Y_3 < x_1]$$

$$= [F(y_2) - F(x_2)][F(x_1) - F(y_1)]$$

$$w_6 = P[x_2 < y_2 < X_3 < y_1 < Y_3 < x_1]$$

$$= P[y_2 < X_3 < y_1]P[y_1 < Y_3 < x_1]$$

$$= [F(y_1) - F(y_2)][F(x_1) - F(y_1)]$$

$$w_7 = P[X_3 < x_2 < y_2 < Y_3 < y_1 < x_1]$$

$$= P[X_3 < x_2]P[y_2 < Y_3 < y_1]$$

$$= F(x_2)[F(y_1) - F(y_2)]$$

$$w_8 = P[x_2 < X_3 < y_2 < Y_3 < y_1 < x_1]$$

$$= P[x_2 < X_3 < y_2]P[y_2 < Y_3 < y_1]$$

$$= [F(y_2) - F(x_2)][F(y_1) - F(y_2)]$$

$$w_9 = P[x_2 < y_2 < X_3 < Y_3 < y_1 < x_1]$$

$$= P[y_2 < X_3 < Y_3 < y_1]$$

$$= \frac{[F(y_1) - F(y_2)]^2}{2}$$

$$w_{10} = P[X_3 < x_2 < Y_3 < y_2 < y_1 < x_1]$$

$$= P[X_3 < x_2 < Y_3 < y_2]$$

$$= P[X_3 < x_2]P[x_2 < Y_3 < y_2]$$

$$= F(x_2)[F(y_2) - F(x_2)]$$

$$w_{11} = P[x_2 < X_3 < Y_3 < y_2 < y_1 < x_1]$$

$$= P[x_2 < X_3 < Y_3 < y_2]$$

$$= \frac{[F(y_2) - F(x_2)]^2}{2}$$

$$w_{12} = P[x_2 < Y_3 < X_3 < y_2 < y_1 < x_1]$$

$$= P[x_2 < Y_3 < X_3 < y_2]$$

$$= \frac{[F(y_2) - F(x_2)]^2}{2}$$

$$w_{13} = P[X_3 < Y_3 < x_2 < y_2 < y_1 < x_1]$$

$$= P[X_3 < Y_3 < x_2]$$

$$= \frac{F^2(x_2)}{2}$$

$$w_{14} = P[Y_3 < X_3 < x_2 < y_2 < y_1 < x_1]$$

$$= P[Y_3 < X_3 < x_2]$$

$$= \frac{F^2(x_2)}{2}$$

$$w_{15} = P[Y_3 < x_2 < X_3 < y_2 < y_1 < x_1]$$

$$= P[Y_3 < x_2 < X_3 < y_2]$$

$$= P[Y_3 < x_2]P[x_2 < X_3 < y_2]$$

$$= F(x_2)[F(y_2) - F(x_2)].$$

Adding 15 terms we get for case 12) $x_2 < y_2 < y_1 < x_1$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = w_1 + w_2 + \ldots\ldots + w_{15}$$

$$= F(y_1) + \frac{1}{2}F^2(y_2) - \frac{1}{2}F^2(y_1).$$

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{x_1}\int_{-\infty}^{y_1}\int_{-\infty}^{y_2} P^2[Med(x_1,x_2,X_3)<Med(y_1,y_2,Y_3)]dF(x_2)dF(y_2)dF(y_1)dF(x_1)$$

$$-\infty<x_2<y_2<y_1<x_1<\infty$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{x_1}\int_{-\infty}^{y_1}\int_{-\infty}^{y_2}\left(F(y_1)+\frac{1}{2}F^2(y_2)-\frac{1}{2}F^2(y_1)\right)^2 dF(x_2)dF(y_2)dF(y_1)dF(x_1)$$

$$-\infty<x_2<y_2<y_1<x_1<\infty$$

$$=\frac{11}{960}.$$

When $y_1<x_1<y_2<x_2$, we have

$$P[Med(x_1,x_2,X_3)<Med(y_1,y_2,Y_3)]=x_1+x_2+.....+x_9$$

where

$x_1=P[X_3<y_1<x_1<y_2<x_2<Y_3]$ $\quad$ $x_2=P[y_1<X_3<x_1<y_2<x_2<Y_3]$

$x_3=P[y_1<x_1<X_3<y_2<x_2<Y_3]$ $\quad$ $x_4=P[X_3<y_1<x_1<y_2<Y_3<x_2]$

$x_5=P[y_1<X_3<x_1<y_2<Y_3<x_2]$ $\quad$ $x_6=P[y_1<x_1<X_3<y_2<Y_3<x_2]$

$x_7=P[X_3<y_1<x_1<Y_3<y_2<x_2]$ $\quad$ $x_8=P[y_1<X_3<x_1<Y_3<y_2<x_2]$

$x_9=P[y_1<x_1<X_3<Y_3<y_2<x_2]$.

Further

$$x_1=P[X_3<y_1<x_1<y_2<x_2<Y_3]$$

$$=P[X_3<y_1]P[x_2<Y_3]$$

$$=F(y_1)\overline{F}(x_2)$$

$$x_2=P[y_1<X_3<x_1<y_2<x_2<Y_3]$$

$$=P[y_1<X_3<x_1]P[x_2<Y_3]$$

$$= [F(x_1) - F(y_1)]\overline{F}(x_2)$$

$$x_3 = P[y_1 < x_1 < X_3 < y_2 < x_2 < Y_3]$$

$$= P[x_1 < X_3 < y_2]P[x_2 < Y_3]$$

$$= [F(y_2) - F(x_1)]\overline{F}(x_2)$$

$$x_4 = P[X_3 < y_1 < x_1 < y_2 < Y_3 < x_2]$$

$$= P[X_3 < y_1]P[y_2 < Y_3 < x_2]$$

$$= F(y_1)[F(x_2) - F(y_2)]$$

$$x_5 = P[y_1 < X_3 < x_1 < y_2 < Y_3 < x_2]$$

$$= P[y_1 < X_3 < x_1]P[y_2 < Y_3 < x_2]$$

$$= [F(x_1) - F(y_1)][F(x_2) - F(y_2)]$$

$$x_6 = P[y_1 < x_1 < X_3 < y_2 < Y_3 < x_2]$$

$$= P[x_1 < X_3 < y_2]P[y_2 < Y_3 < x_2]$$

$$= [F(y_2) - F(x_1)][F(x_2) - F(y_2)]$$

$$x_7 = P[X_3 < y_1 < x_1 < Y_3 < y_2 < x_2]$$

$$= P[X_3 < y_1]P[x_1 < Y_3 < y_2]$$

$$= F(y_1)[F(y_2) - F(x_1)]$$

$$x_8 = P[y_1 < X_3 < x_1 < Y_3 < y_2 < x_2]$$

$$= P[y_1 < X_3 < x_1]P[x_1 < Y_3 < y_2]$$

$$= [F(x_1) - F(y_1)][F(y_2) - F(x_1)]$$

$$x_9 = P[y_1 < x_1 < X_3 < Y_3 < y_2 < x_2]$$

$$= P[x_1 < X_3 < Y_3 < y_2]$$

$$= \frac{[F(y_2) - F(x_1)]^2}{2}$$

Adding 9 terms in this case i.e. $y_1 < x_1 < y_2 < x_2$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = x_1 + x_2 + ... + x_9$$

$$= F(y_2) - \frac{1}{2}F^2(x_1) - \frac{1}{2}F^2(y_2).$$

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{x_2}\int_{-\infty}^{y_2}\int_{-\infty}^{x_1} P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(y_1)dF(x_1)dF(y_2)dF(x_2)$$

$$-\infty < y_1 < x_1 < y_2 < x_2 < \infty$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{x_2}\int_{-\infty}^{y_2}\int_{-\infty}^{x_1}\left(F(y_2) - \frac{1}{2}F^2(x_1) - \frac{1}{2}F^2(y_2)\right)^2 dF(y_1)dF(x_1)dF(y_2)dF(x_2)$$

$$-\infty < y_1 < x_1 < y_2 < x_2 < \infty$$

$$= \frac{9}{2240}.$$

Further consider case 14: $y_1 < x_2 < y_2 < x_1$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = y_1 + y_2 + ... + y_9$$

where

$y_1 = P[X_3 < y_1 < x_2 < y_2 < x_1 < Y_3]$ $\quad$ $y_2 = P[y_1 < X_3 < x_2 < y_2 < x_1 < Y_3]$

$y_3 = P[y_1 < x_2 < X_3 < y_2 < x_1 < Y_3]$ $\quad$ $y_4 = P[X_3 < y_1 < x_2 < y_2 < Y_3 < x_1]$

$y_5 = P[y_1 < X_3 < x_2 < y_2 < Y_3 < x_1]$ $\quad$ $y_6 = P[y_1 < x_2 < X_3 < y_2 < Y_3 < x_1]$

$y_7 = P[X_3 < y_1 < x_2 < Y_3 < y_2 < x_1]$ $\quad$ $y_8 = P[y_1 < X_3 < x_2 < Y_3 < y_2 < x_1]$

$y_9 = P[y_1 < x_2 < X_3 < Y_3 < y_2 < x_1]$.

Now Consider

$$y_1 = P[X_3 < y_1 < x_2 < y_2 < x_1 < Y_3]$$

$$= P[X_3 < y_1]P[x_1 < Y_3]$$

$$= [F(y_1)]\overline{F}(x_1)$$

$$y_2 = P[y_1 < X_3 < x_2 < y_2 < x_1 < Y_3]$$

$$= P[y_1 < X_3 < x_2]P[x_1 < Y_3]$$

$$= [F(x_2) - F(y_1)]\overline{F}(x_1)$$

$$y_3 = P[y_1 < x_2 < X_3 < y_2 < x_1 < Y_3]$$

$$= P[x_2 < X_3 < y_2]P[x_1 < Y_3]$$

$$= [F(y_2) - F(x_2)]\overline{F}(x_1)$$

$$y_4 = P[X_3 < y_1 < x_2 < y_2 < Y_3 < x_1]$$

$$= P[X_3 < y_1]P[y_2 < Y_3 < x_1]$$

$$= [F(y_1)][F(x_1) - F(y_2)]$$

$$y_5 = P[y_1 < X_3 < x_2 < y_2 < Y_3 < x_1]$$

$$= P[y_1 < X_3 < x_2]P[y_2 < Y_3 < x_1]$$

$$= [F(x_2) - F(y_1)][F(x_1) - F(y_2)]$$

$$y_6 = P[y_1 < x_2 < X_3 < y_2 < Y_3 < x_1]$$

$$= P[x_2 < X_3 < y_2]P[y_2 < Y_3 < x_1]$$

$$= [F(y_2) - F(x_2)][F(x_1) - F(y_2)]$$

$$y_7 = P[X_3 < y_1 < x_2 < Y_3 < y_2 < x_1]$$

$$= P[X_3 < y_1]P[x_2 < Y_3 < y_2]$$

$$= [F(y_1)][F(y_2) - F(x_2)]$$

$$y_8 = P[y_1 < X_3 < x_2 < Y_3 < y_2 < x_1]$$

$$= P[y_1 < X_3 < x_2]P[x_2 < Y_3 < y_2]$$

$$= [F(x_2) - F(y_1)][F(y_2) - F(x_2)]$$

$$y_9 = P[y_1 < x_2 < X_3 < Y_3 < y_2 < x_1]$$

$$= P[x_2 < X_3 < Y_3 < y_2]$$

$$= \frac{[F(y_2) - F(x_2)]^2}{2}.$$

Adding 9 terms we have for 14: $y_1 < x_2 < y_2 < x_1$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = y_1 + y_2 + \ldots\ldots + y_9$$

$$= F(y_2) - \frac{1}{2}F^2(x_2) - \frac{1}{2}F^2(y_2).$$

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{x_1}\int_{-\infty}^{y_2}\int_{-\infty}^{x_2} P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(y_1)dF(x_2)dF(y_2)dF(x_1)$$

$$-\infty < y_1 < x_2 < y_2 < x_1 < \infty$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{x_1}\int_{-\infty}^{y_2}\int_{-\infty}^{x_2}\left(F(y_2) - \frac{1}{2}F^2(x_2) - \frac{1}{2}F^2(y_2)\right)^2 dF(y_1)dF(x_2)dF(y_2)dF(x_1)$$

$$-\infty < y_1 < x_2 < y_2 < x_1 < \infty$$

$$= \frac{9}{2240}.$$

Similarly in the case 15: $y_1 < x_1 < x_2 < y_2$ and

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = z_1 + z_2 + \ldots\ldots + z_{15}$$

where

$z_1 = P[X_3 < y_1 < x_1 < x_2 < y_2 < Y_3]$

$z_2 = P[y_1 < X_3 < x_1 < x_2 < y_2 < Y_3]$

$z_3 = P[y_1 < x_1 < X_3 < x_2 < y_2 < Y_3]$

$z_4 = P[y_1 < x_1 < x_2 < X_3 < y_2 < Y_3]$

$z_5 = P[y_1 < x_1 < x_2 < y_2 < X_3 < Y_3]$

$z_6 = P[y_1 < x_1 < x_2 < y_2 < Y_3 < X_3]$

$z_7 = P[X_3 < y_1 < x_1 < x_2 < Y_3 < y_2]$

$z_8 = P[y_1 < X_3 < x_1 < x_2 < Y_3 < y_2]$

$z_9 = P[y_1 < x_1 < X_3 < x_2 < Y_3 < y_2]$

$z_{10} = P[y_1 < x_1 < x_2 < X_3 < Y_3 < y_2]$

$z_{11} = P[y_1 < x_1 < x_2 < Y_3 < X_3 < y_2]$

$z_{12} = P[y_1 < x_1 < x_2 < Y_3 < y_2 < X_3]$

$z_{13} = P[X_3 < y_1 < x_1 < Y_3 < x_2 < y_2]$

$z_{14} = P[y_1 < X_3 < x_1 < Y_3 < x_2 < y_2]$

$z_{15} = P[y_1 < x_1 < X_3 < Y_3 < x_2 < y_2]$.

Now Consider

$$z_1 = P[X_3 < y_1 < x_1 < x_2 < y_2 < Y_3]$$

$$= P[X_3 < y_1]P[y_2 < Y_3]$$

$$= [F(y_1)]\overline{F}(y_2)$$

$$z_2 = P[y_1 < X_3 < x_1 < x_2 < y_2 < Y_3]$$

$$= P[y_1 < X_3 < x_1]P[y_2 < Y_3]$$

$$= [F(x_1) - F(y_1)]\overline{F}(y_2)$$

$$z_3 = P[y_1 < x_1 < X_3 < x_2 < y_2 < Y_3]$$

$$= P[x_1 < X_3 < x_2]P[y_2 < Y_3]$$

$$= [F(x_2) - F(x_1)]\overline{F}(y_2)$$

$$z_4 = P[y_1 < x_1 < x_2 < X_3 < y_2 < Y_3]$$

$$= P[x_2 < X_3 < y_2]P[y_2 < Y_3]$$

$$= [F(y_2) - F(x_2)]\overline{F}(y_2)$$

$$z_5 = P[y_1 < x_1 < x_2 < y_2 < X_3 < Y_3]$$

$$= P[y_2 < X_3 < Y_3]$$

$$= \frac{\overline{F}^2(y_2)}{2}$$

$$z_6 = P[y_1 < x_1 < x_2 < y_2 < Y_3 < X_3]$$

$$= P[y_2 < Y_3 < X_3]$$

$$= \frac{\overline{F}^2(y_2)}{2}$$

$$z_7 = P[X_3 < y_1 < x_1 < x_2 < Y_3 < y_2]$$

$$= P[X_3 < y_1]P[x_2 < Y_3 < y_2]$$

$$= [F(y_1)][F(y_2) - F(x_2)]$$

$$z_8 = P[y_1 < X_3 < x_1 < x_2 < Y_3 < y_2]$$

$$= P[y_1 < X_3 < x_1]P[x_2 < Y_3 < y_2]$$

$$= [F(x_1) - F(y_1)][F(y_2) - F(x_2)]$$

$$z_9 = P[y_1 < x_1 < X_3 < x_2 < Y_3 < y_2]$$

$$= P[x_1 < X_3 < x_2]P[x_2 < Y_3 < y_2]$$

$$= [F(x_2) - F(x_1)][F(y_2) - F(x_2)]$$

$$z_{10} = P[y_1 < x_1 < x_2 < X_3 < Y_3 < y_2]$$

$$= P[x_2 < X_3 < Y_3 < y_2]$$

$$= \frac{[F(y_2) - F(x_2)]^2}{2}$$

$$z_{11} = P[y_1 < x_1 < x_2 < Y_3 < X_3 < y_2]$$

$$= P[x_2 < Y_3 < X_3 < y_2]$$

$$= \frac{[F(y_2) - F(x_2)]^2}{2}$$

$$z_{12} = P[y_1 < x_1 < x_2 < Y_3 < y_2 < X_3]$$

$$= P[x_2 < Y_3 < y_2]P[y_2 < X_3]$$

$$= [F(y_2) - F(x_2)]\overline{F}(y_2)$$

$$z_{13} = P[X_3 < y_1 < x_1 < Y_3 < x_2 < y_2]$$

$$= P[X_3 < y_1]P[x_1 < Y_3 < x_2]$$

$$= [F(y_1)][F(x_2) - F(x_1)]$$

$$z_{14} = P[y_1 < X_3 < x_1 < Y_3 < x_2 < y_2]$$

$$= P[y_1 < X_3 < x_1]P[x_1 < Y_3 < x_2]$$

$$= [F(x_1) - F(y_1)][F(x_2) - F(x_1)]$$

$$z_{15} = P[y_1 < x_1 < X_3 < Y_3 < x_2 < y_2]$$

$$= P[x_1 < X_3 < Y_3 < x_2]$$

$$= \frac{[F(x_2) - F(x_1)]^2}{2}.$$

Adding 15 terms in this case 15: $y_1 < x_1 < x_2 < y_2$, we have

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = z_1 + z_2 + \ldots\ldots + z_{15}$$

$$= 1 - \frac{1}{2}F^2(x_1) - F(x_2) + \frac{1}{2}F^2(x_2).$$

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_2}\int_{-\infty}^{x_2}\int_{-\infty}^{x_1} P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(y_1)dF(x_1)dF(x_2)dF(y_2)$$

$$-\infty < y_1 < x_1 < x_2 < y_2 < \infty$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{y_2}\int_{-\infty}^{x_2}\int_{-\infty}^{x_1}\left(1-\frac{1}{2}F^2(x_1)-F(x_2)+\frac{1}{2}F^2(x_2)\right)^2 dF(y_1)dF(x_1)dF(x_2)dF(y_2)$$

$$-\infty < y_1 < x_1 < x_2 < y_2 < \infty$$

$$= \frac{11}{960}.$$

Now consider the case 16: $y_1 < x_2 < x_1 < y_2$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = A_1 + A_2 + \ldots\ldots + A_{15}$$

where

$A_1 = P[X_3 < y_1 < x_2 < x_1 < y_2 < Y_3]$ $\quad$ $A_2 = P[y_1 < X_3 < x_2 < x_1 < y_2 < Y_3]$

$A_3 = P[y_1 < x_2 < X_3 < x_1 < y_2 < Y_3]$ $\quad$ $A_4 = P[y_1 < x_2 < x_1 < X_3 < y_2 < Y_3$

$A_5 = P[y_1 < x_2 < x_1 < y_2 < X_3 < Y_3]$ $\quad$ $A_6 = P[y_1 < x_2 < x_1 < y_2 < Y_3 < X_3]$

$A_7 = P[X_3 < y_1 < x_2 < x_1 < Y_3 < y_2]$ $\quad$ $A_8 = P[y_1 < X_3 < x_2 < x_1 < Y_3 < y_2]$

$A_9 = P[y_1 < x_2 < X_3 < x_1 < Y_3 < y_2]$ $\quad$ $A_{10} = P[y_1 < x_2 < x_1 < X_3 < Y_3 < y_2]$

$A_{11} = P[y_1 < x_2 < x_1 < Y_3 < X_3 < y_2]$ $\quad$ $A_{12} = P[y_1 < x_2 < x_1 < Y_3 < y_2 < X_3]$

$A_{13} = P[X_3 < y_1 < x_2 < Y_3 < x_1 < y_2]$ $\quad$ $A_{14} = P[y_1 < X_3 < x_2 < Y_3 < x_1 < y_2]$

$A_{15} = P[y_1 < x_2 < X_3 < Y_3 < x_1 < y_2]$.

Now Consider

$$A_1 = P[X_3 < y_1 < x_2 < x_1 < y_2 < Y_3]$$

$$= P[X_3 < y_1]P[y_2 < Y_3]$$

$$= [F(y_1)][\overline{F}(y_2)]$$

$$A_2 = P[y_1 < X_3 < x_2 < x_1 < y_2 < Y_3]$$

$$= P[y_1 < X_3 < x_2]P[y_2 < Y_3]$$

$$= [F(x_2) - F(y_1)]\overline{F}(y_2)$$

$$A_3 = P[y_1 < x_2 < X_3 < x_1 < y_2 < Y_3]$$

$$= P[x_2 < X_3 < x_1]P[y_2 < Y_3]$$

$$= [F(x_1) - F(x_2)]\overline{F}(y_2)$$

$$A_4 = P[y_1 < x_2 < x_1 < X_3 < y_2 < Y_3]$$

$$= P[x_1 < X_3 < y_2]P[y_2 < Y_3]$$

$$= [F(y_2) - F(x_1)]\overline{F}(y_2)$$

$$A_5 = P[y_1 < x_2 < x_1 < y_2 < X_3 < Y_3]$$

$$= P[y_2 < X_3 < Y_3]$$

$$= \frac{\overline{F}^2(y_2)}{2}$$

$$A_6 = P[y_1 < x_2 < x_1 < y_2 < Y_3 < X_3]$$

$$= P[y_2 < Y_3 < X_3]$$

$$= \frac{\overline{F}^2(y_2)}{2}$$

$$A_7 = P[X_3 < y_1 < x_2 < x_1 < Y_3 < y_2]$$

$$= P[X_3 < y_1]P[x_1 < Y_3 < y_2]$$

$$= [F(y_1)][F(y_2) - F(x_1)]$$

$$A_8 = P[y_1 < X_3 < x_2 < x_1 < Y_3 < y_2]$$

$$= P[y_1 < X_3 < x_2]P[x_1 < Y_3 < y_2]$$

$$= [F(x_2) - F(y_1)][F(y_2) - F(x_1)]$$

$$A_9 = P[y_1 < x_2 < X_3 < x_1 < Y_3 < y_2]$$

$$= P[x_2 < X_3 < x_1]P[x_1 < Y_3 < y_2]$$

$$= [F(x_1) - F(x_2)][F(y_2) - F(x_1)]$$

$$A_{10} = P[y_1 < x_2 < x_1 < X_3 < Y_3 < y_2]$$

$$= P[x_1 < X_3 < Y_3 < y_2]$$

$$= \frac{[F(y_2) - F(x_1)]^2}{2}$$

$$A_{11} = P[y_1 < x_2 < x_1 < Y_3 < X_3 < y_2]$$

$$= P[x_1 < Y_3 < X_3 < y_2]$$

$$= \frac{[F(y_2) - F(x_1)]^2}{2}$$

$$A_{12} = P[y_1 < x_2 < x_1 < Y_3 < y_2 < X_3]$$

$$= P[x_1 < Y_3 < y_2]P[y_2 < X_3]$$

$$= [F(y_2) - F(x_1)]\overline{F}(y_2)$$

$$A_{13} = P[X_3 < y_1 < x_2 < Y_3 < x_1 < y_2]$$

$$= P[X_3 < y_1]P[x_2 < Y_3 < x_1]$$

$$= [F(y_1)][F(x_1) - F(x_2)]$$

$$A_{14} = P[y_1 < X_3 < x_2 < Y_3 < x_1 < y_2]$$

$$= P[y_1 < X_3 < x_2]P[x_2 < Y_3 < x_1]$$

$$= [F(x_2) - F(y_1)][F(x_1) - F(x_2)]$$

$$A_{15} = P[y_1 < x_2 < X_3 < Y_3 < x_1 < y_2]$$

$$= P[x_2 < X_3 < Y_3 < x_1]$$

$$= \frac{[F(x_1) - F(x_2)]^2}{2}.$$

Adding the 15 terms we have for case 16: $y_1 < x_2 < x_1 < y_2$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = A_1 + A_2 + \ldots\ldots + A_{15}$$

$$= 1 - \frac{1}{2}F^2(x_2) - F(x_1) + \frac{1}{2}F^2(x_1).$$

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_2}\int_{-\infty}^{x_1}\int_{-\infty}^{x_2} P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(y_1)dF(x_2)dF(x_1)dF(y_2)$$

$$-\infty < y_1 < x_2 < x_1 < y_2 < \infty$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{y_2}\int_{-\infty}^{x_1}\int_{-\infty}^{x_2}\left(1 - \frac{1}{2}F^2(x_2) - F(x_1) + \frac{1}{2}F^2(x_1)\right)^2 dF(y_1)dF(x_2)dF(x_1)dF(y_2)$$

$$-\infty < y_1 < x_2 < x_1 < y_2 < \infty$$

$$= \frac{11}{960}.$$

Further consider case 17: $y_2 < x_1 < y_1 < x_2$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = B_1 + B_2 + \ldots\ldots + B_9$$

where

$B_1 = P[X_3 < y_2 < x_1 < y_1 < x_2 < Y_3]$ $\quad$ $B_2 = P[y_2 < X_3 < x_1 < y_1 < x_2 < Y_3]$

$B_3 = P[y_2 < x_1 < X_3 < y_1 < x_2 < Y_3]$ $\quad$ $B_4 = P[X_3 < y_2 < x_1 < y_1 < Y_3 < x_2]$

$B_5 = P[y_2 < X_3 < x_1 < y_1 < Y_3 < x_2]$ $\quad$ $B_6 = P[y_2 < x_1 < X_3 < y_1 < Y_3 < x_2]$

$B_7 = P[X_3 < y_2 < x_1 < Y_3 < y_1 < x_2]$ $\quad$ $B_8 = P[y_2 < X_3 < x_1 < Y_3 < y_1 < x_2]$

$B_9 = P[y_2 < x_1 < X_3 < Y_3 < y_1 < x_2]$.

Now Consider

$$B_1 = P[X_3 < y_2 < x_1 < y_1 < x_2 < Y_3]$$

$$= P[X_3 < y_2]P[x_2 < Y_3]$$

$$= F(y_2)\overline{F}(x_2)$$

$$B_2 = P[y_2 < X_3 < x_1 < y_1 < x_2 < Y_3]$$

$$= P[y_2 < X_3 < x_1]P[x_2 < Y_3]$$

$$= [F(x_1) - F(y_2)]\overline{F}(x_2)$$

$$B_3 = P[y_2 < x_1 < X_3 < y_1 < x_2 < Y_3]$$

$$= P[x_1 < X_3 < y_1]P[x_2 < Y_3]$$

$$= [F(y_1) - F(x_1)]\overline{F}(x_2)$$

$$B_4 = P[X_3 < y_2 < x_1 < y_1 < Y_3 < x_2]$$

$$= P[X_3 < y_2]P[y_1 < Y_3 < x_2]$$

$$= F(y_2)[F(x_2) - F(y_1)]$$

$$B_5 = P[y_2 < X_3 < x_1 < y_1 < Y_3 < x_2]$$

$$= P[y_2 < X_3 < x_1]P[y_1 < Y_3 < x_2]$$

$$= [F(x_1) - F(y_2)][F(x_2) - F(y_1)]$$

$$B_6 = P[y_2 < x_1 < X_3 < y_1 < Y_3 < x_2]$$

$$= P[x_1 < X_3 < y_1]P[y_1 < Y_3 < x_2]$$

$$= [F(y_1) - F(x_1)][F(x_2) - F(y_1)]$$

$$B_7 = P[X_3 < y_2 < x_1 < Y_3 < y_1 < x_2]$$

$$= P[X_3 < y_2]P[x_1 < Y_3 < y_1]$$

$$= F(y_2)[F(y_1) - F(x_1)]$$

$$B_8 = P[y_2 < X_3 < x_1 < Y_3 < y_1 < x_2]$$

$$= P[y_2 < X_3 < x_1]P[x_1 < Y_3 < y_1]$$

$$= [F(x_1) - F(y_2)][F(y_1) - F(x_1)]$$

$$B_9 = P[y_2 < x_1 < X_3 < Y_3 < y_1 < x_2]$$

$$= P[x_1 < X_3 < Y_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_1)]^2}{2}.$$

Adding the 9 terms we have for case 17: $y_2 < x_1 < y_1 < x_2$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = B_1 + B_2 + ... + B_9$$

$$= F(y_1) - \frac{1}{2}F^2(y_1) - \frac{1}{2}F^2(x_1).$$

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{x_2}\int_{-\infty}^{y_1}\int_{-\infty}^{x_1} P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(y_2)dF(x_1)dF(y_1)dF(x_2)$$

$$-\infty < y_2 < x_1 < y_1 < x_2 < \infty$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{x_2}\int_{-\infty}^{y_1}\int_{-\infty}^{x_1}\left(F(y_1) - \frac{1}{2}F^2(y_1) - \frac{1}{2}F^2(x_1)\right)^2 dF(y_2)dF(x_1)dF(y_1)dF(x_2)$$

$$-\infty < y_2 < x_1 < y_1 < x_2 < \infty$$

$$= \frac{9}{2240}.$$

Similarly consider case 18: $y_2 < x_2 < y_1 < x_1$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = C_1 + C_2 + ... + C_9$$

where

$$C_1 = P[X_3 < y_2 < x_2 < y_1 < x_1 < Y_3] \qquad C_2 = P[y_2 < X_3 < x_2 < y_1 < x_1 < Y_3]$$

$$C_3 = P[y_2 < x_2 < X_3 < y_1 < x_1 < Y_3] \qquad C_4 = P[X_3 < y_2 < x_2 < y_1 < Y_3 < x_1]$$

$$C_5 = P[y_2 < X_3 < x_2 < y_1 < Y_3 < x_1] \qquad C_6 = P[y_2 < x_2 < X_3 < y_1 < Y_3 < x_1]$$

$$C_7 = P[X_3 < y_2 < x_2 < Y_3 < y_1 < x_1] \qquad C_8 \doteq P[y_2 < X_3 < x_2 < Y_3 < y_1 < x_1]$$

$$C_9 = P[y_2 < x_2 < X_3 < Y_3 < y_1 < x_1].$$

Now consider

$$C_1 = P[X_3 < y_2 < x_2 < y_1 < x_1 < Y_3]$$

$$= P[X_3 < y_2]P[x_1 < Y_3]$$

$$= F(y_2)\overline{F}(x_1)$$

$$C_2 = P[y_2 < X_3 < x_2 < y_1 < x_1 < Y_3]$$

$$= P[y_2 < X_3 < x_2]P[x_1 < Y_3]$$

$$= [F(x_2) - F(y_2)]\overline{F}(x_1)$$

$$C_3 = P[y_2 < x_2 < X_3 < y_1 < x_1 < Y_3]$$

$$= P[x_2 < X_3 < y_1]P[x_1 < Y_3]$$

$$= [F(y_1) - F(x_2)]\overline{F}(x_1)$$

$$C_4 = P[X_3 < y_2 < x_2 < y_1 < Y_3 < x_1]$$

$$= P[X_3 < y_2]P[y_1 < Y_3 < x_1]$$

$$= F(y_2)[F(x_1) - F(y_1)]$$

$$C_5 = P[y_2 < X_3 < x_2 < y_1 < Y_3 < x_1]$$

$$= P[y_2 < X_3 < x_2]P[y_1 < Y_3 < x_1]$$

$$= [F(x_2) - F(y_2)][F(x_1) - F(y_1)]$$

$$C_6 = P[y_2 < x_2 < X_3 < y_1 < Y_3 < x_1]$$

$$= P[x_2 < X_3 < y_1]P[y_1 < Y_3 < x_1]$$

$$= [F(y_1) - F(x_2)][F(x_1) - F(y_1)]$$

$$C_7 = P[X_3 < y_2 < x_2 < Y_3 < y_1 < x_1]$$

$$= P[X_3 < y_2]P[x_2 < Y_3 < y_1]$$

$$= F(y_2)[F(y_1) - F(x_2)]$$

$$C_8 = P[y_2 < X_3 < x_2 < Y_3 < y_1 < x_1]$$

$$= P[y_2 < X_3 < x_2]P[x_2 < Y_3 < y_1]$$

$$= [F(x_2) - F(y_2)][F(y_1) - F(x_2)]$$

$$C_9 = P[y_2 < x_2 < X_3 < Y_3 < y_1 < x_1]$$

$$= P[x_2 < X_3 < Y_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_2)]^2}{2} .$$

Adding the 9 terms we have for case 18: $y_2 < x_2 < y_1 < x_1$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = C_1 + C_2 + ... + C_9$$

$$= F(y_1) - \frac{1}{2}F^2(y_1) - \frac{1}{2}F^2(x_2) .$$

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{x_1}\int_{-\infty}^{y_1}\int_{-\infty}^{x_2} P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(y_2)dF(x_1)dF(y_1)dF(x_2)$$

$$-\infty < y_2 < x_2 < y_1 < x_1 < \infty$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{x_1}\int_{-\infty}^{y_1}\left(F(y_1) - \frac{1}{2}F^2(y_1) - \frac{1}{2}F^2(x_2) \right)^2 dF(y_2)dF(x_1)dF(y_1)dF(x_2)$$

$$-\infty < y_2 < x_2 < y_1 < x_1 < \infty$$

$$= \frac{9}{2240} .$$

Further consider case 19: $y_2 < x_1 < x_2 < y_1$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = D_1 + D_2 + ... + D_{15}$$

where

$D_1 = P[X_3 < y_2 < x_1 < x_2 < y_1 < Y_3]$

$D_2 = P[y_2 < X_3 < x_1 < x_2 < y_1 < Y_3]$

$D_3 = P[y_2 < x_1 < X_3 < x_2 < y_1 < Y_3]$

$D_4 = P[y_2 < x_1 < x_2 < X_3 < y_1 < Y_3]$

$D_5 = P[y_2 < x_1 < x_2 < y_1 < X_3 < Y_3]$

$D_6 = P[y_2 < x_1 < x_2 < y_1 < Y_3 < X_3]$

$D_7 = P[X_3 < y_2 < x_1 < x_2 < Y_3 < y_1]$

$D_8 = P[y_2 < X_3 < x_1 < x_2 < Y_3 < y_1]$

$D_9 = P[y_2 < x_1 < X_3 < x_2 < Y_3 < y_1]$

$D_{10} = P[y_2 < x_1 < x_2 < X_3 < Y_3 < y_1]$

$D_{11} = P[y_2 < x_1 < x_2 < Y_3 < X_3 < y_1]$

$D_{12} = P[y_2 < x_1 < x_2 < Y_3 < y_1 < X_3]$

$D_{13} = P[X_3 < y_2 < x_1 < Y_3 < x_2 < y_1]$

$D_{14} = P[y_2 < X_3 < x_1 < Y_3 < x_2 < y_1]$

$D_{15} = P[y_2 < x_1 < X_3 < Y_3 < x_2 < y_1]$.

Now Consider

$$D_1 = P[X_3 < y_2 < x_1 < x_2 < y_1 < Y_3]$$

$$= P[X_3 < y_2]P[y_1 < Y_3]$$

$$= [F(y_2)]\overline{F}(y_1)$$

$$D_2 = P[y_2 < X_3 < x_1 < x_2 < y_1 < Y_3]$$

$$= P[y_2 < X_3 < x_1]P[y_1 < Y_3]$$

$$= [F(x_1) - F(y_2)]\overline{F}(y_1)$$

$$D_3 = P[y_2 < x_1 < X_3 < x_2 < y_1 < Y_3]$$

$$= P[x_1 < X_3 < x_2]P[y_1 < Y_3]$$

$$= [F(x_2) - F(x_1)]\overline{F}(y_1)$$

$$D_4 = P[y_2 < x_1 < x_2 < X_3 < y_1 < Y_3]$$

$$= P[x_2 < X_3 < y_1]P[y_1 < Y_3]$$

$$= [F(y_1) - F(x_2)]\overline{F}(y_1)$$

$$D_5 = P[y_2 < x_1 < x_2 < y_1 < X_3 < Y_3]$$

$$= P[y_1 < X_3 < Y_3]$$

$$= \frac{\overline{F}^2(y_1)}{2}$$

$$D_6 = P[y_2 < x_1 < x_2 < y_1 < Y_3 < X_3]$$

$$= P[y_1 < Y_3 < X_3]$$

$$= \frac{\overline{F}^2(y_1)}{2}$$

$$D_7 = P[X_3 < y_2 < x_1 < x_2 < Y_3 < y_1]$$

$$= P[X_3 < y_2]P[x_2 < Y_3 < y_1]$$

$$= [F(y_2)][F(y_1) - F(x_2)]$$

$$D_8 = P[y_2 < X_3 < x_1 < x_2 < Y_3 < y_1]$$

$$= P[y_2 < X_3 < x_1]P[x_2 < Y_3 < y_1]$$

$$= [F(x_1) - F(y_2)][F(y_1) - F(x_2)]$$

$$D_9 = P[y_2 < x_1 < X_3 < x_2 < Y_3 < y_1]$$

$$= P[x_1 < X_3 < x_2]P[x_2 < Y_3 < y_1]$$

$$= [F(x_2) - F(x_1)][F(y_1) - F(x_2)]$$

$$D_{10} = P[y_2 < x_1 < x_2 < X_3 < Y_3 < y_1]$$

$$= P[x_2 < X_3 < Y_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_2)]^2}{2}$$

$$D_{11} = P[y_2 < x_1 < x_2 < Y_3 < X_3 < y_1]$$

$$= P[x_2 < Y_3 < X_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_2)]^2}{2}$$

$$D_{12} = P[y_2 < x_1 < x_2 < Y_3 < y_1 < X_3]$$

$$= P[x_2 < Y_3 < y_1]P[y_1 < X_3]$$

$$= [F(y_1) - F(x_2)]\overline{F}(y_1)$$

$$D_{13} = P[X_3 < y_2 < x_1 < Y_3 < x_2 < y_1]$$

$$= P[X_3 < y_2]P[x_1 < Y_3 < x_2]$$

$$= [F(y_2)][F(x_2) - F(x_1)]$$

$$D_{14} = P[y_2 < X_3 < x_1 < Y_3 < x_2 < y_1]$$

$$= P[y_2 < X_3 < x_1]P[x_1 < Y_3 < x_2]$$

$$= [F(x_1) - F(y_2)][F(x_2) - F(x_1)]$$

$$D_{15} = P[y_2 < x_1 < X_3 < Y_3 < x_2 < y_1]$$

$$= P[x_1 < X_3 < Y_3 < x_2]$$

$$= \frac{[F(x_2) - F(x_1)]^2}{2}.$$

Adding 15 terms in this case we have for 19: $y_2 < x_1 < x_2 < y_1$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = D_1 + D_2 + ... + D_{15}$$

$$= 1 + \frac{1}{2}F^2(x_2) - F(x_2) - \frac{1}{2}F^2(x_1).$$

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_1}\int_{-\infty}^{x_2}\int_{-\infty}^{x_1} P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(y_2)dF(x_1)dF(x_2)dF(y_1)$$

$$-\infty < y_2 < x_1 < x_2 < y_1 < \infty$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{y_1}\int_{-\infty}^{x_2}\int_{-\infty}^{x_1}\left(1+\frac{1}{2}F^2(x_2)-F(x_2)-\frac{1}{2}F^2(x_1)\right)^2 dF(y_2)dF(x_1)dF(x_2)dF(y_1)$$

$$-\infty < y_2 < x_1 < x_2 < y_1 < \infty$$

$$= \frac{11}{960}.$$

Consider case 20: $y_2 < x_2 < x_1 < y_1$.

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = E_1 + E_2 + ... + E_{15}$$

where

$E_1 = P[X_3 < y_2 < x_2 < x_1 < y_1 < Y_3]$ $\quad$ $E_2 = P[y_2 < X_3 < x_2 < x_1 < y_1 < Y_3]$

$E_3 = P[y_2 < x_2 < X_3 < x_1 < y_1 < Y_3]$ $\quad$ $E_4 = P[y_2 < x_2 < x_1 < X_3 < y_1 < Y_3]$

$E_5 = P[y_2 < x_2 < x_1 < y_1 < X_3 < Y_3]$ $\quad$ $E_6 = P[y_2 < x_2 < x_1 < y_1 < Y_3 < X_3]$

$E_7 = P[X_3 < y_2 < x_2 < x_1 < Y_3 < y_1]$ $\quad$ $E_8 = P[y_2 < X_3 < x_2 < x_1 < Y_3 < y_1]$

$E_9 = P[y_2 < x_2 < X_3 < x_1 < Y_3 < y_1]$ $\quad$ $E_{10} = P[y_2 < x_2 < x_1 < X_3 < Y_3 < y_1]$

$E_{11} = P[y_2 < x_2 < x_1 < Y_3 < X_3 < y_1]$ $\quad$ $E_{12} = P[y_2 < x_2 < x_1 < Y_3 < y_1 < X_3]$

$E_{13} = P[X_3 < y_2 < x_2 < Y_3 < x_1 < y_1]$ $\quad$ $E_{14} = P[y_2 < X_3 < x_2 < Y_3 < x_1 < y_1]$

$E_{15} = P[y_2 < x_2 < X_3 < Y_3 < x_1 < y_1]$.

Now Consider

$$E_1 = P[X_3 < y_2 < x_2 < x_1 < y_1 < Y_3]$$

$$= P[X_3 < y_2]P[y_1 < Y_3]$$

$$= [F(y_2)]\overline{F}(y_1)$$

$$E_2 = P[y_2 < X_3 < x_2 < x_1 < y_1 < Y_3]$$

$$= P[y_2 < X_3 < x_2]P[y_1 < Y_3]$$

$$= [F(x_2) - F(y_2)]\overline{F}(y_1)$$

$$E_3 = P[y_2 < x_2 < X_3 < x_1 < y_1 < Y_3]$$

$$= P[x_2 < X_3 < x_1]P[y_1 < Y_3]$$

$$= [F(x_1) - F(x_2)]\overline{F}(y_1)$$

$$E_4 = P[y_2 < x_2 < x_1 < X_3 < y_1 < Y_3]$$

$$= P[x_1 < X_3 < y_1]P[y_1 < Y_3]$$

$$= [F(y_1) - F(x_1)]\overline{F}(y_1)$$

$$E_5 = P[y_2 < x_2 < x_1 < y_1 < X_3 < Y_3]$$

$$= P[y_1 < X_3 < Y_3]$$

$$= \frac{\overline{F}^2(y_1)}{2}$$

$$E_6 = P[y_2 < x_2 < x_1 < y_1 < Y_3 < X_3]$$

$$= P[y_1 < Y_3 < X_3]$$

$$= \frac{\overline{F}^2(y_1)}{2}$$

$$E_7 = P[X_3 < y_2 < x_2 < x_1 < Y_3 < y_1]$$

$$= P[X_3 < y_2]P[x_1 < Y_3 < y_1]$$

$$= [F(y_2)][F(y_1) - F(x_1)]$$

$$E_8 = P[y_2 < X_3 < x_2 < x_1 < Y_3 < y_1]$$

$$= P[y_2 < X_3 < x_2]P[x_1 < Y_3 < y_1]$$

$$= [F(x_2) - F(y_2)][F(y_1) - F(x_1)]$$

$$E_9 = P[y_2 < x_2 < X_3 < x_1 < Y_3 < y_1]$$

$$= P[x_2 < X_3 < x_1]P[x_1 < Y_3 < y_1]$$

$$= [F(x_1) - F(x_2)][F(y_1) - F(x_1)]$$

$$E_{10} = P[y_2 < x_2 < x_1 < X_3 < Y_3 < y_1]$$

$$= P[x_1 < X_3 < Y_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_1)]^2}{2}$$

$$E_{11} = P[y_2 < x_2 < x_1 < Y_3 < X_3 < y_1]$$

$$= P[x_1 < Y_3 < X_3 < y_1]$$

$$= \frac{[F(y_1) - F(x_1)]^2}{2}$$

$$E_{12} = P[y_2 < x_2 < x_1 < Y_3 < y_1 < X_3]$$

$$= P[x_1 < Y_3 < y_1]P[y_1 < X_3]$$

$$= [F(y_1) - F(x_1)][\overline{F}(y_1)$$

$$E_{13} = P[X_3 < y_2 < x_2 < Y_3 < x_1 < y_1]$$

$$= P[X_3 < y_2]P[x_2 < Y_3 < x_1]$$

$$= [F(y_2)][F(x_1) - F(x_2)]$$

$$E_{14} = P[y_2 < X_3 < x_2 < Y_3 < x_1 < y_1]$$

$$= P[y_2 < X_3 < x_2]P[x_2 < Y_3 < x_1]$$

$$= [F(x_2) - F(y_2)][F(x_1) - F(x_2)]$$

$$E_{15} = P[y_2 < x_2 < X_3 < Y_3 < x_1 < y_1]$$

$$= P[x_2 < X_3 < Y_3 < x_1]$$

$$= \frac{[F(x_1) - F(x_2)]^2}{2}.$$

Adding 15 terms we get for case 20: $y_2 < x_2 < x_1 < y_1$

$$P[Med(x_1,x_2,X_3)<Med(y_1,y_2,Y_3)]=E_1+E_2+.....+E_{15}$$

$$=1+\frac{1}{2}F^2(x_1)-F(x_1)-\frac{1}{2}F^2(x_2).$$

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_1}\int_{-\infty}^{x_1}\int_{-\infty}^{x_2}P^2[Med(x_1,x_2,X_3)<Med(y_1,y_2,Y_3)]dF(y_2)dF(x_2)dF(x_1)dF(y_1)$$

$$-\infty<y_2<x_2<x_1<y_1<\infty$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{y_1}\int_{-\infty}^{x_1}\int_{-\infty}^{x_2}\left(1+\frac{1}{2}F^2(x_1)-F(x_1)-\frac{1}{2}F^2(x_2)\right)^2dF(y_2)dF(x_2)dF(x_1)dF(y_1)$$

$$-\infty<y_2<x_2<x_1<y_1<\infty$$

$$=\frac{11}{960}.$$

Now adding the 20 terms we get

$$P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_2,X_4)<Med(Y_1,Y_2,Y_4)]$$

$$=\frac{5}{14}.$$

Thus,

$$\xi_{2,2}=P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_2,X_4)<Med(Y_1,Y_2,Y_4)]-\frac{1}{4}$$

$$=\frac{5}{14}-\frac{1}{4}$$

$$=\frac{3}{28}.$$

$$\text{Var}(U)=\frac{1}{\binom{n}{3}\binom{m}{3}}\sum_{c=0}^{3}\sum_{d=0}^{3}\binom{3}{c}\binom{m-3}{3-c}\binom{3}{d}\binom{n-3}{3-d}\xi_{c,d}$$

$$=\frac{1}{\binom{n}{3}^2}\sum_{c=0}^{3}\sum_{d=0}^{3}\binom{3}{c}\binom{n-3}{3-c}\binom{3}{d}\binom{n-3}{3-d}\xi_{c,d}$$

$$=\frac{1}{\binom{n}{3}^2}\left\{\begin{array}{l}\left\{\frac{(n-3)^2(n-4)^2(n-5)}{2}\right\}(\xi_{1,0}+\xi_{0,1})+\left\{(n-3)^2(n-4)(n-5)\right\}(\xi_{2,0}+\xi_{0,2})\\+\left\{\frac{(n-3)(n-4)(n-5)}{3}\right\}(\xi_{3,0}+\xi_{0,3})+\left\{\frac{9(n-3)^2(n-4)^2}{4}\right\}(\xi_{1,1})\\+\left\{9(n-3)^2(n-4)\right\}(\xi_{1,2}+\xi_{2,1})+\left\{9(n-3)^2\right\}(\xi_{2,2})\\+\left\{3(n-3)(n-4)\right\}(\xi_{1,3}+\xi_{3,1})+\left\{3(n-3)\right\}(\xi_{2,3}+\xi_{3,2})+(\xi_{3,3})\end{array}\right\}$$

$$=\frac{1}{\binom{n}{3}^2}\left\{\begin{array}{l}\left\{\frac{(n-3)^2(n-4)^2(n-5)}{2}\right\}\left(\frac{131}{5775}+\frac{131}{5775}\right)+\left\{(n-3)^2(n-4)(n-5)\right\}\left(\frac{97}{2100}+\frac{97}{2100}\right)\\+\left\{\frac{(n-3)(n-4)(n-5)}{3}\right\}\left(\frac{1}{12}+\frac{1}{12}\right)+\left\{\frac{9(n-3)^2(n-4)^2}{4}\right\}\left(\frac{2323}{56700}\right)\\+\left\{9(n-3)^2(n-4)\right\}\left(\frac{19}{252}+\frac{19}{252}\right)+\left\{9(n-3)^2\right\}\left(\frac{3}{28}\right)+\left\{3(n-3)(n-4)\right\}\left(\frac{17}{140}+\frac{17}{140}\right)\\+\left\{3(n-3)\right\}\left(\frac{83}{140}+\frac{83}{140}\right)+\frac{1}{4}\end{array}\right\}$$

We now consider the case where $H_0: F(x)=F^k(x)$

Under the alternative hypothesis we have $F(x)=F^{k_0}(x)$ and

$H_1: is\ G(x)=F^{\frac{1}{k_0}}(x)$

Now we consider the mean and variance of the U-statistic under this set up.

2.4.MEAN AND THE VARIANCE WHEN $F(x)=F^{k_0}(x)$ and $G(x)=F^{\frac{1}{k_0}}(x)$.

2.4.1.MEAN

Now we have

$$\mu = E(U) = E(h)$$

$$= E(h(X_1, X_2, X_3; Y_1, Y_2, Y_3))$$

$$= P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3)]$$

$$= \int_{-\infty}^{\infty} F_{m_1}(x) dF_{m_2}(x)$$

where

$$F_{m_1}(x) = \sum_{i=2}^{3} \binom{3}{i} F^i(x)\left[1 - F^i(x)\right]^{3-i}$$

$$= \sum_{i=2}^{3} \binom{3}{i} F^{ik_0}(x)\left[1 - F^{k_0}(x)\right]^{3-i}$$

$$= 3F^{2k_0}(x) - 2F^{3k_0}(x)$$

and

$$F_{m_2}(x) = \sum_{i=2}^{3} \binom{3}{i} F^{\frac{i}{k_0}}(x)\left[1 - F^{\frac{1}{k_0}}(x)\right]^{3-i}$$

$$= 3F^{\frac{2}{k_0}}(x) - 2F^{\frac{3}{k_0}}(x).$$

Further

$$\frac{d}{dx} F_{m_2}(x) = \frac{d}{dx}\left[3F^{\frac{2}{k_0}}(x) - 2F^{\frac{3}{k_0}}(x)\right]$$

$$= \frac{6}{k_0} F^{\frac{2}{k_0}-1}(x) dF(x) - \frac{6}{k_0} F^{\frac{3}{k_0}-1}(x) dF(x)$$

Thus

$$E(U)=\int_{-\infty}^{\infty}\left[3F^{2k_0}(x)-2F^{3k_0}(x)\right]\left[\frac{6}{k_0}F^{\frac{2}{k_0}-1}(x)dF(x)-\frac{6}{k_0}F^{\frac{3}{k_0}-1}(x)dF(x)\right]dx.$$

Letting $F(x)=u$ we have

$$E(U)=\int_{0}^{1}\left[3u^{2k_0}-2u^{3k_0}\right]\left[\frac{6}{k_0}u^{\frac{2}{k_0}-1}-\frac{6}{k_0}u^{\frac{3}{k_0}-1}\right]du$$

Simplifying we get

$$\mu=E(U)=6\left[\frac{3}{\left(2k_0^{\,2}+2\right)}-\frac{2}{\left(3k_0^{\,2}+2\right)}-\frac{3}{\left(2k_0^{\,2}+3\right)}+\frac{2}{\left(3k_0^{\,2}+3\right)}\right]$$

Thus,

$$\mu=E(U)=\frac{\left(19k_0^{\,2}+6\right)}{\left(k_0^{\,2}+1\right)\left(3k_0^{\,2}+2\right)\left(2k_0^{\,2}+3\right)}$$

2.4.2. VARIANCE

Now consider variance of U. Variance of U is given by

$$\text{Var(U)}=\frac{1}{\binom{n}{3}^2}\sum_{c=0}^{3}\sum_{d=0}^{3}\binom{3}{c}\binom{m-3}{3-c}\binom{3}{d}\binom{n-3}{3-d}\xi^*_{c,d} \qquad (2.4.2.1)$$

where

$\xi^*_{c,d}$ =Covariance between the two kernels wherein c x observations and d y observations are in common between the two kernels.

Thus we have

$$\xi^*_{0,0}=0$$

$$\xi^*_{1,0}=Cov\left[h(X_1,X_2,X_3;Y_1,Y_2,Y_3),h(X_1,X_4,X_5;Y_4,Y_5,Y_6)\right]$$

$$\xi^*_{0,1}=Cov\left[h(X_1,X_2,X_3;Y_1,Y_2,Y_3),h(X_4,X_5,X_6;Y_1,Y_4,Y_5)\right]$$

$$\xi^*_{2,0} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_2, X_4; Y_4, Y_5, Y_6)]$$

$$\xi^*_{0,2} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_4, X_5, X_6; Y_1, Y_2, Y_4)]$$

$$\xi^*_{3,0} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_2, X_3; Y_4, Y_5, Y_6)]$$

$$\xi^*_{0,3} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_4, X_5, X_6; Y_1, Y_2, Y_3)]$$

$$\xi^*_{1,2} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_4, X_5; Y_1, Y_2, Y_4)] \qquad (2.4.2.2)$$

$$\xi^*_{2,1} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_2, X_4; Y_1, Y_4, Y_5)]$$

$$\xi^*_{1,3} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_4, X_5; Y_1, Y_2, Y_3)]$$

$$\xi^*_{3,1} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_2, X_3; Y_1, Y_4, Y_5)]$$

$$\xi^*_{2,3} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_2, X_4; Y_1, Y_2, Y_3)]$$

$$\xi^*_{3,2} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_2, X_3; Y_1, Y_2, Y_4)]$$

$$\xi^*_{1,1} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_4, X_5; Y_1, Y_4, Y_5)]$$

$$\xi^*_{2,2} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_2, X_4; Y_1, Y_2, Y_4)]$$

$$\xi^*_{3,3} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_2, X_3; Y_1, Y_2, Y_3)]$$

Now consider

$$\xi^*_{1,0} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_4, X_5; Y_4, Y_5, Y_6)]$$

$$= E[h(X_1, X_2, X_3; Y_1, Y_2, Y_3) h(X_1, X_4, X_5; Y_4, Y_5, Y_6)] - \{E[h(X_1, X_2, X_3; Y_1, Y_2, Y_3)]\}^2$$

$$= P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_4, X_5) < Med(Y_4, Y_5, Y_6)] - [E(U)]^2$$

Now

$$P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_4, X_5) < Med(Y_4, Y_5, Y_6)]$$

$$= \int_{-\infty}^{\infty} P^2[Med(x, X_2, X_3) < Med(Y_1, Y_2, Y_3)] dF(x)$$

Now

$$P[Med(x,X_2,X_3)<Med(Y_1,Y_2,Y_3)]$$

$$=2P[x<Med(Y_1,Y_2,Y_3),X_2<x<X_3]+2P[X_2<Med(Y_1,Y_2,Y_3),x<X_2<X_3]+$$

$$2P[X_2<Med(Y_1,Y_2,Y_3),X_3<X_2<x]$$

$$=2P[X_2<x<X_3<Med(Y_1,Y_2,Y_3)]+2P[X_2<x<Med(Y_1,Y_2,Y_3)<X_3]+$$

$$2P[x<X_2<X_3<Med(Y_1,Y_2,Y_3)]+2P[x<X_2<Med(Y_1,Y_2,Y_3)<X_3]+$$

$$2P[X_3<X_2<x<Med(Y_1,Y_2,Y_3)]+2P[X_3<X_2<Med(Y_1,Y_2,Y_3)<x]$$

(2.4.2.2a).

Now consider the first term of (2.4.2.2a).

$$P[X_2<x<X_3<Med(Y_1,Y_2,Y_3)]$$

$$=P[X_2<x]P[x<X_3<Med(Y_1,Y_2,Y_3)]$$

$$=F^{k_0}(x)\left[\int_x^{\infty}P[x_3<Med(Y_1,Y_2,Y_3)]dF(x_3)\right]$$

Now

$$P[x_3<Med(Y_1,Y_2,Y_3)]=\sum_{i=0}^{1}\binom{3}{i}F^{\frac{i}{k_0}}(x_3)\left[1-F^{\frac{1}{k_0}}(x_3)\right]^{3-i}$$

$$=\left[1-F^{\frac{1}{k_0}}(x_3)\right]^3+3F^{\frac{1}{k_0}}(x_3)\left[1-F^{\frac{1}{k_0}}(x_3)\right]^2$$

Thus

$$P[X_2<x<X_3<Med(Y_1,Y_2,Y_3)]=F^{k_0}(x)\left[\int_x^{\infty}\left[\left[1-F^{\frac{1}{k_0}}(x_3)\right]^3+3F^{\frac{1}{k_0}}(x_3)\left[1-F^{\frac{1}{k_0}}(x_3)\right]^2\right]dF^{k_0}(x_3)\right]$$

Letting $F(x_3)=w$ we get

$$P[X_2<x<X_3<Med(Y_1,Y_2,Y_3)]=F^{k_0}(x)\left[\int_{F(x)}^{1}\left[\left[1-w^{\frac{1}{k_0}}\right]^3+3w^{\frac{1}{k_0}}\left[1-w^{\frac{1}{k_0}}\right]^2\right]k_0w^{k_0-1}dw\right]$$

Simplifying we get

$$P[X_2 < x < X_3 < Med(Y_1,Y_2,Y_3)] = F^{k_0}(x)[1-F^{k_0}(x)] - \frac{3k_0^{\,2}F^{k_0}(x)}{2+k_0^{\,2}}\left[1-F^{\frac{2}{k_0}+k_0}(x)\right] + \frac{2k_0^{\,2}F^{k_0}(x)}{3+k_0^{\,2}}\left[1-F^{\frac{3}{k_0}+k_0}(x)\right].$$

consider next the second term of (2.4.2.2a).

$$P[X_2 < x < Med(Y_1,Y_2,Y_3) < X_3]$$

$$= P[X_2 < x]P[x < Med(Y_1,Y_2,Y_3) < X_3]$$

$$= F^{k_0}(x)\left[\int_x^{\infty} P(z < X_3)6G(z)\overline{G}(z)dG(z)\right]$$

$$= F^{k_0}(x)\left[6\int_x^{\infty}[1-F^{k_0}(z)]F^{\frac{1}{k_0}}(z)\left[1-F^{\frac{1}{k_0}}(z)\right]dF^{\frac{1}{k_0}}(z)\right]$$

$$= F^{k_0}(x)\left[6\int_{F(x)}^{1}[1-m^{k_0}]m^{\frac{1}{k_0}}\left[1-m^{\frac{1}{k_0}}\right]\frac{1}{k_0}m^{\frac{1}{k_0}-1}dm\right]$$

where $F(z) = m$

Further simplification yields

$$P[X_2 < x < Med(Y_1,Y_2,Y_3) < X_3] = F^{k_0}(x)\left[1-3F^{\frac{2}{k_0}}(x) - \frac{6}{k_0^{\,2}+2}\left(1-F^{k_0+\frac{2}{k_0}}(x)\right) + 2F^{\frac{3}{k_0}}(x) + \frac{6}{k_0^{\,2}+3}\left(1-F^{k_0+\frac{3}{k_0}}(x)\right)\right].$$

Now consider the third term of (2.4.2.2a)

$$P[x < X_2 < X_3 < Med(Y_1,Y_2,Y_3)]$$

$$= \int_x^{\infty} P(x_2 < X_3 < Med(Y_1,Y_2,Y_3))dF(x_2)$$

$$= \int_x^{\infty}\left[\int_{x_2}^{\infty} P(x_3 < Med(Y_1,Y_2,Y_3))dF(x_3)\right]dF(x_2)$$

$$= \int_x^{\infty}\int_{x_2}^{\infty}\left[\left[1-F^{\frac{1}{k_0}}(x_3)\right]^3 + 3F^{\frac{1}{k_0}}(x_3)\left[1-F^{\frac{1}{k_0}}(x_3)\right]^2\right]dF^{k_0}(x_3)dF^{k_0}(x_2)$$

Simplifying we get

$$P[x < X_2 < X_3 < Med(Y_1, Y_2, Y_3)] = \frac{6(1-F^{k_0}(x))}{(2+k_0^2)(3+k_0^2)} - \frac{1}{2}[1-F^{2k_0}(x)] + \frac{3k_0^4\left(1-F^{\frac{2}{k_0}+2k_0}(x)\right)}{(2+k_0^2)(2+2k_0^2)} - \frac{2k_0^4\left(1-F^{\frac{3}{k_0}+2k_0}(x)\right)}{(3+k_0^2)(3+2k_0^2)}.$$

Further the fourth term of (2.4.2.2a)

$$P[x < X_2 < Med(Y_1, Y_2, Y_3) < X_3]$$

$$= \int_x^\infty P(x_2 < Med(Y_1, Y_2, Y_3) < X_3) dF(x_2)$$

$$= \int_x^\infty \left[\int_{x_2}^\infty P(z < X_3) 6G(z)\overline{G}(z) dG(z) \right] dF(x_2)$$

$$= \int_x^\infty \int_{x_2}^\infty (1-F^{k_0}(z)) 6F^{\frac{1}{k_0}}(z)\left(1-F^{\frac{1}{k_0}}(z)\right) dF^{\frac{1}{k_0}}(z) dF^{k_0}(x_2)$$

Simplifying we get

$$P[x < X_2 < Med(Y_1, Y_2, Y_3) < X_3] = 1 - F^{k_0}(x) - \frac{3k_0^2}{(k_0^2+2)}\left(1-F^{\frac{2}{k_0}+k_0}(x)\right) - \frac{6}{(k_0^2+2)}(1-F^{k_0}(x))$$
$$+ \frac{6k_0^2}{(k_0^2+2)(2k_0^2+2)}\left(1-F^{\frac{2}{k_0}+2k_0}(x)\right) + \frac{2k_0^2}{(k_0^2+3)}\left(1-F^{\frac{3}{k_0}+k_0}(x)\right) + \frac{6}{(k_0^2+3)}(1-F^{k_0}(x))$$
$$- \frac{6k_0^2}{(k_0^2+3)(2k_0^2+3)}\left(1-F^{\frac{3}{k_0}+2k_0}(x)\right).$$

Similarly the fifth term of (2.4.2.2a) is

$$P[X_3 < X_2 < x < Med(Y_1, Y_2, Y_3)]$$

$$= P[X_3 < X_2 < x] P[x < Med(Y_1, Y_2, Y_3)]$$

$$= \left[\int_{-\infty}^x P(X_3 < x_2) dF(x_2) \right] \left[\sum_{i=0}^1 \binom{3}{i} F^{\frac{i}{k_0}}(x)\left(1-F^{\frac{1}{k_0}}(x)\right)^{3-i} \right]$$

$$=\left[\int_{-\infty}^{x} F^{k_0}(x_2)dF^{k_0}(x_2)\right]\left[\left(1-F^{\frac{1}{k_0}}(x)\right)^3+3F^{\frac{1}{k_0}}(x)\left(1-F^{\frac{1}{k_0}}(x)\right)^2\right]$$

Letting $F(x_2)=v$ we get

$$=\left[\int_{0}^{F(x)} v^{k_0}k_0v^{k_0-1}dv\right]\left[1-3F^{\frac{1}{k_0}}(x)+3F^{\frac{2}{k_0}}(x)-F^{\frac{3}{k_0}}(x)+3F^{\frac{1}{k_0}}(x)\left(1-2F^{\frac{1}{k_0}}(x)+F^{\frac{2}{k_0}}(x)\right)\right]$$

Simplifying we get

$$P[X_3<X_2<x<Med(Y_1,Y_2,Y_3)]=\frac{F^{2k_0}(x)}{2}-\frac{3}{2}F^{2k_0+\frac{2}{k_0}}(x)+F^{2k_0+\frac{3}{k_0}}(x).$$

The sixth term of (2.4.2.2a)

$$P[X_3<X_2<Med(Y_1,Y_2,Y_3)<x]$$

$$=\int_{-\infty}^{x} P(X_3<X_2<z)6G(z)\overline{G}(z)dG(z)$$

$$=\int_{-\infty}^{x}\left[\int_{-\infty}^{z} P(X_3<x_2)dF(x_2)6G(z)\overline{G}(z)dG(z)\right]$$

$$=\int_{-\infty}^{x}\int_{-\infty}^{z} F^{k_0}(x_2)dF^{k_0}(x_2)6F^{\frac{1}{k_0}}(z)\left(1-F^{\frac{1}{k_0}}(z)\right)dF^{\frac{1}{k_0}}(z)$$

Simplifying we get

$$P[X_3<X_2<Med(Y_1,Y_2,Y_3)<x]=\frac{3F^{2k_0+\frac{2}{k_0}}(x)}{\left(2k_0^{\,2}+2\right)}-\frac{3F^{2k_0+\frac{3}{k_0}}(x)}{\left(2k_0^{\,2}+3\right)}$$

Now adding the six terms and simplifying and multiplying by 2 we get

$$P[Med(x,X_2,X_3)<Med(Y_1,Y_2,Y_3)]$$

$$=\frac{6F^{2k_0+\frac{2}{k_0}}(x)}{\left(k_0^{\,2}+1\right)}-\frac{12F^{2k_0+\frac{3}{k_0}}(x)}{\left(2k_0^{\,2}+3\right)}+\frac{12F^{k_0+\frac{3}{k_0}}(x)}{\left(k_0^{\,2}+3\right)}-\frac{3k_0^{\,4}}{\left(k_0^{\,2}+1\right)\left(k_0^{\,2}+2\right)}+\frac{4k_0^{\,4}}{\left(k_0^{\,2}+3\right)\left(2k_0^{\,2}+3\right)}+1-\frac{12F^{k_0+\frac{2}{k_0}}(x)}{\left(k_0^{\,2}+2\right)}.$$

Thus we get

$$P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_4,X_5)<Med(Y_4,Y_5,Y_6)]$$

$$=\int_{-\infty}^{\infty}P^2[Med(x,X_2,X_3)<Med(Y_1,Y_2,Y_3)]dF^{k_0}(x)$$

$$=\int_{-\infty}^{\infty}\left(\begin{array}{l}\dfrac{6F^{2k_0+\frac{2}{k_0}}(x)}{(k_0^2+1)}-\dfrac{12F^{2k_0+\frac{3}{k_0}}(x)}{(2k_0^2+3)}+\dfrac{12F^{k_0+\frac{3}{k_0}}(x)}{(k_0^2+3)}-\dfrac{3k_0^4}{(k_0^2+1)(k_0^2+2)}+\dfrac{4k_0^4}{(k_0^2+3)(2k_0^2+3)}\\+1-\dfrac{12F^{k_0+\frac{2}{k_0}}(x)}{(k_0^2+2)}\end{array}\right)^2 dF^{k_0}(x)$$

$$=\int_0^1\left(\begin{array}{l}\dfrac{6u^{2k_0+\frac{2}{k_0}}}{(k_0^2+1)}-\dfrac{12u^{2k_0+\frac{3}{k_0}}}{(2k_0^2+3)}+\dfrac{12u^{k_0+\frac{3}{k_0}}}{(k_0^2+3)}-\dfrac{3k_0^4}{(k_0^2+1)(k_0^2+2)}+\dfrac{4k_0^4}{(k_0^2+3)(2k_0^2+3)}\\+1-\dfrac{12u^{k_0+\frac{2}{k_0}}}{(k_0^2+2)}\end{array}\right)^2 k_0u^{k_0-1}du$$

Letting $u=F(x)$

$$=\frac{3(265188k_0^{14}+1955069k_0^{12}+6004301k_0^{10}+9893734k_0^8+9360188k_0^6+5014520k_0^4+1377600k_0^2+144000)}{5[(3k_0^2+2)(4k_0^2+5)(3k_0^2+4)(k_0^2+2)(5k_0^2+6)(2k_0^2+3)^2(5k_0^2+4)(k_0^2+1)^2(3k_0^2+5)]}$$

Thus finally we have

$$\xi^*_{1,0}=P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_4,X_5)<Med(Y_4,Y_5,Y_6)]-[E(U)]^2$$

$$=\left[\frac{3(265188k_0^{14}+1955069k_0^{12}+6004301k_0^{10}+9893734k_0^8+9360188k_0^6+5014520k_0^4+1377600k_0^2+144000)}{5[(3k_0^2+2)(4k_0^2+5)(3k_0^2+4)(k_0^2+2)(5k_0^2+6)(2k_0^2+3)^2(5k_0^2+4)(k_0^2+1)^2(3k_0^2+5)]}\right]-\left[\frac{(9k_0^2+6)}{(k_0^2+1)(3k_0^2+2)(2k_0^2+3)}\right]^2$$

$$=\frac{8}{5}k_0^6\frac{(95274k_0^{10}+594828k_0^8+1470841k_0^6+1798869k_0^4+1086732k_0^2+258956)}{[(3k_0^2+2)^2(4k_0^2+5)(3k_0^2+4)(k_0^2+2)(5k_0^2+6)(2k_0^2+3)^2(5k_0^2+4)(k_0^2+1)^2(3k_0^2+5)]}$$

Now consider

$$\xi^*_{0,1}=Cov[h(X_1,X_2,X_3;Y_1,Y_2,Y_3),h(X_4,X_5,X_6;Y_1,Y_4,Y_5)]$$

$$=E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)h(X_4,X_5,X_6;Y_1,Y_4,Y_5)]-\{E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)]\}^2$$

$$=P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_4,X_5,X_6)<Med(Y_1,Y_4,Y_5)]-[E(U)]^2$$

Now

$$P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_4,X_5,X_6)<Med(Y_1,Y_4,Y_5)]$$

$$=\int_{-\infty}^{\infty}P^2[Med(X_1,X_2,X_3)<Med(y,Y_2,Y_3)]dG(y)$$

Further

$$P[Med(X_1,X_2,X_3)<Med(y,Y_2,Y_3)]$$

$$=2P[Med(X_1,X_2,X_3)<y,Y_2<y<Y_3]+2P[Med(X_1,X_2,X_3)<Y_2,y<Y_2<Y_3]$$
$$+2P[Med(X_1,X_2,X_3)<Y_2,Y_3<Y_2<y]$$

$$=2P[Med(X_1,X_2,X_3)<Y_2<y<Y_3]+2P[Y_2<Med(X_1,X_2,X_3)<y<Y_3]+$$

$$2P[Med(X_1,X_2,X_3)<y<Y_2<Y_3]+2P[y<Med(X_1,X_2,X_3)<Y_2<Y_3]+$$

$$2P[Med(X_1,X_2,X_3)<Y_3<Y_2<y]+2P[Y_3<Med(X_1,X_2,X_3)<Y_2<y]$$

(2.4.2.2b)

Now consider the first term of (2.4.2.2b).

$$P[Med(X_1,X_2,X_3)<Y_2<y<Y_3]$$

$$=P[Med(X_1,X_2,X_3)<Y_2<y]P[y<Y_3]$$

$$=\left[\int_{-\infty}^{y}P(Med(X_1,X_2,X_3)<y_2)dG(y_2)\right]\left[\overline{G}(y)\right]$$

$$=\left[\int_{-\infty}^{y}\sum_{i=2}^{3}\binom{3}{i}G^i(y_2)\overline{G}^{3-i}(y_2)dG(y_2)\right]\left[\overline{G}(y)\right]$$

$$=\left[\int_{-\infty}^{y}\sum_{i=2}^{3}\binom{3}{i}F^{ik_0}(y_2)\left(1-F^{k_0}(y_2)\right)^{3-i}dF^{\frac{1}{k_0}}(y_2)\right]\left[1-F^{\frac{1}{k_0}}(y)\right]$$

$$=\left[\int_{0}^{F(y)}\left[3v^{2k_0}\left(1-v^{k_0}\right)+v^{3k_0}\right]\frac{1}{k_0}v^{\frac{1}{k_0}-1}dv\right]\left[1-F^{\frac{1}{k_0}}(y)\right] \qquad \text{where } F(y_2)=v$$

$$=\left[\frac{3F^{2k_0+\frac{1}{k_0}}(y)}{\left(2k_0^{\,2}+1\right)}-\frac{2F^{3k_0+\frac{1}{k_0}}(y)}{\left(3k_0^{\,2}+1\right)}\right]\left[1-F^{\frac{1}{k_0}}(y)\right]$$

Now consider the second term of (2.4.2.2b).

$$P[Y_2 < Med(X_1, X_2, X_3) < y < Y_3]$$

$$= P[Y_2 < Med(X_1, X_2, X_3) < y]P[y < Y_3]$$

$$= \left[\int_{-\infty}^{y} P(Y_2 < z) 6F(z)\overline{F}(z) dF(z)\right]\left[\overline{G}(y)\right]$$

$$= \left[6\int_{-\infty}^{y} F^{\frac{1}{k_0}}(z)F^{k_0}(z)\left(1-F^{k_0}(z)\right) dF^{k_0}(z)\right]\left[1-F^{\frac{1}{k_0}}(y)\right]$$

$$= \left[6\int_{0}^{G(y)} m^{\frac{1}{k_0}} m^{k_0}\left(1-m^{k_0}\right) k_0 m^{k_0-1} dm\right]\left[1-G^{\frac{1}{k_0}}(y)\right] \qquad \text{where } F(z)=m$$

$$= 6k_0{}^2\left[\frac{F^{2k_0+\frac{1}{k_0}}(y)}{\left(2k_0{}^2+1\right)} - \frac{F^{3k_0+\frac{1}{k_0}}(y)}{\left(3k_0{}^2+1\right)}\right]\left[1-F^{\frac{1}{k_0}}(y)\right]$$

Now consider the third term of (2.4.2.2b).

$$P[Med(X_1, X_2, X_3) < y < Y_2 < Y_3]$$

$$= P[Med(X_1, X_2, X_3) < y]P[y < Y_2 < Y_3]$$

$$= \left[\sum_{i=2}^{3}\binom{3}{i} G^i(y)\overline{G}^{3-i}(y)\right]\left[\frac{\overline{G}^2(y)}{2}\right]$$

$$= \left[3G^2(y)\left(1-G(y)\right)+G^3(y)\right]\left[\frac{\overline{G}^2(y)}{2}\right]$$

$$= \left[3F^{2k_0}(y)\left(1-F^{k_0}(y)\right)+F^{3k_0}(y)\right]\left[\frac{\left(1-F^{\frac{1}{k_0}}(y)\right)^2}{2}\right]$$

Now consider the fourth term of (2.4.2.2b).

$$P[y < Med(X_1, X_2, X_3) < Y_2 < Y_3]$$

$$= \int_{y}^{\infty} P(z < Y_2 < Y_3) 6F(z)\overline{F}(z) dF(z)$$

$$= \int_{y}^{\infty} \left[\int_{z}^{\infty} P(y_2 < Y_3) dG(y_2) \right] 6F(z)\overline{F}(z) dF(z)$$

$$= \int_{y}^{\infty} \left[\frac{\overline{F}^{\frac{2}{k_0}}(z)}{2} \right] 6F^{k_0}(z)\overline{F}^{k_0}(z) dF^{k_0}(z)$$

$$= \int_{G(y)}^{1} \frac{\left(1 - m^{\frac{1}{k_0}}\right)^2}{2} 6m^{k_0}\left(1 - m^{k_0}\right) k_0 m^{k_0 - 1} dm \qquad \text{where} \quad F(z) = m$$

$$= \frac{3}{2} - \frac{3}{2}F^{2k_0}(y) - \frac{6k_0^{\,2}}{\left(2k_0^{\,2}+1\right)} + \frac{6k_0^{\,2}F^{2k_0+\frac{1}{k_0}}(y)}{\left(2k_0^{\,2}+1\right)} + \frac{3k_0^{\,2}}{\left(2k_0^{\,2}+2\right)} - \frac{3k_0^{\,2}F^{2k_0+\frac{2}{k_0}}(y)}{\left(2k_0^{\,2}+2\right)} - 1$$

$$+ F^{3k_0}(y) + \frac{6k_0^{\,2}}{\left(3k_0^{\,2}+1\right)} - \frac{6k_0^{\,2}F^{3k_0+\frac{1}{k_0}}(y)}{\left(3k_0^{\,2}+1\right)} - \frac{3k_0^{\,2}}{\left(3k_0^{\,2}+2\right)} + \frac{3k_0^{\,2}F^{3k_0+\frac{2}{k_0}}(y)}{\left(3k_0^{\,2}+2\right)}.$$

Now consider the fifth term of (2.4.2.2b).

$$P[Med(X_1, X_2, X_3) < Y_3 < Y_2 < y]$$

$$= \int_{-\infty}^{y} P[Med(X_1, X_2, X_3) < Y_3 < y_2] dG(y_2)$$

$$= \int_{-\infty}^{y} \int_{-\infty}^{y_2} P[Med(X_1, X_2, X_3) < y_3] dG(y_3) dG(y_2)$$

$$= \int_{-\infty}^{y} \left[\frac{3F^{2k_0+\frac{1}{k_0}}(y_2)}{\left(2k_0^{\,2}+1\right)} - \frac{2F^{3k_0+\frac{1}{k_0}}(y_2)}{\left(3k_0^{\,2}+1\right)} \right] dF^{\frac{1}{k_0}}(y_2)$$

$$= \int_{0}^{F(y)} \left[\frac{3v^{2k_0+\frac{1}{k_0}}}{\left(2k_0^{\,2}+1\right)} - \frac{2v^{3k_0+\frac{1}{k_0}}}{\left(3k_0^{\,2}+1\right)} \right] \frac{1}{k_0} v^{\frac{1}{k_0}-1} dv$$

where $F(y_2) = v$

$$= \frac{3F^{2k_0+\frac{2}{k_0}}(y)}{(2k_0^2+1)(2k_0^2+2)} - \frac{2F^{3k_0+\frac{2}{k_0}}(y)}{(3k_0^2+1)(3k_0^2+2)}.$$

Now consider the sixth term of (2.4.2.2b).

$$P[Y_3 < Med(X_1, X_2, X_3) < Y_2 < y]$$

$$= \int_{-\infty}^{y} P(Y_3 < Med(X_1, X_2, X_3) < y_2) dG(y_2)$$

$$= \int_{-\infty}^{y} \left[\int_{-\infty}^{y_2} P(Y_3 < z) 6F(z)\bar{F}(z) dF(z) \right] dG(y_2)$$

$$= \int_{-\infty}^{y} 6k_0^2 \left[\frac{F^{2k_0+\frac{1}{k_0}}(y_2)}{(2k_0^2+1)} - \frac{F^{3k_0+\frac{1}{k_0}}(y_2)}{(3k_0^2+1)} \right] dF^{\frac{1}{k_0}}(y_2)$$

$$= \int_{0}^{F(y)} 6k_0^2 \left[\frac{v^{2k_0+\frac{1}{k_0}}}{(2k_0^2+1)} - \frac{v^{3k_0+\frac{1}{k_0}}}{(3k_0^2+1)} \right] \frac{1}{k_0} v^{\frac{1}{k_0}-1} dv \qquad \text{where } F(y_2) = v$$

$$= \frac{6k_0^2 F^{2k_0+\frac{2}{k_0}}(y)}{(2k_0^2+1)(2k_0^2+2)} - \frac{6k_0^2 F^{3k_0+\frac{2}{k_0}}(y)}{(3k_0^2+1)(3k_0^2+2)}.$$

Now adding the six terms and simplifying and multiplying by 2 we get

$$P[Med(X_1, X_2, X_3) < Med(y, Y_2, Y_3)]$$

$$= \frac{12k_0^2 F^{2k_0+\frac{1}{k_0}}(y)}{(2k_0^2+1)} - \frac{6k_0^2 F^{2k_0+\frac{2}{k_0}}(y)}{(k_0^2+1)} - \frac{12k_0^2 F^{3k_0+\frac{1}{k_0}}(y)}{(3k_0^2+1)}$$

$$+ \frac{12k_0^2 F^{3k_0+\frac{2}{k_0}}(y)}{(3k_0^2+2)} - \frac{12k_0^4}{(2k_0^2+1)(3k_0^2+1)} + \frac{3k_0^4}{(k_0^2+1)(3k_0^2+2)} + 1$$

Thus we get

$$P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_4, X_5, X_6) < Med(Y_1, Y_4, Y_5)]$$

$$= \int_{-\infty}^{\infty} P^2[Med(X_1, X_2, X_3) < Med(y, Y_2, Y_3)] dG(y)$$

$$= \int_{-\infty}^{\infty} \left(\begin{array}{l} \dfrac{12k_0^{\,2} F^{2k_0+\frac{1}{k_0}}(y)}{(2k_0^{\,2}+1)} - \dfrac{6k_0^{\,2} F^{2k_0+\frac{2}{k_0}}(y)}{(k_0^{\,2}+1)} - \dfrac{12k_0^{\,2} F^{3k_0+\frac{1}{k_0}}(y)}{(3k_0^{\,2}+1)} + \dfrac{12k_0^{\,2} F^{3k_0+\frac{2}{k_0}}(y)}{(3k_0^{\,2}+2)} \\ - \dfrac{12k_0^{\,4}}{(2k_0^{\,2}+1)(3k_0^{\,2}+1)} + \dfrac{3k_0^{\,4}}{(k_0^{\,2}+1)(3k_0^{\,2}+2)} + 1 \end{array} \right)^2 dF^{\frac{1}{k_0}}(y) dy$$

$$= \int_{-\infty}^{\infty} \left(\begin{array}{l} \dfrac{12k_0^{\,2} v^{2k_0+\frac{1}{k_0}}}{(2k_0^{\,2}+1)} - \dfrac{6k_0^{\,2} v^{2k_0+\frac{2}{k_0}}}{(k_0^{\,2}+1)} - \dfrac{12k_0^{\,2} v^{3k_0+\frac{1}{k_0}}}{(3k_0^{\,2}+1)} + \dfrac{12k_0^{\,2} v^{3k_0+\frac{2}{k_0}}}{(3k_0^{\,2}+2)} \\ - \dfrac{12k_0^{\,4}}{(2k_0^{\,2}+1)(3k_0^{\,2}+1)} + \dfrac{3k_0^{\,4}}{(k_0^{\,2}+1)(3k_0^{\,2}+2)} + 1 \end{array} \right)^2 \frac{1}{k_0} v^{\frac{1}{k_0}-1} dv$$

where $F(y) = v$

$$= -\frac{1}{5} \frac{\left(\begin{array}{l} 413580240k_0^{17} - 1424205944k_0^{14} + 4779000k_0^{7} - 8451840k_0^{4} + 264224160k_0^{19} + 76848608k_0^{18} - 904552112k_0^{16} \\ + 409840920k_0^{15} + 266671980k_0^{13} - 1162641132k_0^{12} + 113918760k_0^{11} - 595425847k_0^{10} + 30798540k_0^{9} - 202869394k_0^{8} \\ + 324000k_0^{5} - 48249335k_0^{6} + 31104000k_0^{26} + 204249600k_0^{24} + 15552000k_0^{23} + 540371520k_0^{22} + 96940800k_0^{21} \\ + 653741376k_0^{20} - 1021500k_0^{2} - 54000 \end{array} \right)}{\left[(2k_0^{\,2}+3)(5k_0^{\,2}+4)(5k_0^{\,2}+3)(6k_0^{\,2}+5)(3k_0^{\,2}+2)^2 (3k_0^{\,2}+1)(4k_0^{\,2}+5)(k_0^{\,2}+1)^2 (4k_0^{\,2}+3)(2k_0^{\,2}+1)^2 \right]}$$

Thus, finally we have

$$\xi^*_{0,1} = P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_4, X_5, X_6) < Med(Y_1, Y_4, Y_5)] - [E(U)]^2$$

$$= \left[-\frac{1}{5} \frac{\left(\begin{array}{l} 413580240k_0^{17} - 1424205944k_0^{14} + 4779000k_0^{7} - 8451840k_0^{4} + 264224160k_0^{19} + 76848608k_0^{18} - 904552112k_0^{16} \\ + 409840920k_0^{15} + 266671980k_0^{13} - 1162641132k_0^{12} + 113918760k_0^{11} - 595425847k_0^{10} + 30798540k_0^{9} - 202869394k_0^{8} \\ + 324000k_0^{5} - 48249335k_0^{6} + 31104000k_0^{26} + 204249600k_0^{24} + 15552000k_0^{23} + 540371520k_0^{22} + 96940800k_0^{21} \\ + 653741376k_0^{20} - 1021500k_0^{2} - 54000 \end{array} \right)}{\left[(2k_0^{\,2}+3)(5k_0^{\,2}+4)(5k_0^{\,2}+3)(6k_0^{\,2}+5)(3k_0^{\,2}+2)^2 (3k_0^{\,2}+1)(4k_0^{\,2}+5)(k_0^{\,2}+1)^2 (4k_0^{\,2}+3)(2k_0^{\,2}+1)^2 \right]} \right] - \left[\frac{(9k_0^{\,2}+6)}{(k_0^{\,2}+1)(3k_0^{\,2}+2)(2k_0^{\,2}+3)} \right]^2$$

$$= -\frac{4}{5} k_0^{\,5} \frac{\left(\begin{array}{l} 440716680k_0^{10} + 100838340k_0^{6} + 256963365k_0^{8} - 308791521k_0^{5} - 818091770k_0^{7} \\ - 1170961026k_0^{11} - 314087200k_0^{13} + 404958240k_0^{14} - 1302708109k_0^{9} + 3746250k_0^{2} \\ + 515105640k_0^{12} + 243000 - 6075000k_0 + 541726336k_0^{15} + 60134400k_0^{18} + 732149328k_0^{17} \\ + 204817680k_0^{16} + 125452800k_0^{21} + 423372960k_0^{19} + 25488405k_0^{4} - 65658798k_0^{3} \\ + 15552000k_0^{23} + 7776000k_0^{20} \end{array} \right)}{\left[(2k_0^{\,2}+3)^2 (5k_0^{\,2}+4)(5k_0^{\,2}+3)(6k_0^{\,2}+5)(3k_0^{\,2}+2)^2 (3k_0^{\,2}+1)(4k_0^{\,2}+5)(k_0^{\,2}+1)^2 (4k_0^{\,2}+3)(2k_0^{\,2}+1)^2 \right]}.$$

Now consider

$$\xi^*_{2,0} = Cov[h(X_1,X_2,X_3;Y_1,Y_2,Y_3), h(X_1,X_2,X_4;Y_4,Y_5,Y_6)]$$

$$= E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)h(X_1,X_2,X_4;Y_4,Y_5,Y_6)] - \{E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)]\}^2$$

$$= P[Med(X_1,X_2,X_3) < Med(Y_1,Y_2,Y_3), Med(X_1,X_2,X_4) < Med(Y_4,Y_5,Y_6)] - [E(U)]^2$$

Now

$$P[Med(X_1,X_2,X_3) < Med(Y_1,Y_2,Y_3), Med(X_1,X_2,X_4) < Med(Y_4,Y_5,Y_6)]$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P^2[Med(x_1,x_2,X_3) < Med(Y_1,Y_2,Y_3)]dF(x_1)dF(x_2)$$
$$= \iint_{x_1<x_2} P^2[Med(x_1,x_2,X_3) < Med(Y_1,Y_2,Y_3)]dF(x_1)dF(x_2)$$
$$+ \iint_{x_2<x_1} P^2[Med(x_1,x_2,X_3) < Med(Y_1,Y_2,Y_3)]dF(x_1)dF(x_2)$$

(2.4.2.2c)

Now for $x_1 < x_2$

$$P[Med(x_1,x_2,X_3) < Med(Y_1,Y_2,Y_3)]$$
$$= P[x_1 < x_2 < X_3 < Med(Y_1,Y_2,Y_3)] + P[x_1 < x_2 < Med(Y_1,Y_2,Y_3) < X_3] +$$

$$P[X_3 < x_1 < x_2 < Med(Y_1,Y_2,Y_3)] + P[X_3 < x_1 < Med(Y_1,Y_2,Y_3) < x_2]$$
$$+ P[x_1 < X_3 < x_2 < Med(Y_1,Y_2,Y_3)] + P[x_1 < X_3 < Med(Y_1,Y_2,Y_3) < x_2] \quad \text{(2.4.2.2d).}$$

Now consider the first term of (2.4.2.2d).

$$P[x_1 < x_2 < X_3 < Med(Y_1,Y_2,Y_3)]$$

$$= \int_{x_2}^{\infty} P[Med(Y_1,Y_2,Y_3) > x_3]dF(x_3)$$

$$= \int_{x_2}^{\infty}\left[1 - 3F^{\frac{2}{k_0}}(x_3) + 2F^{\frac{3}{k_0}}(x_3)\right]dF^{k_0}(x_3)$$

$$= \int_{F(x_2)}^{1}\left[1 - 3w^{\frac{2}{k_0}} + 2w^{\frac{3}{k_0}}\right]k_0 w^{k_0-1}dw \qquad \text{where } F(x_3) = w$$

$$=\left[1-F^{k_0}(x_2)\right]-\frac{3k_0^{\,2}}{\left(k_0^{\,2}+2\right)}\left[1-F^{\frac{2}{k_0}+k_0}(x_2)\right]+\frac{2k_0^{\,2}}{\left(k_0^{\,2}+3\right)}\left[1-F^{\frac{3}{k_0}+k_0}(x_2)\right].$$

Now consider the second term of (2.4.2.2d).

$$P[x_1 < x_2 < Med(Y_1,Y_2,Y_3) < X_3]$$

$$=\int_{x_2}^{\infty} P(z<X_3)6F(z)\overline{F}(z)dF(z)$$

$$=6\int_{x_2}^{\infty}\overline{F}(z)F(z)\overline{F}(z)dF(z)$$

$$=6\int_{x_2}^{\infty}\overline{F}^{k_0}(z)F^{\frac{1}{k_0}}(z)\left(1-F^{\frac{1}{k_0}}(z)\right)dF^{\frac{1}{k_0}}(z)$$

$$=6\int_{F(x_2)}^{1}\left(1-m^{k_0}\right)m^{\frac{1}{k_0}}\left(1-m^{\frac{1}{k_0}}\right)\frac{1}{k_0}m^{\frac{1}{k_0}-1}dm$$

where $F(z)=m$ we

$$=3-3F^{\frac{2}{k_0}}(x_2)-2+2F^{\frac{3}{k_0}}(x_2)-\frac{6}{\left(k_0^{\,2}+2\right)}+\frac{6F^{\frac{2}{k_0}+k_0}(x_2)}{\left(k_0^{\,2}+2\right)}+\frac{6}{\left(k_0^{\,2}+3\right)}-\frac{6F^{\frac{3}{k_0}+k_0}(x_2)}{\left(k_0^{\,2}+3\right)}.$$

Now consider the third term of (2.4.2.2d).

$$P[X_3 < x_1 < x_2 < Med(Y_1,Y_2,Y_3)]$$

$$=P[X_3<x_1]P[x_2<Med(Y_1,Y_2,Y_3)]$$

$$=F(x_1)\sum_{i=0}^{1}\binom{3}{i}G^i(x_2)\overline{G}^{3-i}(x_2)$$

$$=F^{k_0}(x_1)\left[1-3F^{\frac{2}{k_0}}(x_2)+2F^{\frac{3}{k_0}}(x_2)\right].$$

Now consider the fourth term of (2.4.2.2d).

$$P[X_3 < x_1 < Med(Y_1,Y_2,Y_3) < x_2]$$

$$=P[X_3<x_1]P[x_1<Med(Y_1,Y_2,Y_3)<x_2]$$

$$= F^{k_0}(x_1)\left[P[Med(Y_1,Y_2,Y_3)< x_2]- P[Med(Y_1,Y_2,Y_3)< x_1]\right]$$

$$= F^{k_0}(x_1)\left[3F^{\frac{2}{k_0}}(x_2)-2F^{\frac{3}{k_0}}(x_2)-\left[3F^{\frac{2}{k_0}}(x_1)-2F^{\frac{3}{k_0}}(x_1)\right]\right]$$

Now consider the fifth term of (2.4.2.2d).

$$P[x_1 < X_3 < x_2 < Med\,(Y_1,Y_2,Y_3)]$$

$$= P[x_1 < X_3 < x_2]P[x_2 < Med(Y_1,Y_2,Y_3)]$$

$$= \left[F^{k_0}(x_2)- F^{k_0}(x_1)\right]\left[1-3F^{\frac{2}{k_0}}(x_2)-2F^{\frac{3}{k_0}}(x_2)\right]$$

Finally consider the sixth term of (2.4.2.2d).

$$P[x_1 < X_3 < Med(Y_1,Y_2,Y_3)< x_2]$$

$$= \iint\limits_{x_1<x_3<z<x_2} 6G(z)\overline{G}(z)dF(x_3)dG(z)$$

$$= \int_{x_1}^{x_2}\int_{x_1}^{z} 6dF^{k_0}(x_3)F^{\frac{1}{k_0}}(z)\left(1-F^{\frac{1}{k_0}}(z)\right)dF^{\frac{1}{k_0}}(z)$$

$$= \int_{x_1}^{x_2}\int_{F(x_1)}^{F(z)} 6k_0 w^{k_0-1}dwF^{\frac{1}{k_0}}(z)\left(1-F^{\frac{1}{k_0}}(z)\right)dF^{\frac{1}{k_0}}(z) \quad \text{where } F(x_3)=w$$

$$= 6\int_{F(x_1)}^{F(x_2)}\left[m^{k_0} - F^{k_0}(x_1)\right]\left[m^{\frac{1}{k_0}} - m^{\frac{2}{k_0}}\right]\frac{1}{k_0}m^{\frac{1}{k_0}-1}dm \quad \text{where } G(z)=m$$

$$= \frac{6}{k_0}\left[\frac{k_0}{(k_0^{\,2}+2)}\left(F^{k_0+\frac{2}{k_0}}(x_2)-F^{k_0+\frac{2}{k_0}}(x_1)\right)\right]-\frac{k_0}{(k_0^{\,2}+3)}\left(F^{k_0+\frac{3}{k_0}}(x_2)-F^{k_0+\frac{3}{k_0}}(x_1)\right)$$

$$-\frac{F^{k_0}(x_1)k_0}{2}\left(F^{\frac{2}{k_0}}(x_2)-F^{\frac{2}{k_0}}(x_1)\right)+\frac{F^{k_0}(x_1)k_0}{3}\left(F^{\frac{3}{k_0}}(x_2)-F^{\frac{3}{k_0}}(x_1)\right).$$

Thus by adding the six terms and simplifying we have

$$P[Med(x_1,x_2,X_3)<Med(Y_1,Y_2,Y_3)]$$

$$=1+\frac{6}{(k_0^2+2)}F^{\frac{2}{k_0}+k_0}(x_2)-\frac{6}{(k_0^2+3)}F^{\frac{3}{k_0}+k_0}(x_2)-\frac{6}{(k_0^2+2)}F^{\frac{2}{k_0}+k_0}(x_1)$$

$$+\frac{6}{(k_0^2+3)}F^{\frac{3}{k_0}+k_0}(x_1)-3F^{\frac{2}{k_0}}(x_2)+2F^{\frac{3}{k_0}}(x_2)$$

Thus we have the first term of (2.4.2.2c)

$$\int_{-\infty}^{\infty}\int_{-\infty}^{x_2}P^2[Med(x_1,x_2,X_3)<Med(Y_1,Y_2,Y_3)]dF(x_1)dF(x_2)$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{F(x_2)}\left[\begin{array}{l}1+\frac{6}{(k_0^2+2)}F^{\frac{2}{k_0}+k_0}(x_2)-\frac{6}{(k_0^2+3)}F^{\frac{3}{k_0}+k_0}(x_2)-\frac{6}{(k_0^2+2)}F^{\frac{2}{k_0}+k_0}(x_1)\\+\frac{6}{(k_0^2+3)}F^{\frac{3}{k_0}+k_0}(x_1)-3F^{\frac{2}{k_0}}(x_2)+2F^{\frac{3}{k_0}}(x_2)\end{array}\right]^2 dF(x_1)dF(x_2)$$

$$=\int_0^1\int_0^v\left[\begin{array}{l}1+\frac{6}{(k_0^2+2)}v^{\frac{2}{k_0}+k_0}-\frac{6}{(k_0^2+3)}v^{\frac{3}{k_0}+k_0}-\frac{6}{(k_0^2+2)}u^{\frac{2}{k_0}+k_0}\\+\frac{6}{(k_0^2+3)}u^{\frac{3}{k_0}+k_0}-3v^{\frac{2}{k_0}}+2v^{\frac{3}{k_0}}\end{array}\right]^2 k_0u^{k_0-1}k_0v^{k_0-1}dudv$$

where $F(x_2)=v \quad F(x_1)=u$

$$=\frac{\left(\begin{array}{l}-7704k_0^{13}+7704k_0^{14}+792k_0^{16}+110037k_0^4-11400k_0^3-45550k_0^5+159372k_0^6-74686k_0^7\\+142451k_0^8-64346k_0^9+84580k_0^{10}-30722k_0^{11}+33134k_0^{12}+40050k_0^2-792k_0^{15}+5400\end{array}\right)}{\left[(2k_0^2+3)^2(k_0^2+1)^2(k_0^2+2)(k_0^2+3)(3k_0^2+4)(3k_0^2+2)(3k_0^2+5)(2k_0^2+5)\right]}$$

Now consider case ii) $x_1 > x_2$

Interchanging x_1 and x_2 in the first case we have the second term of (2.4.2.2c)

$$\int_{-\infty}^{\infty}\int_{-\infty}^{x_1}P^2[Med(x_1,x_2,X_3)<Med(Y_1,Y_2,Y_3)]dF(x_2)dF(x_1)$$

$$= \int_0^1 \int_0^{F(x_1)} \left[\begin{array}{l} 1 + \frac{6}{\left(k_0^{\,2}+2\right)} F^{\frac{2}{k_0}+k_0}(x_1) - \frac{6}{\left(k_0^{\,2}+3\right)} F^{\frac{3}{k_0}+k_0}(x_1) - \frac{6}{\left(k_0^{\,2}+2\right)} F^{\frac{2}{k_0}+k_0}(x_2) \\ + \frac{6}{\left(k_0^{\,2}+3\right)} F^{\frac{3}{k_0}+k_0}(x_2) - 3F^{\frac{2}{k_0}}(x_1) + 2F^{\frac{3}{k_0}}(x_1) \end{array} \right]^2 dF(x_2)\,dF(x_1)$$

$$= \frac{\left(\begin{array}{l} -7704k_0^{\,13} + 7704k_0^{\,14} + 792k_0^{\,16} + 110037k_0^{\,4} - 11400k_0^{\,3} - 45550k_0^{\,5} + 159372k_0^{\,6} - 74686k_0^{\,7} \\ + 142451k_0^{\,8} - 64346k_0^{\,9} + 84580k_0^{\,10} - 30722k_0^{\,11} + 33134k_0^{\,12} + 40050k_0^{\,2} - 792k_0^{\,15} + 5400 \end{array} \right)}{\left[\left(2k_0^{\,2}+3\right)^2 \left(k_0^{\,2}+1\right)^2 \left(k_0^{\,2}+2\right)\left(k_0^{\,2}+3\right)\left(3k_0^{\,2}+4\right)\left(3k_0^{\,2}+2\right)\left(3k_0^{\,2}+5\right)\left(2k_0^{\,2}+5\right) \right]}.$$

Now by adding the first and the second term of (2.4.2.2c) we have

$$P\left[Med(X_1,X_2,X_3) < Med(Y_1,Y_2,Y_3), Med(X_1,X_2,X_4) < Med(Y_4,Y_5,Y_6)\right]$$

$$= \left[\iint_{x_1<x_2} P^2\left[Med(x_1,x_2,X_3) < Med(Y_1,Y_2,Y_3)\right] dF(x_1)\,dF(x_2) \right]$$

$$+ \left[\iint_{x_2<x_1} P^2\left[Med(x_1,x_2,X_3) < Med(Y_1,Y_2,Y_3)\right] dF(x_1)\,dF(x_2) \right]$$

$$= 2\left(\frac{\left(\begin{array}{l} -7704\,k_0^{\,13} + 7704\,k_0^{\,14} + 792\,k_0^{\,16} + 110037\,k_0^{\,4} - 11400\,k_0^{\,3} - 45550\,k_0^{\,5} + 159372\,k_0^{\,6} - 74686\,k_0^{\,7} \\ + 142451\,k_0^{\,8} - 64346\,k_0^{\,9} + 84580\,k_0^{\,10} - 30722\,k_0^{\,11} + 33134\,k_0^{\,12} + 40050\,k_0^{\,2} - 792\,k_0^{\,15} + 5400 \end{array} \right)}{\left[\left(2k_0^{\,2}+3\right)^2 \left(k_0^{\,2}+1\right)^2 \left(k_0^{\,2}+2\right)\left(k_0^{\,2}+3\right)\left(3k_0^{\,2}+4\right)\left(3k_0^{\,2}+2\right)\left(3k_0^{\,2}+5\right)\left(2k_0^{\,2}+5\right) \right]} \right),$$

Thus finally we have

$$\xi^*_{2,0} = P\left[Med(X_1,X_2,X_3) < Med(Y_1,Y_2,Y_3), Med(X_1,X_2,X_4) < Med(Y_4,Y_5,Y_6)\right] - \left[E(U)\right]^2$$

$$= \left[2\left(\frac{\left(\begin{array}{l} -7704k_0^{\,13} + 7704k_0^{\,14} + 792k_0^{\,16} + 110037k_0^{\,4} - 11400k_0^{\,3} - 45550k_0^{\,5} + 159372k_0^{\,6} - 74686k_0^{\,7} \\ + 142451k_0^{\,8} - 64346k_0^{\,9} + 84580k_0^{\,10} - 30722k_0^{\,11} + 33134k_0^{\,12} + 40050k_0^{\,2} - 792k_0^{\,15} + 5400 \end{array} \right)}{\left[\left(2k_0^{\,2}+3\right)^2 \left(k_0^{\,2}+1\right)^2 \left(k_0^{\,2}+2\right)\left(k_0^{\,2}+3\right)\left(3k_0^{\,2}+4\right)\left(3k_0^{\,2}+2\right)\left(3k_0^{\,2}+5\right)\left(2k_0^{\,2}+5\right) \right]} \right) \right] - \left[\frac{\left(9k_0^{\,2}+6\right)}{\left(k_0^{\,2}+1\right)\left(3k_0^{\,2}+2\right)\left(2k_0^{\,2}+3\right)} \right]^2$$

$$= k_0^{\,3} \frac{\left(\begin{array}{l} -49392k_0^{\,12} + 567683k_0^{\,9} - 508964k_0^{\,8} + 868428k_0^{\,7} - 705500k_0^{\,6} + 775439k_0^{\,5} - 572044k_0^{\,4} \\ + 352420k_0^{\,3} - 250600k_0^{\,2} - 45600 + 53964k_0 + 49392k_0^{\,13} + 223122k_0^{\,11} - 215148k_0^{\,10} + 4752k_0^{\,15} - 4752k_0^{\,14} \end{array} \right)}{\left[\left(2k_0^{\,2}+3\right)^2 \left(k_0^{\,2}+1\right)^2 \left(k_0^{\,2}+2\right)\left(k_0^{\,2}+3\right)\left(3k_0^{\,2}+4\right)\left(3k_0^{\,2}+2\right)^2\left(3k_0^{\,2}+5\right)\left(2k_0^{\,2}+5\right) \right]}.$$

Now consider

$$\xi^*_{0,2} = Cov\left[h(X_1,X_2,X_3;Y_1,Y_2,Y_3), h(X_4,X_5,X_6;Y_1,Y_2,Y_4)\right]$$

$$= E[h(X_1, X_2, X_3; Y_1, Y_2, Y_3)h(X_4, X_5, X_6; Y_1, Y_2, Y_4)] - \{E[h(X_1, X_2, X_3; Y_1, Y_2, Y_3)]\}^2$$

$$= P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_4, X_5, X_6) < Med(Y_1, Y_2, Y_4)] - [E(U)]^2$$

Now

$$P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_4, X_5, X_6) < Med(Y_1, Y_2, Y_4)]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P^2[Med(X_1, X_2, X_3) < Med(y_1, y_2, Y_3)]dG(y_1)dG(y_2)$$

$$= \iint_{y_1<y_2} P^2[Med(X_1, X_2, X_3) < Med(y_1, y_2, Y_3)]dG(y_1)dG(y_2)$$
$$+ \iint_{y_2<y_1} P^2[Med(X_1, X_2, X_3) < Med(y_1, y_2, Y_3)]dG(y_1)dG(y_2) \qquad (2.4.2.2f)$$

Now consider case i) $y_1 < y_2$.

$$\begin{aligned} &P[Med(X_1, X_2, X_3) < Med(y_1, y_2, Y_3)] \\ &= P[Med(X_1, X_2, X_3) < y_1 < y_2 < Y_3] + P[Y_3 < Med(X_1, X_2, X_3) < y_1 < y_2] \\ &+ P[Med(X_1, X_2, X_3) < Y_3 < y_1 < y_2] + P[Med(X_1, X_2, X_3) < y_1 < Y_3 < y_2] \\ &+ P[y_1 < Med(X_1, X_2, X_3) < y_2 < Y_3] + P[y_1 < Med(X_1, X_2, X_3) < Y_3 < y_2] \end{aligned} \qquad (2.4.2.2g)$$

Now consider the first term of (2.4.2.2g)

$$P[Med(X_1, X_2, X_3) < y_1 < y_2 < Y_3]$$

$$= P[Med(X_1, X_2, X_3) < y_1]P[y_2 < Y_3]$$

$$= \left[\sum_{i=2}^{3}\binom{3}{i}F^i(y_1)\overline{F}^{3-i}(y_1)\right][\overline{G}(y_2)]$$

$$= [3F^2(y_1)(1 - F(y_1) + F^3(y_1)][1 - G(y_2)]$$

$$= \left[3F^{2k_0}(y_1)(1 - F^{k_0}(y_1) + F^{3k_0}(y_1)\right]\left[1 - F^{\frac{1}{k_0}}(y_2)\right]$$

$$= \left[3F^{2k_0}(y_1) - 2F^{3k_0}(y_1)\right]\left[1 - F^{\frac{1}{k_0}}(y_2)\right]$$

$$= 3F^{2k_0}(y_1) - 3F^{2k_0}(y_1)F^{\frac{1}{k_0}}(y_2) - 2F^{3k_0}(y_1) + 2F^{3k_0}(y_1)F^{\frac{1}{k_0}}(y_2).$$

Now consider the second term of (2.4.2.2g).

$$P[Y_3 < Med(X_1, X_2, X_3) < y_1 < y_2]$$

$$= \int_{-\infty}^{y_1} P(Y_3 < z) 6F(z)\overline{F}(z) dF(z)$$

$$= 6\int_{-\infty}^{y_1} F^{\frac{1}{k_0}}(z) F^{k_0}(z)\left(1 - F^{k_0}(z)\right) dF^{k_0}(z)$$

$$= 6\int_{0}^{F(y_1)} m^{\frac{1}{k_0}} m^{k_0}\left(1 - m^{k_0}\right) k_0 m^{k_0 - 1} dm \qquad \text{where } F(z) = m$$

$$= 6k_0\left[\frac{F^{2k_0 + \frac{1}{k_0}}(y_1)}{\left(2k_0 + \frac{1}{k_0}\right)} - \frac{F^{3k_0 + \frac{1}{k_0}}(y_1)}{\left(3k_0 + \frac{1}{k_0}\right)}\right].$$

Now consider the third term of (2.4.2.2g).

$$P[Med(X_1, X_2, X_3) < Y_3 < y_1 < y_2]$$

$$= \int_{-\infty}^{y_1} P(Med(X_1, X_2, X_3) < y_3) dG(y_3)$$

$$= \int_{-\infty}^{y_1} \left[3F^{2k_0}(y_3) - 2F^{3k_0}(y_3)\right] dF^{\frac{1}{k_0}}(y_3)$$

$$= \frac{3F^{2k_0 + \frac{1}{k_0}}(y_1)}{\left(2k_0^{\,2} + 1\right)} - \frac{2F^{3k_0 + \frac{1}{k_0}}(y_1)}{\left(3k_0^{\,2} + 1\right)}.$$

Now consider the fourth term of (2.4.2.2g).

$$P[Med(X_1, X_2, X_3) < y_1 < Y_3 < y_2]$$

$$= P[Med(X_1, X_2, X_3) < y_1]P[y_1 < Y_3 < y_2]$$

$$= \sum_{i=2}^{3} \binom{3}{i} F^i(y_1) F^{3-i}(y_1)[G(y_2) - G(y_1)]$$

$$= \left[3F^{2k_0}(y_1) - 2F^{3k_0}(y_1)\right]\left[F^{\frac{1}{k_0}}(y_2) - F^{\frac{1}{k_0}}(y_1)\right]$$

$$= 3F^{2k_0}(y_1)F^{\frac{1}{k_0}}(y_2) - 3F^{2k_0+\frac{1}{k_0}}(y_1) - 2F^{3k_0}(y_1)F^{\frac{1}{k_0}}(y_2) + 2F^{3k_0+\frac{1}{k_0}}(y_1).$$

Now consider the fifth term of (2.4.2.2g).

$$P[y_1 < Med(X_1, X_2, X_3) < y_2 < Y_3]$$

$$= P[y_1 < Med(X_1, X_2, X_3) < y_2]P[y_2 < Y_3]$$

$$= [P[Med(X_1, X_2, X_3) < y_2] - P[Med(X_1, X_2, X_3) < y_1]][\bar{G}(y_2)]$$

$$= \begin{bmatrix} \left[3F^{2k_0}(y_2)\left(1 - F^{k_0}(y_2)\right) + F^{3k_0}(y_2)\right] \\ -\left[3F^{2k_0}(y_1)\left(1 - F^{k_0}(y_1)\right) + F^{3k_0}(y_1)\right] \end{bmatrix}\left[1 - F^{\frac{1}{k_0}}(y_2)\right]$$

$$= \left[\left[3F^{2k_0}(y_2) - 2F^{3k_0}(y_2)\right] - 3F^{2k_0}(y_1) + 2F^{3k_0}(y_1)\right]\left[1 - F^{\frac{1}{k_0}}(y_2)\right]$$

$$= 3F^{2k_0}(y_2) - 2F^{3k_0}(y_2) - 3F^{2k_0}(y_1) + 2F^{3k_0}(y_1) - 3F^{2k_0+\frac{1}{k_0}}(y_2)$$

$$+ 2F^{3k_0+\frac{1}{k_0}}(y_2) + 3F^{2k_0}(y_1)F^{\frac{1}{k_0}}(y_2) - 2F^{3k_0}(y_1)F^{\frac{1}{k_0}}(y_2)$$

Finally consider the sixth term of (2.4.2.2g).

$$P[y_1 < Med(X_1, X_2, X_3) < Y_3 < y_2]$$

$$= \iint_{y_1 < z < y_3 < y_2} 6F(z)\bar{F}(z)dG(y_3)dF(z)$$

$$= \int_{y_1}^{y_2}\int_{z}^{y_2} 6dG(y_3)F(z)\bar{F}(z)dF(z)$$

$$= 6\int_{y_1}^{y_2}\int_{z}^{y_2} dF^{\frac{1}{k_0}}(y_3)F^{k_0}(z)\left(1-F^{k_0}(z)\right)dF^{k_0}(z)$$

$$= 6\int_{y_1}^{y_2}\int_{F(z)}^{F(y_2)} \frac{1}{k_0} w^{\frac{1}{k_0}-1} dw F^{k_0}(z)\left(1-F^{k_0}(z)\right)dF^{k_0}(z) \quad \text{where } F(y_3)=w$$

$$= 6\int_{y_1}^{y_2}\left[F^{\frac{1}{k_0}}(y_2)-F^{\frac{1}{k_0}}(z)\right]\left[F^{k_0}(z)-F^{2k_0}(z)\right]dF^{k_0}(z)$$

$$= 6\int_{F(y_1)}^{F(y_2)}\left[F^{\frac{1}{k_0}}(y_2)m^{k_0}-F^{\frac{1}{k_0}}(y_2)m^{2k_0}-m^{\frac{1}{k_0}+k_0}+m^{\frac{1}{k_0}+2k_0}\right]k_0 m^{k_0-1}dm \qquad \text{where } F(z)=m$$

$$= 3F^{2k_0+\frac{1}{k_0}}(y_2)-3F^{\frac{1}{k_0}}(y_2)F^{2k_0}(y_1)-2F^{3k_0+\frac{1}{k_0}}(y_2)+2F^{\frac{1}{k_0}}(y_2)F^{3k_0}(y_1)$$

$$-\frac{6k_0^{\,2}F^{2k_0+\frac{1}{k_0}}(y_2)}{\left(2k_0^{\,2}+1\right)}+\frac{6k_0^{\,2}F^{2k_0+\frac{1}{k_0}}(y_1)}{\left(2k_0^{\,2}+1\right)}+\frac{6k_0^{\,2}F^{3k_0+\frac{1}{k_0}}(y_2)}{\left(3k_0^{\,2}+1\right)}-\frac{6k_0^{\,2}F^{3k_0+\frac{1}{k_0}}(y_1)}{\left(3k_0^{\,2}+1\right)}.$$

Thus by adding the six terms and simplifying we have,

$$P[Med(X_1,X_2,X_3)<Med(y_1,y_2,Y_3)]$$

$$= \frac{6k_0^{\,2}F^{2k_0+\frac{1}{k_0}}(y_1)}{\left(2k_0^{\,2}+1\right)}-\frac{6k_0^{\,2}F^{3k_0+\frac{1}{k_0}}(y_1)}{\left(3k_0^{\,2}+1\right)}-\frac{6k_0^{\,2}F^{2k_0+\frac{1}{k_0}}(y_2)}{\left(2k_0^{\,2}+1\right)}+\frac{6k_0^{\,2}F^{3k_0+\frac{1}{k_0}}(y_2)}{\left(3k_0^{\,2}+1\right)}+3F^{2k_0}(y_2)-2F^{3k_0}(y_2)$$

Thus we have the first term of (2.4.2.2f)

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{y_2} P^2[Med(X_1,X_2,X_3)<Med(y_1,y_2,Y_3)]dG(y_1)dG(y_2)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{y_2}\left[\frac{6k_0^{\,2}F^{2k_0+\frac{1}{k_0}}(y_1)}{\left(2k_0^{\,2}+1\right)}-\frac{6k_0^{\,2}F^{3k_0+\frac{1}{k_0}}(y_1)}{\left(3k_0^{\,2}+1\right)}-\frac{6k_0^{\,2}F^{2k_0+\frac{1}{k_0}}(y_2)}{\left(2k_0^{\,2}+1\right)}+\frac{6k_0^{\,2}F^{3k_0+\frac{1}{k_0}}(y_2)}{\left(3k_0^{\,2}+1\right)}+3F^{2k_0}(y_2)-2F^{3k_0}(y_2)\right]^2 dF^{\frac{1}{k_0}}(y_1)dF^{\frac{1}{k_0}}(y_2)$$

$$= \int_{0}^{1}\int_{0}^{v}\left[\frac{6k_0^{\,2}u^{2k_0+\frac{1}{k_0}}}{\left(2k_0^{\,2}+1\right)}-\frac{6k_0^{\,2}u^{3k_0+\frac{1}{k_0}}}{\left(3k_0^{\,2}+1\right)}-\frac{6k_0^{\,2}v^{2k_0+\frac{1}{k_0}}}{\left(2k_0^{\,2}+1\right)}+\frac{6k_0^{\,2}v^{3k_0+\frac{1}{k_0}}}{\left(3k_0^{\,2}+1\right)}+3v^{2k_0}-2v^{3k_0}\right]^2 \frac{1}{k_0}u^{\frac{1}{k_0}-1}\frac{1}{k_0}v^{\frac{1}{k_0}-1}dudv$$

Where $F(y_1)=u$, $F(y_2)=v$

$$= \frac{1}{2}\frac{\left(72+1116k_0^{\,2}+7102k_0^{\,4}+23601k_0^{\,6}+43416k_0^{\,8}+42748k_0^{\,10}+19182k_0^{\,12}+2091k_0^{\,14}\right)}{\left[\left(3k_0^{\,2}+2\right)^2\left(2k_0^{\,2}+1\right)\left(4k_0^{\,2}+3\right)\left(k_0^{\,2}+1\right)^2\left(3k_0^{\,2}+1\right)\left(5k_0^{\,2}+3\right)\left(5k_0^{\,2}+2\right)\right]}$$

Now consider case ii) $y_1 > y_2$

Interchanging y_1 and y_2 in the above case we have the second term of (2.4.2.2f)

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_1} P^2[Med(X_1,X_2,X_3) < Med(y_1,y_2,Y_3)]dG(y_2)dG(y_1)$$

$$= \frac{1}{2}\frac{\left(72+1116k_0^{\,2}+7102k_0^{\,4}+23601k_0^{\,6}+43416k_0^{\,8}+42748k_0^{\,10}+19182k_0^{\,12}+2091k_0^{\,14}\right)}{\left[\left(3k_0^{\,2}+2\right)^2\left(2k_0^{\,2}+1\right)\left(4k_0^{\,2}+3\right)\left(k_0^{\,2}+1\right)^2\left(3k_0^{\,2}+1\right)\left(5k_0^{\,2}+3\right)\left(5k_0^{\,2}+2\right)\right]}$$

Now by adding the first and the second term of (2.4.2.2f) we have

$$P[Med(X_1,X_2,X_3) < Med(Y_1,Y_2,Y_3), Med(X_4,X_5,X_6) < Med(Y_1,Y_2,Y_4)]$$

$$= \left[\iint_{y_1<y_2} P^2[Med(X_1,X_2,X_3) < Med(y_1,y_2,Y_3)]dG(y_1)dG(y_2)\right]$$

$$+ \left[\iint_{y_2<y_1} P^2[Med(X_1,X_2,X_3) < Med(y_1,y_2,Y_3)]dG(y_2)dG(y_1)\right]$$

$$= \frac{\left(72+1116k_0^{\,2}+7102k_0^{\,4}+23601k_0^{\,6}+43416k_0^{\,8}+42748k_0^{\,10}+19182k_0^{\,12}+2091k_0^{\,14}\right)}{\left[\left(3k_0^{\,2}+2\right)^2\left(2k_0^{\,2}+1\right)\left(4k_0^{\,2}+3\right)\left(k_0^{\,2}+1\right)^2\left(3k_0^{\,2}+1\right)\left(5k_0^{\,2}+3\right)\left(5k_0^{\,2}+2\right)\right]}$$

Thus finally we have

$$\xi^*_{0,2} = P[Med(X_1,X_2,X_3) < Med(Y_1,Y_2,Y_3), Med(X_4,X_5,X_6) < Med(Y_1,Y_2,Y_4)] - [E(U)]^2$$

$$= \left[\frac{\left(72+1116k_0^{\,2}+7102k_0^{\,4}+23601k_0^{\,6}+43416k_0^{\,8}+42748k_0^{\,10}+19182k_0^{\,12}+2091k_0^{\,14}\right)}{\left[\left(3k_0^{\,2}+2\right)^2\left(2k_0^{\,2}+1\right)\left(4k_0^{\,2}+3\right)\left(k_0^{\,2}+1\right)^2\left(3k_0^{\,2}+1\right)\left(5k_0^{\,2}+3\right)\left(5k_0^{\,2}+2\right)\right]}\right] - \left[\frac{\left(9k_0^{\,2}+6\right)}{\left(k_0^{\,2}+1\right)\left(3k_0^{\,2}+2\right)\left(2k_0^{\,2}+3\right)}\right]^2$$

$$= k_0^{\,8}\left[\frac{\left(7974+58791k_0^{\,2}+162928k_0^{\,4}+203395k_0^{\,6}+101820k_0^{\,8}+8364k_0^{\,10}\right)}{\left[\left(3k_0^{\,2}+2\right)^2\left(2k_0^{\,2}+1\right)\left(4k_0^{\,2}+3\right)\left(k_0^{\,2}+1\right)^2\left(3k_0^{\,2}+1\right)\left(5k_0^{\,2}+3\right)\left(5k_0^{\,2}+2\right)\left(2k_0^{\,2}+3\right)^2\right]}\right].$$

Now consider

$$\xi^*_{3,0} = Cov[h(X_1,X_2,X_3;Y_1,Y_2,Y_3), h(X_1,X_2,X_3;Y_4,Y_5,Y_6)]$$

$$= E[h(X_1, X_2, X_3; Y_1, Y_2, Y_3)h(X_1, X_2, X_3; Y_4, Y_5, Y_6)] - \{E[h(X_1, X_2, X_3; Y_1, Y_2, Y_3)]\}^2$$

$$= P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_2, X_3) < Med(Y_4, Y_5, Y_6)] - [E(U)]^2 .$$

Now

$$P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_2, X_3) < Med(Y_4, Y_5, Y_6)]$$

$$= \int P^2[x < Med(Y_1, Y_2, Y_3)]6F(x)\overline{F}(x)dF(x)$$

$$= \int_{-\infty}^{\infty} \left[1 - 3F^{\frac{2}{k_0}}(x) + 2F^{\frac{3}{k_0}}(x)\right]^2 6F^{k_0}(x)\left[1 - F^{k_0}(x)\right]dF^{k_0}(x)$$

$$= 6\int_0^1 \left[1 - 3u^{\frac{2}{k_0}} + 2u^{\frac{3}{k_0}}\right]^2 \left[u^{k_0} - u^{2k_0}\right]k_0 u^{k_0 - 1}du$$

where $F(x) = u$

Simplifying we get

$$P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_2, X_3) < Med(Y_4, Y_5, Y_6)] =$$

$$= 2\left[\frac{\left(10350k_0^{\,2} + 9328k_0^{\,6} + 1899k_0^{\,8} + 15823k_0^{\,4} + 1800\right)}{\left(k_0^{\,2} + 2\right)\left(k_0^{\,2} + 3\right)\left(k_0^{\,2} + 1\right)\left(2k_0^{\,2} + 3\right)\left(2k_0^{\,2} + 5\right)\left(3k_0^{\,2} + 4\right)\left(3k_0^{\,2} + 2\right)\left(3k_0^{\,2} + 5\right)}\right].$$

Thus

$$\xi^*_{3,0} = P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_2, X_3) < Med(Y_4, Y_5, Y_6)] - [E(U)]^2$$

$$= \left[2\frac{\left(10350k_0^{\,2} + 9328k_0^{\,6} + 1899k_0^{\,8} + 15823k_0^{\,4} + 1800\right)}{\left(k_0^{\,2} + 2\right)\left(k_0^{\,2} + 3\right)\left(k_0^{\,2} + 1\right)\left(2k_0^{\,2} + 3\right)\left(2k_0^{\,2} + 5\right)\left(3k_0^{\,2} + 4\right)\left(3k_0^{\,2} + 2\right)\left(3k_0^{\,2} + 5\right)}\right] - \left[\frac{\left(9k_0^{\,2} + 6\right)}{\left(k_0^{\,2} + 1\right)\left(3k_0^{\,2} + 2\right)\left(2k_0^{\,2} + 3\right)}\right]^2$$

$$\xi^*_{3,0} = k_0^{\,4}\left[\frac{\left(352129k_0^{\,4} + 182820k_0^{\,2} + 25092 + 111765k_0^{\,8} + 291904k_0^{\,6} + 16290k_0^{\,10}\right)}{\left(k_0^{\,2} + 2\right)\left(k_0^{\,2} + 3\right)\left(k_0^{\,2} + 1\right)^2\left(2k_0^{\,2} + 3\right)^2\left(2k_0^{\,2} + 5\right)\left(3k_0^{\,2} + 4\right)\left(3k_0^{\,2} + 2\right)\left(3k_0^{\,2} + 5\right)}\right].$$

Now consider

$$\xi^*_{0,3} = Cov[h(X_1,X_2,X_3;Y_1,Y_2,Y_3), h(X_4,X_5,X_6;Y_1,Y_2,Y_3)]$$

$$= E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)h(X_4,X_5,X_6;Y_1,Y_2,Y_3)] - \{E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)]\}^2$$

$$= P[Med(X_1,X_2,X_3) < Med(Y_1,Y_2,Y_3), Med(X_4,X_5,X_6) < Med(Y_1,Y_2,Y_3)] - [E(U)]^2$$

Now

$$P[Med(X_1,X_2,X_3) < Med(Y_1,Y_2,Y_3), Med(X_4,X_5,X_6) < Med(Y_1,Y_2,Y_3)]$$
$$= \int P^2[Med(X_1,X_2,X_3) < y] 6G(y)\overline{G}(y)dG(y)$$

$$= \int_{-\infty}^{\infty} \left(3F^{2k_0}(y)\left(1-F^{k_0}(y)\right)+F^{3k_0}(y)\right)^2 6F^{\frac{1}{k_0}}(y)\left(1-F^{\frac{1}{k_0}}(y)\right)dF^{\frac{1}{k_0}}(y)$$

$$= \int_0^1 \left(3v^{2k_0}\left(1-v^{k_0}\right)+v^{3k_0}\right)^2 6v^{\frac{1}{k_0}}\left(1-v^{\frac{1}{k_0}}\right)\frac{1}{k_0}v^{\frac{1}{k_0}-1}dv \qquad \text{where } F(y)=v$$

$$P[Med(X_1,X_2,X_3) < Med(Y_1,Y_2,Y_3), Med(X_4,X_5,X_6) < Med(Y_1,Y_2,Y_3)]$$

$$= \frac{\left(18+189k_0^2+664k_0^4+697k_0^6\right)}{\left[\left(2k_0^2+1\right)\left(5k_0^2+2\right)\left(3k_0^2+1\right)\left(4k_0^2+3\right)\left(5k_0^2+3\right)\right]}.$$

Thus

$$\xi^*_{0,3} = P[Med(X_1,X_2,X_3) < Med(Y_1,Y_2,Y_3), Med(X_4,X_5,X_6) < Med(Y_1,Y_2,Y_3)] - [E(U)]^2$$

$$= \left[\frac{\left(18+189k_0^2+664k_0^4+697k_0^6\right)}{\left[\left(2k_0^2+1\right)\left(5k_0^2+2\right)\left(3k_0^2+1\right)\left(4k_0^2+3\right)\left(5k_0^2+3\right)\right]}\right] - \left[\frac{\left(9k_0^2+6\right)}{\left(k_0^2+1\right)\left(3k_0^2+2\right)\left(2k_0^2+3\right)}\right]^2$$

$$\xi^*_{0,3} = k_0^8 \left[\frac{\left(291904k_0^4+111765k_0^2+16290+352129k_0^6+182820k_0^8+25092k_0^{10}\right)}{\left(2k_0^2+1\right)\left(5k_0^2+2\right)\left(3k_0^2+1\right)\left(4k_0^2+3\right)\left(5k_0^2+3\right)\left(k_0^2+1\right)^2\left(3k_0^2+2\right)^2\left(2k_0^2+3\right)^2}\right].$$

Now consider

$$\xi^*_{3,3} = Cov[h(X_1,X_2,X_3;Y_1,Y_2,Y_3), h(X_1,X_2,X_3;Y_1,Y_2,Y_3)]$$

$$= Var[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)]$$

$$= E\left[h^2(X_1, X_2, X_3; Y_1, Y_2, Y_3)\right] - \left[E(U)\right]^2$$

$$= \left[\left[\frac{\left(19k_0^{\,2}+6\right)}{\left(k_0^{\,2}+1\right)\left(3k_0^{\,2}+2\right)\left(2k_0^{\,2}+3\right)}\right]\right] - \left[\left[\frac{\left(19k_0^{\,2}+6\right)}{\left(k_0^{\,2}+1\right)\left(3k_0^{\,2}+2\right)\left(2k_0^{\,2}+3\right)}\right]\right]^2$$

$$= \left(19k_0^{\,2}+6\right)k_0^{\,4}\left[\frac{\left(6k_0^{\,2}+19\right)}{\left(k_0^{\,2}+1\right)^2\left(3k_0^{\,2}+2\right)^2\left(2k_0^{\,2}+3\right)^2}\right].$$

Now consider

$$\xi^*_{1,2} = Cov\left[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_4, X_5; Y_1, Y_2, Y_4)\right]$$

$$= E\left[h(X_1, X_2, X_3; Y_1, Y_2, Y_3)h(X_1, X_4, X_5; Y_1, Y_2, Y_4)\right] - \left\{E\left[h(X_1, X_2, X_3; Y_1, Y_2, Y_3)\right]\right\}^2$$

$$= P\left[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_4, X_5) < Med(Y_1, Y_2, Y_4)\right] - \left[E(U)\right]^2$$

Now

$$P\left[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_4, X_5) < Med(Y_1, Y_2, Y_4)\right]$$

$$= \iiint P^2\left[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)\right] dF(x_1)\, dG(y_1)\, dG(y_2)$$

$$= \iiint_{x_1 < y_1 < y_2} P^2\left[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)\right] dF(x_1)\, dG(y_1)\, dG(y_2)$$

$$+ \iiint_{x_1 < y_2 < y_1} P^2\left[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)\right] dF(x_1)\, dG(y_1)\, dG(y_2)$$

$$+ \iiint_{y_1 < x_1 < y_2} P^2\left[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)\right] dF(x_1)\, dG(y_1)\, dG(y_2)$$

$$+ \iiint_{y_2 < x_1 < y_1} P^2\left[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)\right] dF(x_1)\, dG(y_1)\, dG(y_2)$$

$$+ \iiint_{y_1 < y_2 < x_1} P^2\left[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)\right] dF(x_1)\, dG(y_1)\, dG(y_2)$$

$$+ \iiint_{y_2 < y_1 < x_1} P^2\left[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)\right] dF(x_1)\, dG(y_1)\, dG(y_2) \quad (2.4.2.2I)$$

Now consider the first term of (2.4.2.2I).

$$\iiint_{x_1<y_1<y_2} P^2[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dG(y_1)dG(y_2)$$

We have

$$P[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)] = 2F_1 + 2F_2 + ... + 2F_{48}$$

where

$$F_1 = P[x_1 < y_1 < X_2 < y_2 < X_3 < Y_3]$$

$$= P[y_1 < X_2 < y_2]P[y_2 < X_3 < Y_3]$$

$$= \left[r^{k_0} - v^{k_0}\right]\left[\left(1 - r^{k_0}\right) - \frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)}\left(1 - r^{\frac{1}{k_0}+k_0}\right)\right]$$

$$F_2 = P[x_1 < y_1 < X_2 < y_2 < Y_3 < X_3]$$

$$= P[y_1 < X_2 < y_2]P[y_2 < Y_3 < X_3]$$

$$= \left[r^{k_0} - v^{k_0}\right]\left[\left(1 - r^{\frac{1}{k_0}}\right) - \frac{1}{\left(k_0^{\,2}+1\right)}\left(1 - r^{\frac{1}{k_0}+k_0}\right)\right]$$

$$F_3 = P[x_1 < y_1 < X_2 < X_3 < y_2 < Y_3]$$

$$= P[y_1 < X_2 < X_3 < y_2]P[y_2 < Y_3]$$

$$= \left[\int_{y_1}^{y_2}\int_{y_1}^{x_3} dF(x_2)dF(x_3)\right]\left[1 - r^{\frac{1}{k_0}}\right]$$

$$= \left[\int_{v}^{r}\int_{v}^{n} k_0 m^{k_0-1} k_0 n^{k_0-1} dmdn\right]\left[1 - r^{\frac{1}{k_0}}\right]$$

$$= \left[\frac{1}{2}\left(r^{2k_0} - v^{2k_0}\right) - v^{k_0}\left(r^{k_0} - v^{k_0}\right)\right]\left[1 - r^{\frac{1}{k_0}}\right]$$

$$F_4 = P[x_1 < y_1 < X_2 < Y_3 < y_2 < X_3]$$

$$= P[y_1 < X_2 < Y_3 < y_2]P[y_2 < X_3]$$

$$= \left[\int_{y_1}^{y_2} \int_{y_1}^{y_3} dF(x_2) dG(y_3) \right] \left[1 - r^{k_0} \right]$$

$$= \left[\int_{v}^{r} \int_{v}^{s} k_0 m^{k_0 - 1} \frac{1}{k_0} s^{\frac{1}{k_0} - 1} dm ds \right] \left[1 - r^{k_0} \right]$$

$$= \left[\frac{1}{\left(k_0^{\,2} + 1 \right)} \left(r^{\frac{1}{k_0} + k_0} - v^{\frac{1}{k_0} + k_0} \right) - v^{k_0} \left(r^{\frac{1}{k_0}} - v^{\frac{1}{k_0}} \right) \right] \left[1 - r^{k_0} \right]$$

$$F_5 = P\left[x_1 < y_1 < X_2 < X_3 < Y_3 < y_2 \right]$$

$$= P\left[y_1 < X_2 < X_3 < Y_3 < y_2 \right]$$

$$= \int_{y_1}^{y_2} \int_{y_1}^{y_3} \int_{y_1}^{x_3} dF(x_2) dF(x_3) dG(y_3)$$

$$= \int_{v}^{r} \int_{v}^{s} \int_{v}^{n} k_0 m^{k_0 - 1} k_0 n^{k_0 - 1} \frac{1}{k_0} s^{\frac{1}{k_0} - 1} dm dn ds$$

$$= \frac{\left(r^{\frac{1}{k_0} + 2k_0} - v^{\frac{1}{k_0} + 2k_0} \right)}{2\left(2k_0^{\,2} + 1 \right)} - \frac{v^{2k_0}}{2} \left[r^{\frac{1}{k_0}} - v^{\frac{1}{k_0}} \right] - \frac{v^{k_0}}{\left(k_0^{\,2} + 1 \right)} \left[r^{\frac{1}{k_0} + k_0} - v^{\frac{1}{k_0} + k_0} \right] + v^{2k_0} \left[r^{\frac{1}{k_0}} - v^{\frac{1}{k_0}} \right]$$

$$F_6 = P\left[x_1 < y_1 < X_2 < Y_3 < X_3 < y_2 \right]$$

$$= P\left[y_1 < X_2 < Y_3 < X_3 < y_2 \right]$$

$$= \int_{y_1}^{y_2} \int_{y_1}^{x_3} \int_{y_1}^{y_3} dF(x_2) dG(y_3) dF(x_3)$$

$$= \int_{v}^{r} \int_{v}^{n} \int_{v}^{s} k_0 m^{k_0 - 1} \frac{1}{k_0} s^{\frac{1}{k_0} - 1} k_0 n^{k_0 - 1} dm ds dn$$

$$= \frac{k_0^{\,2} \left(r^{\frac{1}{k_0} + 2k_0} - v^{\frac{1}{k_0} + 2k_0} \right)}{\left(k_0^{\,2} + 1 \right)\left(2k_0^{\,2} + 1 \right)} - \frac{v^{\frac{1}{k_0} + k_0}}{\left(k_0^{\,2} + 1 \right)} \left[r^{k_0} - v^{k_0} \right] - \frac{k_0^{\,2} v^{k_0}}{\left(k_0^{\,2} + 1 \right)} \left[r^{\frac{1}{k_0} + k_0} - v^{\frac{1}{k_0} + k_0} \right] + v^{\frac{1}{k_0} + k_0} \left[r^{k_0} - v^{k_0} \right]$$

$$F_7 = P\left[x_1 < X_2 < y_1 < y_2 < X_3 < Y_3 \right]$$

$$= P[x_1 < X_2 < y_1]P[y_2 < X_3 < Y_3]$$

$$= \left[v^{k_0} - u^{k_0}\right]\left[\left(1 - r^{k_0}\right) - \frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)}\left[1 - r^{\frac{1}{k_0}+1}\right]\right]$$

$$F_8 = P[x_1 < X_2 < y_1 < y_2 < Y_3 < X_3]$$

$$= P[x_1 < X_2 < y_1]P[y_2 < Y_3 < X_3]$$

$$= \left[v^{k_0} - u^{k_0}\right]\left[\left(1 - r^{\frac{1}{k_0}}\right) - \frac{1}{\left(k_0^{\,2}+1\right)}\left[1 - r^{\frac{1}{k_0}+k_0}\right]\right]$$

$$F_9 = P[x_1 < X_2 < y_1 < X_3 < y_2 < Y_3]$$

$$= P[x_1 < X_2 < y_1]P[y_1 < X_3 < y_2]P[y_2 < Y_3]$$

$$= \left[v^{k_0} - u^{k_0}\right]\left[r^{k_0} - v^{k_0}\right]\left(1 - r^{\frac{1}{k_0}}\right)$$

$$F_{10} = P[x_1 < X_2 < y_1 < Y_3 < y_2 < X_3]$$

$$= P[x_1 < X_2 < y_1]P[y_1 < Y_3 < y_2]P[y_2 < X_3]$$

$$= \left[v^{k_0} - u^{k_0}\right]\left[r^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right]\left(1 - r^{k_0}\right)$$

$$F_{11} = P[x_1 < X_2 < y_1 < X_3 < Y_3 < y_2]$$

$$= P[x_1 < X_2 < y_1]P[y_1 < X_3 < Y_3 < y_2]$$

$$= \left[v^{k_0} - u^{k_0}\right]\left[\frac{1}{\left(k_0^{\,2}+1\right)}\left[r^{\frac{1}{k_0}+k_0} - v^{\frac{1}{k_0}+k_0}\right] - v^{k_0}\left[r^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right]\right]$$

$$F_{12} = P[x_1 < X_2 < y_1 < Y_3 < X_3 < y_2]$$

$$= P[x_1 < X_2 < y_1]P[y_1 < Y_3 < X_3 < y_2]$$

$$= \left[v^{k_0} - u^{k_0}\right]\int_{y_1}^{y_2}\int_{y_1}^{x_3} dF(y_3)dF(x_3)$$

$$= \left[v^{k_0} - u^{k_0}\right] \int_{v}^{r} \int_{v}^{n} \frac{1}{k_0} s^{\frac{1}{k_0}-1} k_0 n^{k_0-1} dsdn$$

$$= \left[v^{k_0} - u^{k_0}\right] \left[\frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)}\left[r^{\frac{1}{k_0}+k_0} - v^{\frac{1}{k_0}+k_0}\right] - v^{\frac{1}{k_0}}\left[r^{k_0} - v^{k_0}\right]\right]$$

$$F_{13} = P[x_1 < X_2 < X_3 < y_1 < y_2 < Y_3]$$

$$= P[x_1 < X_2 < X_3 < y_1]P[y_2 < Y_3]$$

$$= \left[\frac{1}{2}\left(v^{2k_0} - u^{2k_0}\right) - u^{k_0}\left(v^{k_0} - u^{k_0}\right)\right]\left[1 - r^{\frac{1}{k_0}}\right]$$

$$F_{14} = P[x_1 < X_2 < Y_3 < y_1 < y_2 < X_3]$$

$$= P[x_1 < X_2 < Y_3 < y_1]P[y_2 < X_3]$$

$$= \left[\frac{1}{\left(k_0^{\,2}+1\right)}\left(v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right) - u^{k_0}\left(v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\right]\left[1 - r^{k_0}\right]$$

$$F_{15} = P[x_1 < Y_3 < X_2 < y_1 < y_2 < X_3]$$

$$= P[x_1 < Y_3 < X_2 < y_1]P[y_2 < X_3]$$

$$= \left[\frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)}\left(v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right) - u^{\frac{1}{k_0}}\left(v^{k_0} - u^{k_0}\right)\right]\left[1 - r^{k_0}\right]$$

$$F_{16} = P[x_1 < X_2 < X_3 < y_1 < Y_3 < y_2]$$

$$= P[x_1 < X_2 < X_3 < y_1]P[y_1 < Y_3 < y_2]$$

$$= \left[\frac{1}{2}\left(v^{2k_0} - u^{2k_0}\right) - u^{k_0}\left(v^{k_0} - u^{k_0}\right)\right]\left[r^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right]$$

$$F_{17} = P[x_1 < X_2 < Y_3 < y_1 < X_3 < y_2]$$

$$= P[x_1 < X_2 < Y_3 < y_1]P[y_1 < X_3 < y_2]$$

$$= \left[\frac{1}{\left(k_0^{\,2}+1\right)} \left(v^{\frac{1}{k_0}+1} - u^{\frac{1}{k_0}+1} \right) - u^{k_0} \left(v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}} \right) \right] \left[r^{k_0} - v^{k_0} \right]$$

$$F_{18} = P[x_1 < Y_3 < X_2 < y_1 < X_3 < y_2]$$

$$= P[x_1 < Y_3 < X_2 < y_1] P[y_1 < X_3 < y_2]$$

$$= \left[\frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)} \left(v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0} \right) - u^{\frac{1}{k_0}} \left(v^{k_0} - u^{k_0} \right) \right] \left[r^{k_0} - v^{k_0} \right]$$

$$F_{19} = P[x_1 < X_2 < X_3 < Y_3 < y_1 < y_2]$$

$$= P[x_1 < X_2 < X_3 < Y_3 < y_1]$$

$$= \left[\frac{1}{2\left(2k_0^{\,2}+1\right)} \left(v^{\frac{1}{k_0}+2k_0} - u^{\frac{1}{k_0}+2k_0} \right) + \frac{u^{2k_0}}{2} \left(v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}} \right) - \frac{u^{k_0}}{\left(k_0^{\,2}+1\right)} \left(v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0} \right) \right]$$

$$F_{20} = P[x_1 < X_2 < Y_3 < X_3 < y_1 < y_2]$$

$$= P[x_1 < X_2 < Y_3 < X_3 < y_1]$$

$$= \left[\frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)\left(2k_0^{\,2}+1\right)} \left(v^{\frac{1}{k_0}+2k_0} - u^{\frac{1}{k_0}+2k_0} \right) + \frac{k_0^{\,2} u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)} \left(v^{k_0} - u^{k_0} \right) - \frac{k_0^{\,2} u^{k_0}}{\left(k_0^{\,2}+1\right)} \left(v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0} \right) \right]$$

$$F_{21} = P[x_1 < Y_3 < X_2 < X_3 < y_1 < y_2]$$

$$= P[x_1 < Y_3 < X_2 < X_3 < y_1]$$

$$= \left[\frac{k_0^{\,4}}{\left(k_0^{\,2}+1\right)\left(2k_0^{\,2}+1\right)} \left(v^{\frac{1}{k_0}+2k_0} - u^{\frac{1}{k_0}+2k_0} \right) - \frac{k_0^{\,2} u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)} \left(v^{k_0} - u^{k_0} \right) - \frac{u^{\frac{1}{k_0}}}{2} \left(v^{2k_0} - u^{2k_0} \right) + u^{\frac{1}{k_0}+k_0} \left(v^{k_0} - u^{k_0} \right) \right]$$

$$F_{22} = P[X_2 < x_1 < y_1 < y_2 < X_3 < Y_3]$$

$$= P[X_2 < x_1] P[y_2 < X_3 < Y_3]$$

$$= u^{k_0} \left[\left(1 - r^{k_0}\right) - \frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)} \left(1 - r^{\frac{1}{k_0}+1} \right) \right]$$

$$F_{23} = P[X_2 < x_1 < y_1 < y_2 < Y_3 < X_3]$$

$$= P[X_2 < x_1]P[y_2 < Y_3 < X_3]$$

$$= u^{k_0}\left[\left(1 - r^{\frac{1}{k_0}}\right) - \frac{1}{(k_0^{\,2}+1)}\left(1 - r^{\frac{1}{k_0}+k_0}\right)\right]$$

$$F_{24} = P[X_2 < x_1 < y_1 < X_3 < y_2 < Y_3]$$

$$= P[X_2 < x_1]P[y_1 < X_3 < y_2]P[y_2 < Y_3]$$

$$= u^{k_0}\left[r^{k_0} - v^{k_0}\left[1 - r^{\frac{1}{k_0}}\right]\right.$$

$$F_{25} = P[X_2 < x_1 < y_1 < Y_3 < y_2 < X_3]$$

$$= P[X_2 < x_1]P[y_1 < Y_3 < y_2]P[y_2 < X_3]$$

$$= u^{k_0}\left[r^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right]\left[1 - r^{k_0}\right]$$

$$F_{26} = P[X_2 < x_1 < y_1 < X_3 < Y_3 < y_2]$$

$$= P[X_2 < x_1]P[y_1 < X_3 < Y_3 < y_2]$$

$$= u^{k_0}\left[\frac{1}{(k_0^{\,2}+1)}\left[r^{\frac{1}{k_0}+k_0} - v^{\frac{1}{k_0}+k_0}\right] - v^{k_0}\left[r^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right]\right]$$

$$F_{27} = P[X_2 < x_1 < y_1 < Y_3 < X_3 < y_2]$$

$$= P[X_2 < x_1]P[y_1 < Y_3 < X_3 < y_2]$$

$$= u^{k_0}\int_{y_1}^{y_2}\int_{y_1}^{x_3} dG(y_3)dF(x_3)$$

$$= u^{k_0}\int_{v}^{r}\int_{v}^{n}\frac{1}{k_0}s^{\frac{1}{k_0}-1}k_0 n^{k_0-1}dsdn$$

$$= u^{k_0}\left[\frac{{k_0}^2}{({k_0}^2+1)}\left[r^{\frac{1}{k_0}+k_0} - v^{\frac{1}{k_0}+k_0}\right] - v^{\frac{1}{k_0}}\left[r^{k_0} - v^{k_0}\right]\right]$$

$$F_{28} = P[X_2 < x_1 < X_3 < y_1 < y_2 < Y_3]$$

$$= P[X_2 < x_1]P[x_1 < X_3 < y_1]P[y_2 < Y_3]$$

$$= u^{k_0}\left[v^{k_0} - u^{k_0}\left[1 - r^{\frac{1}{k_0}}\right]\right.$$

$$F_{29} = P[X_2 < x_1 < Y_3 < y_1 < y_2 < X_3]$$

$$= P[X_2 < x_1]P[x_1 < Y_3 < y_1]P[y_2 < X_3]$$

$$= u^{k_0}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right]\left[1 - r^{k_0}\right]$$

$$F_{30} = P[Y_3 < x_1 < X_2 < y_1 < y_2 < X_3]$$

$$= P[Y_3 < x_1]P[x_1 < X_2 < y_1]P[y_2 < X_3]$$

$$= u^{\frac{1}{k_0}}\left[v^{k_0} - u^{k_0}\right]\left[1 - r^{k_0}\right]$$

$$F_{31} = P[X_2 < x_1 < X_3 < y_1 < Y_3 < y_2]$$

$$= P[X_2 < x_1]P[x_1 < X_3 < y_1]P[y_1 < Y_3 < y_2]$$

$$= u^{k_0}\left[v^{k_0} - u^{k_0}\left[r^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right]\right.$$

$$F_{32} = P[X_2 < x_1 < Y_3 < y_1 < X_3 < y_2]$$

$$= P[X_2 < x_1]P[x_1 < Y_3 < y_1]P[y_1 < X_3 < y_2]$$

$$= u^{k_0}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right]\left[r^{k_0} - v^{k_0}\right]$$

$$F_{33} = P[Y_3 < x_1 < X_2 < y_1 < X_3 < y_2]$$

$$= P[Y_3 < x_1]P[x_1 < X_2 < y_1]P[y_1 < X_3 < y_2]$$

$$= u^{\frac{1}{k_0}}\left[v^{k_0} - u^{k_0}\right]\left[r^{k_0} - v^{k_0}\right]$$

$$F_{34} = P[X_2 < x_1 < X_3 < Y_3 < y_1 < y_2]$$

$$= P[X_2 < x_1]P[x_1 < X_3 < Y_3 < y_1]$$

$$= u^{k_0}\left[\frac{1}{\left(k_0^{\,2}+1\right)}\left[v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right] - u^{k_0}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right]\right]$$

$$F_{35} = P[X_2 < x_1 < Y_3 < X_3 < y_1 < y_2]$$

$$= P[X_2 < x_1]P[x_1 < Y_3 < X_3 < y_1]$$

$$= u^{k_0}\left[\frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)}\left[v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right] - u^{\frac{1}{k_0}}\left[v^{k_0} - u^{k_0}\right]\right]$$

$$F_{36} = P[Y_3 < x_1 < X_2 < X_3 < y_1 < y_2]$$

$$= P[Y_3 < x_1]P[x_1 < X_2 < X_3 < y_1]$$

$$= u^{\frac{1}{k_0}}\int_{x_1}^{y_1}\int_{x_1}^{x_3} dF(x_2)dF(x_3)$$

$$= u^{\frac{1}{k_0}}\int_{u}^{v}\int_{u}^{n} k_0 m^{k_0-1} k_0 n^{k_0-1} dmdn$$

$$= u^{\frac{1}{k_0}}\left[\frac{1}{2}\left[v^{2k_0} - u^{2k_0}\right] - u^{k_0}\left[v^{k_0} - u^{k_0}\right]\right]$$

$$F_{37} = P[X_2 < X_3 < x_1 < y_1 < y_2 < Y_3]$$

$$= P[X_2 < X_3 < x_1]P[y_2 < Y_3]$$

$$= \frac{u^{2k_0}}{2}\left[1 - r^{\frac{1}{k_0}}\right]$$

$$F_{38} = P[X_2 < Y_3 < x_1 < y_1 < y_2 < X_3]$$

$$= P[X_2 < Y_3 < x_1]P[y_2 < X_3]$$

$$= \frac{u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}\left[1 - r^{k_0}\right]$$

$$F_{39} = P[Y_3 < X_2 < x_1 < y_1 < y_2 < X_3]$$

$$= P[Y_3 < X_2 < x_1]P[y_2 < X_3]$$

$$= \frac{k_0^{\,2} u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}\left[1 - r^{k_0}\right]$$

$$F_{40} = P[X_2 < X_3 < x_1 < y_1 < Y_3 < y_2]$$

$$= P[X_2 < X_3 < x_1]P[y_1 < Y_3 < y_2]$$

$$= \frac{u^{2k_0}}{2}\left[r^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right]$$

$$F_{41} = P[X_2 < Y_3 < x_1 < y_1 < X_3 < y_2]$$

$$= P[X_2 < Y_3 < x_1]P[y_1 < X_3 < y_2]$$

$$= \frac{u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}\left[r^{k_0} - v^{k_0}\right]$$

$$F_{42} = P[Y_3 < X_2 < x_1 < y_1 < X_3 < y_2]$$

$$= P[Y_3 < X_2 < x_1]P[y_1 < X_3 < y_2]$$

$$= \frac{k_0^{\,2} u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}\left[r^{k_0} - v^{k_0}\right]$$

$$F_{43} = P[X_2 < X_3 < x_1 < Y_3 < y_1 < y_2]$$

$$= P[X_2 < X_3 < x_1]P[x_1 < Y_3 < y_1]$$

$$= \frac{u^{2k_0}}{2}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right]$$

$$F_{44} = P[X_2 < Y_3 < x_1 < X_3 < y_1 < y_2]$$

$$= P[X_2 < Y_3 < x_1]P[x_1 < X_3 < y_1]$$

$$= \frac{u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}\left[v^{k_0} - u^{k_0}\right]$$

$$F_{45} = P[Y_3 < X_2 < x_1 < X_3 < y_1 < y_2]$$

$$= P[Y_3 < X_2 < x_1]P[x_1 < X_3 < y_1]$$

$$= \frac{k_0^{\,2}u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}\left[v^{k_0} - u^{k_0}\right]$$

$$F_{46} = P[X_2 < X_3 < Y_3 < x_1 < y_1 < y_2]$$

$$= P[X_2 < X_3 < Y_3 < x_1]$$

$$= \frac{u^{\frac{1}{k_0}+2k_0}}{2\left(2k_0^{\,2}+1\right)}$$

$$F_{47} = P[X_2 < Y_3 < X_3 < x_1 < y_1 < y_2]$$

$$= P[X_2 < Y_3 < X_3 < x_1]$$

$$= \frac{k_0^{\,2}u^{\frac{1}{k_0}+2k_0}}{\left(k_0^{\,2}+1\right)\left(2k_0^{\,2}+1\right)}$$

$$F_{48} = P[Y_3 < X_2 < X_3 < x_1 < y_1 < y_2]$$

$$= P[Y_3 < X_2 < X_3 < x_1]$$

$$= \frac{k_0^{\,4}u^{\frac{1}{k_0}+2k_0}}{\left(k_0^{\,2}+1\right)\left(2k_0^{\,2}+1\right)}$$

where $F(x_1)=u \quad F(y_1)=v$ and $F(y_2)=r$.

Now adding all the 48 terms and simplifying we have

$$P[Med(x_1,X_2,X_3)<Med(y_1,y_2,Y_3)]=2F_1+2F_2+...+2F_{48}$$

$$=2r^{k_0}-r^{2k_0}+\frac{2k_0^{\,2}r^{\frac{1}{k_0}+2k_0}}{(2k_0^{\,2}+1)}-\frac{2k_0^{\,2}r^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)}+\frac{2k_0^{\,2}v^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)}-\frac{2k_0^{\,2}v^{\frac{1}{k_0}+2k_0}}{(2k_0^{\,2}+1)}.$$

Thus integrating we have

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_2}\int_{-\infty}^{y_1}P^2[Med(x_1,X_2,X_3)<Med(y_1,y_2,Y_3)]\,dF(x_1)\,dG(y_1)\,dF(y_2)$$

$$=\int_0^1\int_0^r\int_0^v\left(2r^{k_0}-r^{2k_0}+\frac{2k_0^{\,2}r^{\frac{1}{k_0}+2k_0}}{(2k_0^{\,2}+1)}-\frac{2k_0^{\,2}r^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)}+\frac{2k_0^{\,2}v^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)}-\frac{2k_0^{\,2}v^{\frac{1}{k_0}+2k_0}}{(2k_0^{\,2}+1)}\right)^2 k_0u^{k_0-1}\frac{1}{k_0}v^{\frac{1}{k_0}-1}\frac{1}{k_0}r^{\frac{1}{k_0}-1}\,du\,dv\,dr$$

$$=\frac{1}{3}\frac{(6330k_0^{16}+47447k_0^{14}+155108k_0^{12}+269187k_0^{10}+269725k_0^{8}+161161k_0^{6}+56634k_0^{4}+10800k_0^{2}+864)}{[(4k_0^{2}+3)(5k_0^{2}+3)(3k_0^{2}+4)(5k_0^{2}+4)(2k_0^{2}+1)(5k_0^{2}+2)(3k_0^{2}+2)(k_0^{2}+1)^3]}$$

where $F(x_1)=u \quad G(y_1)=v$ and $G(y_2)=r$

Now consider the second term of (2.4.2.2I).

$$\iiint_{x_1<y_2<y_1}P^2[Med(x_1,X_2,X_3)<Med(y_1,y_2,Y_3)]\,dF(x_1)\,dG(y_1)\,dG(y_2)$$

Interchanging y_1 and y_2 in the above case and simplifying as above we have

$$\iiint_{x_1<y_2<y_1}P^2[Med(x_1,X_2,X_3)<Med(y_1,y_2,Y_3)]\,dF(x_1)\,dG(y_1)\,dG(y_2)$$

$$=\frac{1}{3}\frac{(6330k_0^{16}+47447k_0^{14}+155108k_0^{12}+269187k_0^{10}+269725k_0^{8}+161161k_0^{6}+56634k_0^{4}+10800k_0^{2}+864)}{[(4k_0^{2}+3)(5k_0^{2}+3)(3k_0^{2}+4)(5k_0^{2}+4)(2k_0^{2}+1)(5k_0^{2}+2)(3k_0^{2}+2)(k_0^{2}+1)^3]}$$

Now consider the third term of (2.4.2.2I).

$$\iiint_{y_1<x_1<y_2}P^2[Med(x_1,X_2,X_3)<Med(y_1,y_2,Y_3)]\,dF(x_1)\,dG(y_1)\,dG(y_2)$$

we have

$$P[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)] = 2G_1 + 2G_2 + \ldots + 2G_{30}$$

where

$$G_1 = P[y_1 < x_1 < X_2 < y_2 < X_3 < Y_3]$$

$$= P[x_1 < X_2 < y_2]P[y_2 < X_3 < Y_3]$$

$$= \left[r^{k_0} - u^{k_0}\right]\left[\left(1 - r^{k_0}\right) - \frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)}\left(1 - r^{\frac{1}{k_0}+k_0}\right)\right]$$

$$G_2 = P[y_1 < x_1 < X_2 < y_2 < Y_3 < X_3]$$

$$= P[x_1 < X_2 < y_2]P[y_2 < Y_3 < X_3]$$

$$= \left[r^{k_0} - u^{k_0}\right]\left[\left(1 - r^{\frac{1}{k_0}}\right) - \frac{}{\left(k_0^{\,2}+\right)}\left(1 - r^{\frac{1}{k_0}+k_0}\right)\right]$$

$$G_3 = P[y_1 < x_1 < X_2 < X_3 < y_2 < Y_3]$$

$$= P[x_1 < X_2 < X_3 < y_2]P[y_2 < Y_3]$$

$$= \left[\frac{1}{2}\left(r^{2k_0} - u^{2k_0}\right) - u^{k_0}\left(r^{k_0} - u^{k_0}\right)\right]\left[1 - r^{\frac{1}{k_0}}\right]$$

$$G_4 = P[y_1 < x_1 < X_2 < Y_3 < y_2 < X_3]$$

$$= P[x_1 < X_2 < Y_3 < y_2]P[y_2 < X_3]$$

$$= \left[\frac{1}{\left(k_0^{\,2}+1\right)}\left(r^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right) - u^{k_0}\left(r^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\right]\left[1 - r^{k_0}\right]$$

$$G_5 = P[y_1 < x_1 < X_2 < X_3 < Y_3 < y_2]$$

$$= P[x_1 < X_2 < X_3 < Y_3 < y_2]$$

$$= \left[\frac{1}{2\left(2k_0^{\,2}+1\right)}\left(r^{\frac{1}{k_0}+2k_0} - u^{\frac{1}{k_0}+2k_0}\right) + \frac{u^{2k_0}}{2}\left(r^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right) - \frac{u^{k_0}}{\left(k_0^{\,2}+1\right)}\left(r^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right)\right]$$

$$G_6 = P[y_1 < x_1 < X_2 < Y_3 < X_3 < y_2]$$

$$= P[x_1 < X_2 < Y_3 < X_3 < y_2]$$

$$= \left[\frac{k_0^{\,2}}{(k_0^{\,2}+1)(2k_0^{\,2}+1)} \left(r^{\frac{1}{k_0}+2k_0} - u^{\frac{1}{k_0}+2k_0} \right) + \frac{k_0^{\,2} u^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)} \left(r^{k_0} - u^{k_0} \right) - \frac{k_0^{\,2} u^{k_0}}{(k_0^{\,2}+1)} \left(r^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0} \right) \right]$$

$$G_7 = P[y_1 < X_2 < x_1 < y_2 < X_3 < Y_3]$$

$$= P[y_1 < X_2 < x_1] P[y_2 < X_3 < Y_3]$$

$$= \left[u^{k_0} - v^{k_0} \right] \left[\left(1 - r^{k_0}\right) - \frac{k_0^{\,2}}{(k_0^{\,2}+1)} \left(1 - r^{\frac{1}{k_0}+k_0} \right) \right]$$

$$G_8 = P[y_1 < X_2 < x_1 < y_2 < Y_3 < X_3]$$

$$= P[y_1 < X_2 < x_1] P[y_2 < Y_3 < X_3]$$

$$= \left[u^{k_0} - v^{k_0} \right] \left[\left(1 - r^{\frac{1}{k_0}} \right) - \frac{1}{(k_0^{\,2}+1)} \left(1 - r^{\frac{1}{k_0}+k_0} \right) \right]$$

$$G_9 = P[y_1 < X_2 < x_1 < X_3 < y_2 < Y_3]$$

$$= P[y_1 < X_2 < x_1] P[x_1 < X_3 < y_2] P[y_2 < Y_3]$$

$$= \left(u^{k_0} - v^{k_0} \right) \left(r^{k_0} - u^{k_0} \right) \left(1 - r^{\frac{1}{k_0}} \right)$$

$$G_{10} = P[y_1 < X_2 < x_1 < Y_3 < y_2 < X_3]$$

$$= P[y_1 < X_2 < x_1] P[x_1 < Y_3 < y_2] P[y_2 < X_3]$$

$$= \left(u^{k_0} - v^{k_0} \right) \left(r^{\frac{1}{k_0}} - u^{\frac{1}{k_0}} \right) \left(1 - r^{k_0} \right)$$

$$G_{11} = P[y_1 < X_2 < x_1 < X_3 < Y_3 < y_2]$$

$$= P[y_1 < X_2 < x_1] P[x_1 < X_3 < Y_3 < y_2]$$

$$= \left(u^{k_0} - v^{k_0}\right)\left(\frac{1}{\left(k_0^{\,2}+1\right)}\left(r^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right) - u^{k_0}\left(r^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\right)$$

$$G_{12} = P[y_1 < X_2 < x_1 < Y_3 < X_3 < y_2]$$

$$= P[y_1 < X_2 < x_1]P[x_1 < Y_3 < X_3 < y_2]$$

$$= \left(u^{k_0} - v^{k_0}\right)\left(\frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)}\left(r^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right) - u^{\frac{1}{k_0}}\left(r^{k_0} - u^{k_0}\right)\right)$$

$$G_{13} = P[y_1 < X_2 < X_3 < x_1 < y_2 < Y_3]$$

$$= P[y_1 < X_2 < X_3 < x_1]P[y_2 < Y_3]$$

$$= \left[\int_{y_1}^{x_1}\int_{y_1}^{x_3} dF(x_2)dF(x_3)\right]\left[1 - r^{\frac{1}{k_0}}\right]$$

$$= \left[\int_{v}^{u}\int_{v}^{n} k_0 m^{k_0-1} k_0 n^{k_0-1} dmdn\right]\left[\left[1 - r^{\frac{1}{k_0}}\right]\right]$$

$$= \left[\frac{1}{2}\left(u^{2k_0} - v^{2k_0}\right) - v^{k_0}\left(u^{k_0} - v^{k_0}\right)\right]\left[\left[1 - r^{\frac{1}{k_0}}\right]\right]$$

$$G_{14} = P[y_1 < X_2 < X_3 < x_1 < Y_3 < y_2]$$

$$= P[y_1 < X_2 < X_3 < x_1]P[x_1 < Y_3 < y_2]$$

$$= \left[\frac{1}{2}\left(u^{2k_0} - v^{2k_0}\right) - v^{k_0}\left(u^{k_0} - v^{k_0}\right)\right]\left[\left[r^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right]\right]$$

$$G_{15} = P[y_1 < X_2 < X_3 < Y_3 < x_1 < y_2]$$

$$= P[y_1 < X_2 < X_3 < Y_3 < x_1]$$

$$= \int_{y_1}^{x_1}\int_{y_1}^{y_3}\int_{y_1}^{x_3} dF(x_2)dF(x_3)dG(y_3)$$

$$= \int_v^u \int_v^s \int_v^n k_0 m^{k_0-1} k_0 n^{k_0-1} \frac{1}{k_0} s^{\frac{1}{k_0}-1} dmdnds$$

$$= \left[\frac{1}{2(2k_0^2+1)} \left(u^{\frac{1}{k_0}+2k_0} - v^{\frac{1}{k_0}+2k_0} \right) - \frac{v^{2k_0}}{2} \left(u^{\frac{1}{k_0}} - v^{\frac{1}{k_0}} \right) - \frac{v^{k_0}}{(k_0^2+1)} \left(u^{\frac{1}{k_0}+k_0} - v^{\frac{1}{k_0}+k_0} \right) + v^{2k_0} \left(u^{\frac{1}{k_0}} - v^{\frac{1}{k_0}} \right) \right]$$

$$G_{16} = P[X_2 < y_1 < x_1 < y_2 < X_3 < Y_3]$$

$$= P[X_2 < y_1] P[y_2 < X_3 < Y_3]$$

$$= v^{k_0} \left[(1 - r^{k_0}) - \frac{k_0^2}{(k_0^2+1)} \left(1 - r^{\frac{1}{k_0}+k_0} \right) \right]$$

$$G_{17} = P[X_2 < y_1 < x_1 < y_2 < Y_3 < X_3]$$

$$= P[X_2 < y_1] P[y_2 < Y_3 < X_3]$$

$$= v^{k_0} \left[\left(1 - r^{\frac{1}{k_0}} \right) - \frac{1}{(k_0^2+1)} \left(1 - r^{\frac{1}{k_0}+k_0} \right) \right]$$

$$G_{18} = P[X_2 < y_1 < x_1 < X_3 < y_2 < Y_3]$$

$$= P[X_2 < y_1] P[x_1 < X_3 < y_2] P[y_2 < Y_3]$$

$$= v^{k_0} \left[(r^{k_0} - u^{k_0}) \right] \left[1 - r^{\frac{1}{k_0}} \right]$$

$$G_{19} = P[X_2 < y_1 < x_1 < Y_3 < y_2 < X_3]$$

$$= P[X_2 < y_1] P[x_1 < Y_3 < y_2] P[y_2 < X_3]$$

$$= v^{k_0} \left[\left(r^{\frac{1}{k_0}} - u^{\frac{1}{k_0}} \right) \right] \left[1 - r^{k_0} \right]$$

$$G_{20} = P[X_2 < y_1 < x_1 < X_3 < Y_3 < y_2]$$

$$= P[X_2 < y_1] P[x_1 < X_3 < Y_3 < y_2]$$

$$= v^{k_0}\left(\frac{1}{(k_0^{\,2}+1)}\left(r^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right) - u^{k_0}\left(r^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\right)$$

$$G_{21} = P[X_2 < y_1 < x_1 < Y_3 < X_3 < y_2]$$

$$= P[X_2 < y_1]P[x_1 < Y_3 < X_3 < y_2]$$

$$= v^{k_0}\left(\frac{k_0^{\,2}}{(k_0^{\,2}+1)}\left(r^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right) - u^{\frac{1}{k_0}}\left(r^{k_0} - u^{k_0}\right)\right)$$

$$G_{22} = P[X_2 < y_1 < X_3 < x_1 < y_2 < Y_3]$$

$$= P[X_2 < y_1]P[y_1 < X_3 < x_1]P[y_2 < Y_3]$$

$$= v^{k_0}\left[\left(u^{k_0} - v^{k_0}\right)\right]\left[1 - r^{\frac{1}{k_0}}\right]$$

$$G_{23} = P[X_2 < y_1 < X_3 < x_1 < Y_3 < y_2]$$

$$= P[X_2 < y_1]P[y_1 < X_3 < x_1]P[x_1 < Y_3 < y_2]$$

$$= v^{k_0}\left[\left(u^{k_0} - v^{k_0}\right)\right]\left[r^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right]$$

$$G_{24} = P[X_2 < y_1 < X_3 < Y_3 < x_1 < y_2]$$

$$= P[X_2 < y_1]P[y_1 < X_3 < Y_3 < x_1]$$

$$= v^{k_0}\left(\frac{1}{(k_0^{\,2}+1)}\left(u^{\frac{1}{k_0}+k_0} - v^{\frac{1}{k_0}+k_0}\right) - v^{k_0}\left(u^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right)\right)$$

$$G_{25} = P[X_2 < X_3 < y_1 < x_1 < y_2 < Y_3]$$

$$= P[X_2 < X_3 < y_1]P[y_2 < Y_3]$$

$$= \left[\int_{-\infty}^{y_1} F^{k_0}(x_3)dF^{k_0}(x_3)\right]\left[1 - r^{\frac{1}{k_0}}\right]$$

$$= \left[\int_0^v n^{k_0} k_0 n^{k_0-1} dn\right]\left[1 - r^{\frac{1}{k_0}}\right]$$

$$= \frac{v^{2k_0}}{2}\left[1 - r^{\frac{1}{k_0}}\right]$$

$$G_{26} = P[X_2 < X_3 < y_1 < x_1 < Y_3 < y_2]$$

$$= P[X_2 < X_3 < y_1]P[x_1 < Y_3 < y_2]$$

$$= \frac{v^{2k_0}}{2}\left[r^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right]$$

$$G_{27} = P[X_2 < X_3 < y_1 < Y_3 < x_1 < y_2]$$

$$= P[X_2 < X_3 < y_1]P[y_1 < Y_3 < x_1]$$

$$= \frac{v^{2k_0}}{2}\left[u^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right]$$

$$G_{28} = P[X_2 < X_3 < Y_3 < y_1 < x_1 < y_2]$$

$$= P[X_2 < X_3 < Y_3 < y_1]$$

$$= \int_{-\infty}^{y_1}\int_{-\infty}^{y_3} F^{k_0}(x_3)dF(x_3)dG(y_3)$$

$$= \int_0^v\int_0^s n^{k_0} k_0 n^{k_0-1}\frac{1}{k_0} s^{\frac{1}{k_0}-1} dnds$$

$$= \frac{v^{\frac{1}{k_0}+2k_0}}{2\left(2k_0^{\,2}+1\right)}$$

$$G_{29} = P[X_2 < Y_3 < X_3 < y_1 < x_1 < y_2]$$

$$= P[X_2 < Y_3 < X_3 < y_1]$$

$$= \int_{-\infty}^{y_1}\int_{-\infty}^{x_3} G^{k_0}(y_3)dG(y_3)dF(x_3)$$

$$= \int_0^v \int_0^n s^{k_0} \frac{1}{k_0} s^{\frac{1}{k_0}-1} k_0 n^{k_0-1} ds dn$$

$$= \frac{k_0^{\,2} v^{\frac{1}{k_0}+2k_0}}{(k_0^{\,2}+1)(2k_0^{\,2}+1)}$$

$$G_{30} = P[Y_3 < X_2 < X_3 < y_1 < x_1 < y_2]$$

$$= P[Y_3 < X_2 < X_3 < y_1]$$

$$= \int_{-\infty}^{y_1} \int_{-\infty}^{x_1} F^{k_0}(x_2) dF(x_2) dF(x_3)$$

$$= \int_0^v \int_0^n m^{\frac{1}{k_0}} k_0 m^{k_0-1} k_0 n^{k_0-1} dm dn$$

$$= \frac{k_0^{\,4} v^{\frac{1}{k_0}+2k_0}}{(k_0^{\,2}+1)(2k_0^{\,2}+1)}.$$

Now adding all the 30 terms and simplifying we have

$$P[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)] = 2G_1 + 2G_2 + \ldots + 2G_{30}$$

$$= 2r^{k_0} - r^{2k_0} - \frac{2k_0^{\,2} r^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)} - \frac{2u^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)} + \frac{2k_0^{\,2} r^{\frac{1}{k_0}+2k_0}}{(2k_0^{\,2}+1)} + \frac{2k_0^{\,2} v^{\frac{1}{k_0}+2k_0}}{(2k_0^{\,2}+1)} + \frac{2u^{\frac{1}{k_0}+2k_0}}{(2k_0^{\,2}+1)}$$

where $F(x_1) = u$ $F(y_1) = v$ and $F(y_2) = r$

Thus integrating we have

$$\int_{-\infty}^{\infty} \int_{-\infty}^{y_2} \int_{-\infty}^{x_1} P^2[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)] dG(y_1) dF(x_1) dG(y_2)$$

$$= \int_0^1 \int_0^r \int_0^u \left(2r^{k_0} - r^{2k_0} - \frac{2k_0^{\,2} r^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)} - \frac{2u^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)} + \frac{2k_0^{\,2} r^{\frac{1}{k_0}+2k_0}}{(2k_0^{\,2}+1)} + \frac{2k_0^{\,2} v^{\frac{1}{k_0}+2k_0}}{(2k_0^{\,2}+1)} + \frac{2u^{\frac{1}{k_0}+2k_0}}{(2k_0^{\,2}+1)} \right)^2 \frac{1}{k_0} v^{\frac{1}{k_0}-1} k_0 u^{k_0-1} \frac{1}{k_0} r^{\frac{1}{k_0}-1} dv\, du\, dr$$

where $F(x_1) = u$ $F(y_1) = v$ and $F(y_2) = r$

$$=\frac{1}{3}\frac{k_0^{\,2}\left(270k_0^{\,14}+24440k_0^{\,12}+85228k_0^{\,10}+124231k_0^{\,8}+95717k_0^{\,6}+41010k_0^{\,4}+9264k_0^{\,2}+864\right)}{\left[\left(5k_0^{\,2}+3\right)\left(4k_0^{\,2}+3\right)\left(5k_0^{\,2}+4\right)\left(2k_0^{\,2}+1\right)\left(3k_0^{\,2}+4\right)\left(5k_0^{\,2}+2\right)\left(3k_0^{\,2}+2\right)\left(k_0^{\,2}+1\right)^3\right]}$$

Now consider the fourth term of (2.4.2.2I).

$$\iiint_{y_2<x_1<y_1} P^2[Med(x_1,X_2,X_3)<Med(y_1,y_2,Y_3)]\mathrm{dF}(x_1)\mathrm{dG}(y_1)\mathrm{dG}(y_2)$$

Interchanging y_1 and y_2 in the above case and simplifying we have,

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_1}\int_{-\infty}^{x_1} P^2[Med(x_1,X_2,X_3)<Med(y_1,y_2,Y_3)]\mathrm{dG}(y_2)\mathrm{dF}(x_1)\mathrm{dG}(y_1)$$

$$=\frac{1}{3}\frac{k_0^{\,2}\left(270k_0^{\,14}+24440k_0^{\,12}+85228k_0^{\,10}+124231k_0^{\,8}+95717k_0^{\,6}+41010k_0^{\,4}+9264k_0^{\,2}+864\right)}{\left[\left(5k_0^{\,2}+3\right)\left(4k_0^{\,2}+3\right)\left(5k_0^{\,2}+4\right)\left(2k_0^{\,2}+1\right)\left(3k_0^{\,2}+4\right)\left(5k_0^{\,2}+2\right)\left(3k_0^{\,2}+2\right)\left(k_0^{\,2}+1\right)^3\right]}$$

Now consider the fifth term of (2.4.2.2I).

$$\iiint_{y_1<y_2<x_1} P^2[Med(x_1,X_2,X_3)<Med(y_1,y_2,Y_3)]\mathrm{dF}(x_1)\mathrm{dG}(y_1)\mathrm{dG}(y_2)$$

we have

$$P[Med(x_1,X_2,X_3)<Med(y_1,y_2,Y_3)]=2H_1+2H_2+\ldots+2H_{12}$$

where

$$H_1=P[y_1<X_2<X_3<y_2<x_1<Y_3]$$

$$=P[y_1<X_2<X_3<y_2]P[x_1<Y_3]$$

$$=\left[\frac{1}{2}\left(r^{2k_0}-v^{2k_0}\right)-v^{k_0}\left(r^{k_0}-v^{k_0}\right)\right]\left[1-u^{\frac{1}{k_0}}\right]$$

$$H_2=P[y_1<X_2<X_3<y_2<Y_3<x_1]$$

$$=P[y_1<X_2<X_3<y_2]P[y_2<Y_3<x_1]$$

$$= \left[\frac{1}{2}\left(r^{2k_0} - v^{2k_0}\right) - v^{k_0}\left(r^{k_0} - v^{k_0}\right)\right]\left[u^{\frac{1}{k_0}} - r^{\frac{1}{k_0}}\right]$$

$$H_3 = P[y_1 < X_2 < X_3 < Y_3 < y_2 < x_1]$$

$$= P[y_1 < X_2 < X_3 < Y_3 < y_2]$$

$$= \frac{\left[r^{\frac{1}{k_0}+2k_0} - v^{\frac{1}{k_0}+2k_0}\right]}{2\left(2k_0^{\,2}+1\right)} - \frac{v^{2k_0}}{2}\left[r^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right] - \frac{v^{k_0}}{\left(k_0^{\,2}+1\right)}\left[r^{\frac{1}{k_0}+k_0} - v^{\frac{1}{k_0}+k_0}\right] + v^{2k_0}\left[r^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right]$$

$$H_4 = P[X_2 < y_1 < X_3 < y_2 < x_1 < Y_3]$$

$$= P[X_2 < y_1]P[y_1 < X_3 < y_2]P[x_1 < Y_3]$$

$$= v^{k_0}\left[\left(r^{k_0} - v^{k_0}\right)\right]\left[1 - u^{\frac{1}{k_0}}\right]$$

$$H_5 = P[X_2 < y_1 < X_3 < y_2 < Y_3 < x_1]$$

$$= P[X_2 < y_1]P[y_1 < X_3 < y_2]P[y_2 < Y_3 < x_1]$$

$$= v^{k_0}\left[\left(r^{k_0} - v^{k_0}\right)\right]\left[u^{\frac{1}{k_0}} - r^{\frac{1}{k_0}}\right]$$

$$H_6 = P[X_2 < y_1 < X_3 < Y_3 < y_2 < x_1]$$

$$= P[X_2 < y_1]P[y_1 < X_3 < Y_3 < y_2]$$

$$= v^{k_0}\left[\frac{1}{\left(k_0^{\,2}+1\right)}\left[r^{\frac{1}{k_0}+k_0} - v^{\frac{1}{k_0}+k_0}\right] - v^{k_0}\left[r^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right]\right]$$

$$H_7 = P[X_2 < X_3 < y_1 < y_2 < x_1 < Y_3]$$

$$= P[X_2 < X_3 < y_1]P[x_1 < Y_3]$$

$$= \frac{v^{2k_0}}{2}\left[1 - u^{\frac{1}{k_0}}\right]$$

$$H_8 = P[X_2 < X_3 < y_1 < y_2 < Y_3 < x_1]$$

$$= P[X_2 < X_3 < y_1]P[y_2 < Y_3 < x_1]$$

$$= \frac{v^{2k_0}}{2}\left[u^{\frac{1}{k_0}} - r^{\frac{1}{k_0}}\right]$$

$$H_9 = P[X_2 < X_3 < y_1 < Y_3 < y_2 < x_1]$$

$$= P[X_2 < X_3 < y_1]P[y_1 < Y_3 < y_2]$$

$$= \frac{v^{2k_0}}{2}\left[r^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right]$$

$$H_{10} = P[X_2 < X_3 < Y_3 < y_1 < y_2 < x_1]$$

$$= P[X_2 < X_3 < Y_3 < y_1]$$

$$= \frac{v^{\frac{1}{k_0}+2k_0}}{2\left(2k_0^{\,2}+1\right)}$$

$$H_{11} = P[X_2 < Y_3 < X_3 < y_1 < y_2 < x_1]$$

$$= P[X_2 < Y_3 < X_3 < y_1]$$

$$= \frac{k_0^{\,2} v^{\frac{1}{k_0}+2k_0}}{\left(k_0^{\,2}+1\right)\left(2k_0^{\,2}+1\right)}$$

$$H_{12} = P[Y_3 < X_2 < X_3 < y_1 < y_2 < x_1]$$

$$= P[Y_3 < X_2 < X_3 < y_1]$$

$$= \frac{k_0^{\,4} v^{\frac{1}{k_0}+2k_0}}{\left(k_0^{\,2}+1\right)\left(2k_0^{\,2}+1\right)}.$$

Now adding all the 12 terms and simplifying we have

$$P[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)] = 2H_1 + 2H_2 + \ldots + 2H_{12}$$

$$= r^{2k_0} - \frac{2k_0^{\,2}}{(2k_0^{\,2}+1)} r^{\frac{1}{k_0}+2k_0} + \frac{2k_0^{\,2}}{(2k_0^{\,2}+1)} v^{\frac{1}{k_0}+2k_0}$$

where $F(x_1) = u \quad F(y_1) = v$ and $F(y_2) = r$

Thus integrating we have

$$\int_{-\infty}^{\infty}\int_{-\infty}^{x_1}\int_{-\infty}^{y_2} P^2[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)]\,dG(y_1)\,dG(y_2)\,dF(x_1)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{x_1}\int_{-\infty}^{y_2}\left(r^{2k_0} - \frac{2k_0^{\,2}}{(2k_0^{\,2}+1)} r^{\frac{1}{k_0}+2k_0} + \frac{2k_0^{\,2}}{(2k_0^{\,2}+1)} v^{\frac{1}{k_0}+2k_0} \right)^2 dG(y_1)\,dG(y_2)\,dF(x_1)$$

$$= \int_0^1\int_0^u\int_0^r\left(r^{2k_0} - \frac{2k_0^{\,2}}{(2k_0^{\,2}+1)} r^{\frac{1}{k_0}+2k_0} + \frac{2k_0^{\,2}}{(2k_0^{\,2}+1)} v^{\frac{1}{k_0}+2k_0} \right)^2 \frac{1}{k_0} v^{\frac{1}{k_0}-1} \frac{1}{k_0} r^{\frac{1}{k_0}-1} k_0 u^{k_0-1}\,dv\,dr\,du$$

$$= \frac{1}{2}\left[\frac{k_0^{\,2}(25k_0^{\,8} + 233k_0^{\,6} + 353k_0^{\,4} + 193k_0^{\,2} + 36)}{(5k_0^{\,2}+3)(4k_0^{\,2}+3)(5k_0^{\,2}+4)(k_0^{\,2}+1)^2(2k_0^{\,2}+1)(5k_0^{\,2}+2)}\right].$$

Now consider the sixth term of (2.4.2.2I).

Interchanging y_1 and y_2 in the above case and simplifying we have,

$$\iiint_{y_2<y_1<x_1} P^2[Med(x_1, X_2, X_3) < Med(y_1, y_2, Y_3)]\,dF(x_1)\,dG(y_1)\,dG(y_2)$$

$$= \frac{1}{2}\left[\frac{k_0^{\,2}(25k_0^{\,8} + 233k_0^{\,6} + 353k_0^{\,4} + 193k_0^{\,2} + 36)}{(5k_0^{\,2}+3)(4k_0^{\,2}+3)(5k_0^{\,2}+4)(k_0^{\,2}+1)^2(2k_0^{\,2}+1)(5k_0^{\,2}+2)}\right].$$

Thus finally adding the six terms we have (2.4.2.2I) that is

$$P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_4, X_5) < Med(Y_1, Y_2, Y_4)]$$

$$= \frac{1}{3}\frac{(2775k_0^{\,12} + 26533k_0^{\,10} + 63949k_0^{\,8} + 67741k_0^{\,6} + 35706k_0^{\,4} + 9072k_0^{\,2} + 864)}{[(k_0^{\,2}+1)^2(3k_0^{\,2}+2)(2k_0^{\,2}+1)(5k_0^{\,2}+4)(3k_0^{\,2}+4)(5k_0^{\,2}+3)(4k_0^{\,2}+3)]}.$$

Thus, finally we have

$$\xi^*_{1,2} = P[Med(X_1,X_2,X_3) < Med(Y_1,Y_2,Y_3), Med(X_1,X_4,X_5) < Med(Y_1,Y_2,Y_4)] - [E(U)]^2$$

$$= \left[\frac{1}{3}\frac{(2775k_0^{12} + 26533k_0^{10} + 63949k_0^8 + 67741k_0^6 + 35706k_0^4 + 9072k_0^2 + 864)}{[(k_0^2+1)^2(3k_0^2+2)(2k_0^2+1)(5k_0^2+4)(3k_0^2+4)(5k_0^2+3)(4k_0^2+3)]}\right] - \left[\left[\frac{(9k_0^2+6)}{(k_0^2+1)(3k_0^2+2)(2k_0^2+3)}\right]\right]^2$$

$$= \frac{1}{3}k_0^6\frac{(33300k_0^{12} + 440496k_0^{10} + 1426565k_0^8 + 2031011k_0^6 + 1459820k_0^4 + 520512k_0^2 + 73296)}{[(k_0^2+1)^2(3k_0^2+2)^2(2k_0^2+1)(5k_0^2+4)(3k_0^2+4)(5k_0^2+3)(4k_0^2+3)(2k_0^2+3)^2]}.$$

Now consider

$$\xi^*_{2,1} = Cov[h(X_1,X_2,X_3;Y_1,Y_2,Y_3), h(X_1,X_2,X_4;Y_1,Y_4,Y_5)]$$

$$= E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)h(X_1,X_2,X_4;Y_1,Y_4,Y_5)] - \{E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)]\}^2$$

$$= P[Med(X_1,X_2,X_3) < Med(Y_1,Y_2,Y_3), Med(X_1,X_2,X_4) < Med(Y_1,Y_4,Y_5)] - [E(U)]^2$$

Now

$$P[Med(X_1,X_2,X_3) < Med(Y_1,Y_2,Y_3), Med(X_1,X_2,X_4) < Med(Y_1,Y_4,Y_5)]$$

$$= \iiint P^2[Med(x_1,x_2,X_3) < Med(y_1,Y_2,Y_3)]dF(x_1)dF(x_2)dG(y_1)$$

$$= \iiint_{x_1<x_2<y_1} P^2[Med(x_1,x_2,X_3) < Med(y_1,Y_2,Y_3)]dF(x_1)dF(x_2)dG(y_1)$$
$$+ \iiint_{x_2<x_1<y_1} P^2[Med(x_1,x_2,X_3) < Med(y_1,Y_2,Y_3)]dF(x_2)dF(x_1)dG(y_1) +$$
$$\iiint_{x_1<y_1<x_2} P^2[Med(x_1,x_2,X_3) < Med(y_1,Y_2,Y_3)]dF(x_1)dG(y_1)dF(x_2)$$
$$+ \iiint_{x_2<y_1<x_1} P^2[Med(x_1,x_2,X_3) < Med(y_1,Y_2,Y_3)]dF(x_2)dG(y_1)dF(x_1) \qquad (2.4.2.2J)$$
$$+ \iiint_{y_1<x_1<x_2} P^2[Med(x_1,x_2,X_3) < Med(y_1,Y_2,Y_3)]dG(y_1)dF(x_1)dF(x_2)$$
$$+ \iiint_{y_1<x_2<x_1} P^2[Med(x_1,x_2,X_3) < Med(y_1,Y_2,Y_3)]dG(y_1)dF(x_2)dF(x_1)$$

Now consider the first term of (2.4.2.2J).

$$\iiint_{x_1<x_2<y_1} P^2[Med(x_1,x_2,X_3) < Med(y_1,Y_2,Y_3)]dF(x_1)dF(x_2)dG(y_1)$$

We have

$$P[Med(x_1, x_2, X_3) < Med(y_1, Y_2, Y_3)] = 2I_1 + 2I_2 + ... + 2I_{48}$$

where

$$I_1 = P[x_1 < x_2 < y_1 < X_3 < Y_2 < Y_3]$$

$$= P[y_1 < X_3 < Y_2 < Y_3]$$

$$= \frac{1}{2}\left(1 - v^{k_0}\right) - \frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)}\left(1 - v^{\frac{1}{k_0}+k_0}\right) + \frac{k_0^{\,2}}{2\left(k_0^{\,2}+2\right)}\left(1 - v^{\frac{2}{k_0}+k_0}\right)$$

$$I_2 = P[x_1 < x_2 < y_1 < Y_2 < X_3 < Y_3]$$

$$= P[y_1 < Y_2 < X_3 < Y_3]$$

$$= \frac{1}{\left(k_0^{\,2}+1\right)}\left(1 - v^{k_0}\right) - \frac{1}{\left(k_0^{\,2}+1\right)}\left(1 - v^{\frac{1}{k_0}+k_0}\right) + \frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)\left(k_0^{\,2}+2\right)}\left(1 - v^{\frac{2}{k_0}+k_0}\right)$$

$$I_3 = P[x_1 < x_2 < y_1 < Y_2 < Y_3 < X_3]$$

$$= P[y_1 < Y_2 < Y_3 < X_3]$$

$$= \frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)}\left(1 - v^{\frac{1}{k_0}}\right) - \frac{1}{2}\left(1 - v^{\frac{2}{k_0}}\right) + \frac{1}{\left(k_0^{\,2}+1\right)\left(k_0^{\,2}+2\right)}\left(1 - v^{\frac{2}{k_0}+k_0}\right)$$

$$I_4 = P[x_1 < x_2 < X_3 < y_1 < Y_2 < Y_3]$$

$$= P[x_2 < X_3 < y_1]P[y_1 < Y_2 < Y_3]$$

$$= \left(v^{k_0} - m^{k_0}\right)\left[\left(1 - v^{\frac{1}{k_0}}\right) - \frac{1}{2}\left(1 - v^{\frac{2}{k_0}}\right)\right]$$

$$I_5 = P[x_1 < x_2 < Y_2 < y_1 < X_3 < Y_3]$$

$$= P[x_2 < Y_2 < y_1]P[y_1 < X_3 < Y_3]$$

$$= \left(v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right)\left[\left(1 - v^{k_0}\right) - \frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)}\left(1 - v^{\frac{1}{k_0}+k_0}\right)\right]$$

$$I_6 = P[x_1 < x_2 < Y_2 < y_1 < Y_3 < X_3]$$

$$= P[x_2 < Y_2 < y_1]P[y_1 < Y_3 < X_3]$$

$$= \left(v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right)\left[\left(1 - v^{\frac{1}{k_0}}\right) - \frac{1}{\left(k_0^{\,2}+1\right)}\left(1 - v^{\frac{1}{k_0}+k_0}\right)\right]$$

$$I_7 = P[x_1 < x_2 < X_3 < Y_2 < y_1 < Y_3]$$

$$= P[x_2 < X_3 < Y_2 < y_1]P[y_1 < Y_3]$$

$$= \frac{1}{\left(k_0^{\,2}+1\right)}\left(v^{\frac{1}{k_0}+k_0} - m^{\frac{1}{k_0}+k_0}\right) - m^{k_0}\left[v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right]$$

$$I_8 = P[x_1 < x_2 < Y_2 < X_3 < y_1 < Y_3]$$

$$= P[x_2 < Y_2 < X_3 < y_1]P[y_1 < Y_3]$$

$$= \frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)}\left(v^{\frac{1}{k_0}+k_0} - m^{\frac{1}{k_0}+k_0}\right) - m^{\frac{1}{k_0}}\left[v^{k_0} - m^{k_0}\right]$$

$$I_9 = P[x_1 < x_2 < Y_2 < Y_3 < y_1 < X_3]$$

$$= P[x_2 < Y_2 < Y_3 < y_1]P[y_1 < X_3]$$

$$= \left[\int_{x_2}^{y_1}\left[F^{\frac{1}{k_0}}(y_1) - F^{\frac{1}{k_0}}(y_2)\right]dF^{\frac{1}{k_0}}(y_2)\right]\left[1 - v^{k_0}\right]$$

$$= \left[\int_{m}^{v}\left[v^{\frac{1}{k_0}} - r^{\frac{1}{k_0}}\right]\frac{1}{k_0}r^{\frac{1}{k_0}-1}\,dr\right]\left[1 - v^{k_0}\right]$$

$$= \frac{1}{2}\left(v^{\frac{2}{k_0}} - m^{\frac{2}{k_0}}\right) - m^{\frac{1}{k_0}}\left(v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right)$$

$$I_{10} = P[x_1 < x_2 < X_3 < Y_2 < Y_3 < y_1]$$

$$= P[x_2 < X_3 < Y_2 < Y_3 < y_1]$$

$$= \frac{1}{\left(k_0^{\,2}+1\right)\left(k_0^{\,2}+2\right)}\left(v^{\frac{2}{k_0}+k_0} - m^{\frac{2}{k_0}+k_0}\right) - \frac{m^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}\left(v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right) - \frac{m^{k_0}}{2}\left(v^{\frac{2}{k_0}} - m^{\frac{2}{k_0}}\right) + m^{\frac{1}{k_0}+k_0}\left(v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right)$$

$$I_{11} = P[x_1 < x_2 < Y_2 < X_3 < Y_3 < y_1]$$

$$= P[x_2 < Y_2 < X_3 < Y_3 < y_1]$$

$$= \frac{k_0^{\,2}}{(k_0^{\,2}+1)(k_0^{\,2}+2)}\left(v^{\frac{2}{k_0}+k_0} - m^{\frac{2}{k_0}+k_0}\right) - \frac{k_0^{\,2} m^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)}\left(v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right) - \frac{m^{\frac{1}{k_0}}}{(k_0^{\,2}+1)}\left(v^{\frac{1}{k_0}+k_0} - m^{\frac{1}{k_0}+k_0}\right) + m^{\frac{1}{k_0}+k_0}\left(v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right)$$

$$I_{12} = P[x_1 < x_2 < Y_2 < Y_3 < X_3 < y_1]$$

$$= P[x_2 < Y_2 < Y_3 < X_3 < y_1]$$

$$= \frac{k_0^{\,2}}{2(k_0^{\,2}+2)}\left(v^{\frac{2}{k_0}+k_0} - m^{\frac{2}{k_0}+k_0}\right) - \frac{m^{\frac{2}{k_0}}}{2}\left(v^{k_0} - m^{k_0}\right) - \frac{k_0^{\,2} m^{\frac{1}{k_0}}}{(k_0^{\,2}+1)}\left(v^{\frac{1}{k_0}+k_0} - m^{\frac{1}{k_0}+k_0}\right) + m^{\frac{2}{k_0}}\left(v^{k_0} - m^{k_0}\right)$$

$$I_{13} = P[x_1 < X_3 < x_2 < y_1 < Y_2 < Y_3]$$

$$= P[x_1 < X_3 < x_2]P[y_1 < Y_2 < Y_3]$$

$$= \left(m^{k_0} - u^{k_0}\right)\left[\left(1 - v^{\frac{1}{k_0}}\right) - \frac{1}{2}\left(1 - v^{\frac{2}{k_0}}\right)\right]$$

$$I_{14} = P[x_1 < Y_2 < x_2 < y_1 < X_3 < Y_3]$$

$$= P[x_1 < Y_2 < x_2]P[y_1 < X_3 < Y_3]$$

$$= \left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\left[\left(1 - v^{k_0}\right) - \frac{k_0^{\,2}}{(k_0^{\,2}+1)}\left(1 - v^{\frac{1}{k_0}+k_0}\right)\right]$$

$$I_{15} = P[x_1 < Y_2 < x_2 < y_1 < Y_3 < X_3]$$

$$= P[x_1 < Y_2 < x_2]P[y_1 < Y_3 < X_3]$$

$$= \left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\left[\left(1 - v^{\frac{1}{k_0}}\right) - \frac{1}{(k_0^{\,2}+1)}\left(1 - v^{\frac{1}{k_0}+k_0}\right)\right]$$

$$I_{16} = P[x_1 < X_3 < x_2 < Y_2 < y_1 < Y_3]$$

$$= P[x_1 < X_3 < x_2]P[x_2 < Y_2 < y_1]P[y_1 < Y_3]$$

$$= \left(m^{k_0} - u^{k_0}\right)\left(v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right)\left(1 - v^{\frac{1}{k_0}}\right)$$

$$I_{17} = P[x_1 < Y_2 < x_2 < X_3 < y_1 < Y_3]$$

$$= P[x_1 < Y_2 < x_2]P[x_2 < X_3 < y_1]P[y_1 < Y_3]$$

$$= \left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\left(v^{k_0} - m^{k_0}\right)\left(1 - v^{\frac{1}{k_0}}\right)$$

$$I_{18} = P[x_1 < Y_2 < x_2 < Y_3 < y_1 < X_3]$$

$$= P[x_1 < Y_2 < x_2]P[x_2 < Y_3 < y_1]P[y_1 < X_3]$$

$$= \left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\left(v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right)\left(1 - v^{\frac{1}{k_0}}\right)$$

$$I_{19} = P[x_1 < X_3 < x_2 < Y_2 < Y_3 < y_1]$$

$$= P[x_1 < X_3 < x_2]P[x_2 < Y_2 < Y_3 < y_1]$$

$$= \left(m^{k_0} - u^{k_0}\right)\left[\frac{1}{2}\left(v^{\frac{2}{k_0}} - m^{\frac{2}{k_0}}\right) - \frac{1}{m^{k_0}}\left(v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right)\right]$$

$$I_{20} = P[x_1 < Y_2 < x_2 < X_3 < Y_3 < y_1]$$

$$= P[x_1 < Y_2 < x_2]P[x_2 < X_3 < Y_3 < y_1]$$

$$= \left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\left[\frac{1}{\left(k_0^{\,2}+1\right)}\left(v^{\frac{1}{k_0}+k_0} - m^{\frac{1}{k_0}+k_0}\right) - m^{k_0}\left(v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right)\right]$$

$$I_{21} = P[x_1 < Y_2 < x_2 < Y_3 < X_3 < y_1]$$

$$= P[x_1 < Y_2 < x_2]P[x_2 < Y_3 < X_3 < y_1]$$

$$= \left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\left[\frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)}\left(v^{\frac{1}{k_0}+k_0} - m^{\frac{1}{k_0}+k_0}\right) - m^{\frac{1}{k_0}}\left(v^{k_0} - m^{k_0}\right)\right]$$

$$I_{22} = P[x_1 < X_3 < Y_2 < x_2 < y_1 < Y_3]$$

$$= P[x_1 < X_3 < Y_2 < x_2]P[y_1 < Y_3]$$

$$= \left[\frac{1}{(k_0^{\,2}+1)}\left(m^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right) - u^{k_0}\left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\right]\left[1 - v^{\frac{1}{k_0}}\right]$$

$$I_{23} = P[x_1 < Y_2 < X_3 < x_2 < y_1 < Y_3]$$

$$= P[x_1 < Y_2 < X_3 < x_2]P[y_1 < Y_3]$$

$$= \left[\frac{k_0^{\,2}}{(k_0^{\,2}+1)}\left(m^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right) - u^{\frac{1}{k_0}}\left(m^{k_0} - u^{k_0}\right)\right]\left[1 - v^{\frac{1}{k_0}}\right]$$

$$I_{24} = P[x_1 < X_3 < Y_2 < x_2 < Y_3 < y_1]$$

$$= P[x_1 < X_3 < Y_2 < x_2]P[x_2 < Y_3 < y_1]$$

$$= \left[\frac{1}{(k_0^{\,2}+1)}\left(m^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right) - u^{k_0}\left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\right]\left[v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right]$$

$$I_{25} = P[x_1 < Y_2 < X_3 < x_2 < Y_3 < y_1]$$

$$= P[x_1 < Y_2 < X_3 < x_2]P[x_2 < Y_3 < y_1]$$

$$= \left[\frac{k_0^{\,2}}{(k_0^{\,2}+1)}\left(m^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right) - u^{\frac{1}{k_0}}\left(m^{k_0} - u^{k_0}\right)\right]\left[v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right]$$

$$I_{26} = P[x_1 < X_3 < Y_2 < Y_3 < x_2 < y_1]$$

$$= P[x_1 < X_3 < Y_2 < Y_3 < x_2]$$

$$= \left[\frac{\left(m^{\frac{2}{k_0}+k_0} - u^{\frac{2}{k_0}+k_0}\right)}{(k_0^{\,2}+1)(k_0^{\,2}+2)} - \frac{u^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)}\left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right) - \frac{u^{k_0}}{2}\left(m^{\frac{2}{k_0}} - u^{\frac{2}{k_0}}\right) + u^{\frac{1}{k_0}+k_0}\left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\right]$$

$$I_{27} = P[x_1 < Y_2 < X_3 < Y_3 < x_2 < y_1]$$

$$= P[x_1 < Y_2 < X_3 < Y_3 < x_2]$$

$$= \left[\frac{k_0^{\,2}\left(m^{\frac{2}{k_0}+k_0} - u^{\frac{2}{k_0}+k_0} \right)}{\left(k_0^{\,2}+1\right)\left(k_0^{\,2}+2\right)} - \frac{k_0^{\,2} u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)} \left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}} \right) - \frac{u^{\frac{1}{k_0}}}{\left(k_0^{\,2}+1\right)} \left(m^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0} \right) + u^{\frac{1}{k_0}+k_0} \left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}} \right) \right]$$

$$I_{28} = P[X_3 < x_1 < x_2 < y_1 < Y_2 < Y_3]$$

$$= P[X_3 < x_1] P[y_1 < Y_2 < Y_3]$$

$$= u^{k_0} \left[\left(1 - v^{\frac{1}{k_0}} \right) - \frac{1}{2} \left(1 - v^{\frac{2}{k_0}} \right) \right]$$

$$I_{29} = P[Y_2 < x_1 < x_2 < y_1 < X_3 < Y_3]$$

$$= P[Y_2 < x_1] P[y_1 < X_3 < Y_3]$$

$$= u^{\frac{1}{k_0}} \left[\left(1 - v^{k_0} \right) - \frac{k_0^{\,2}}{(k_0+1)} \left(1 - v^{\frac{1}{k_0}+k_0} \right) \right]$$

$$I_{30} = P[Y_2 < x_1 < x_2 < y_1 < Y_3 < X_3]$$

$$= P[Y_2 < x_1] P[y_1 < Y_3 < X_3]$$

$$= u^{\frac{1}{k_0}} \left[\left(1 - v^{\frac{1}{k_0}} \right) - \frac{1}{(k_0+1)} \left(1 - v^{\frac{1}{k_0}+k_0} \right) \right]$$

$$I_{31} = P[X_3 < x_1 < x_2 < Y_2 < y_1 < Y_3]$$

$$= P[X_3 < x_1] P[x_2 < Y_2 < y_1] P[y_1 < Y_3]$$

$$= u^{k_0} \left(v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}} \right) \left(1 - v^{\frac{1}{k_0}} \right)$$

$$I_{32} = P[Y_2 < x_1 < x_2 < X_3 < y_1 < Y_3]$$

$$= P[Y_2 < x_1] P[x_2 < X_3 < y_1] P[y_1 < Y_3]$$

$$= u^{\frac{1}{k_0}} \left(v^{k_0} - m^{k_0} \right) \left(1 - v^{\frac{1}{k_0}} \right)$$

$$I_{33} = P[Y_2 < x_1 < x_2 < Y_3 < y_1 < X_3]$$

$$= P[Y_2 < x_1]P[x_2 < Y_3 < y_1]P[y_1 < X_3]$$

$$= u^{\frac{1}{k_0}}\left(v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right)\left(1 - v^{k_0}\right)$$

$$I_{34} = P[X_3 < x_1 < x_2 < Y_2 < Y_3 < y_1]$$

$$= P[X_3 < x_1]P[x_2 < Y_2 < Y_3 < y_1]$$

$$= u^{k_0}\left[\frac{1}{2}\left(v^{\frac{2}{k_0}} - m^{\frac{2}{k_0}}\right) - m^{\frac{1}{k_0}}\left(v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right)\right]$$

$$I_{35} = P[Y_2 < x_1 < x_2 < X_3 < Y_3 < y_1]$$

$$= P[Y_2 < x_1]P[x_2 < X_3 < Y_3 < y_1]$$

$$= u^{\frac{1}{k_0}}\left[\frac{1}{\left(k_0^{\,2}+1\right)}\left(v^{\frac{1}{k_0}+k_0} - m^{\frac{1}{k_0}+k_0}\right) - m^{k_0}\left(v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right)\right]$$

$$I_{36} = P[Y_2 < x_1 < x_2 < Y_3 < X_3 < y_1]$$

$$= P[Y_2 < x_1]P[x_2 < Y_3 < X_3 < y_1]$$

$$= u^{\frac{1}{k_0}}\left[\frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)}\left(v^{\frac{1}{k_0}+k_0} - m^{\frac{1}{k_0}+k_0}\right) - m^{\frac{1}{k_0}}\left(v^{k_0} - m^{k_0}\right)\right]$$

$$I_{37} = P[X_3 < x_1 < Y_2 < x_2 < y_1 < Y_3]$$

$$= P[X_3 < x_1]P[x_1 < Y_2 < x_2]P[y_1 < Y_3]$$

$$= u^{k_0}\left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\left(1 - v^{\frac{1}{k_0}}\right)$$

$$I_{38} = P[Y_2 < x_1 < X_3 < x_2 < y_1 < Y_3]$$

$$= P[Y_2 < x_1]P[x_1 < X_3 < x_2]P[y_1 < Y_3]$$

$$= u^{\frac{1}{k_0}}\left(m^{k_0} - u^{k_0}\right)\left(1 - v^{\frac{1}{k_0}}\right)$$

$$I_{39} = P[X_3 < x_1 < Y_2 < x_2 < Y_3 < y_1]$$

$$= P[X_3 < x_1]P[x_1 < Y_2 < x_2]P[x_2 < Y_3 < y_1]$$

$$= u^{k_0}\left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\left(v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right)$$

$$I_{40} = P[Y_2 < x_1 < X_3 < x_2 < Y_3 < y_1]$$

$$= P[Y_2 < x_1]P[x_1 < X_3 < x_2]P[x_2 < Y_3 < y_1]$$

$$= u^{\frac{1}{k_0}}\left(m^{k_0} - u^{k_0}\right)\left(v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right)$$

$$I_{41} = P[X_3 < x_1 < Y_2 < Y_3 < x_2 < y_1]$$

$$= P[X_3 < x_1]P[x_1 < Y_2 < Y_3 < x_2]$$

$$= u^{k_0}\left[\frac{1}{2}\left(m^{\frac{2}{k_0}} - u^{\frac{2}{k_0}}\right) - u^{\frac{1}{k_0}}\left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\right]$$

$$I_{42} = P[Y_2 < x_1 < X_3 < Y_3 < x_2 < y_1]$$

$$= P[Y_2 < x_1]P[x_1 < X_3 < Y_3 < x_2]$$

$$= u^{\frac{1}{k_0}}\left[\frac{1}{\left(k_0^{\,2}+1\right)}\left(m^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right) - u^{k_0}\left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\right]$$

$$I_{43} = P[X_3 < Y_2 < x_1 < x_2 < y_1 < Y_3]$$

$$= P[X_3 < Y_2 < x_2]P[y_1 < Y_3]$$

$$= \frac{u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}\left(1 - v^{\frac{1}{k_0}}\right)$$

$$I_{44} = P[Y_2 < X_3 < x_1 < x_2 < y_1 < Y_3]$$

$$= P[Y_2 < X_3 < x_1]P[y_1 < Y_3]$$

$$= \frac{k_0^{\,2} u^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)}\left(1 - v^{\frac{1}{k_0}}\right)$$

$$I_{45} = P[X_3 < Y_2 < x_1 < x_2 < Y_3 < y_1]$$

$$= P[X_3 < Y_2 < x_1]P[x_2 < Y_3 < y_1]$$

$$= \frac{u^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)}\left(v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right)$$

$$I_{46} = P[Y_2 < X_3 < x_1 < x_2 < Y_3 < y_1]$$

$$= P[Y_2 < X_3 < x_1]P[x_2 < Y_3 < y_1]$$

$$= \frac{k_0^{\,2} u^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)}\left(v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right)$$

$$I_{47} = P[X_3 < Y_2 < x_1 < Y_3 < x_2 < y_1]$$

$$= P[X_3 < Y_2 < x_1]P[x_1 < Y_3 < x_2]$$

$$= \frac{u^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)}\left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)$$

$$I_{48} = P[Y_2 < X_3 < x_1 < Y_3 < x_2 < y_1]$$

$$= P[Y_2 < X_3 < x_1]P[x_1 < Y_3 < x_2]$$

$$= \frac{k_0^{\,2} u^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)}\left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)$$

where $F(x_1) = u$ $F(x_2) = m$ and $F(y_1) = v$

Now adding all the 48 terms and simplifying we have

$$P[Med(x_1, x_2, X_3) < Med(y_1, Y_2, Y_3)] = 2I_1 + 2I_2 + \ldots + 2I_{48}$$

$$=1-m^{\frac{2}{k_0}}-2m^{\frac{2}{k_0}}v^{k_0}+2m^{\frac{2}{k_0}}v^{\frac{1}{k_0}}-2m^{\frac{1}{k_0}}v^{\frac{2}{k_0}}+2m^{\frac{1}{k_0}}v^{\frac{1}{k_0}+k_0}+2u^{\frac{1}{k_0}}v^{\frac{2}{k_0}}-2u^{\frac{1}{k_0}}v^{\frac{1}{k_0}+k_0}+2u^{\frac{1}{k_0}}m^{\frac{1}{k_0}}v^{k_0}$$

$$-2u^{\frac{1}{k_0}}m^{\frac{1}{k_0}}v^{\frac{1}{k_0}}+\frac{2m^{\frac{2}{k_0}+k_0}}{(k_0^{\,2}+2)}-\frac{2u^{\frac{2}{k_0}+k_0}}{(k_0^{\,2}+2)}.$$

Thus integrating we have

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_1}\int_{-\infty}^{x_2}P^2[Med(x_1,x_2,X_3)<Med(y_1,Y_2,Y_3)]\mathrm{dF(x_1)dF(x_2)dG(y_1)}$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{y_1}\int_{-\infty}^{x_2}\left(\begin{array}{l}1-m^{\frac{2}{k_0}}-2m^{\frac{2}{k_0}}v^{k_0}+2m^{\frac{2}{k_0}}v^{\frac{1}{k_0}}-2m^{\frac{1}{k_0}}v^{\frac{2}{k_0}}+2m^{\frac{1}{k_0}}v^{\frac{1}{k_0}+k_0}+2u^{\frac{1}{k_0}}v^{\frac{2}{k_0}}-2u^{\frac{1}{k_0}}v^{\frac{1}{k_0}+k_0}+2u^{\frac{1}{k_0}}m^{\frac{1}{k_0}}v^{k_0}\\ -2u^{\frac{1}{k_0}}m^{\frac{1}{k_0}}v^{\frac{1}{k_0}}+\frac{2m^{\frac{2}{k_0}+k_0}}{(k_0^{\,2}+2)}-\frac{2u^{\frac{2}{k_0}+k_0}}{(k_0^{\,2}+2)}\end{array}\right)^2\mathrm{dF(x_1)dF(x_2)dF(y_1)}$$

where $F(x_1)=u \quad F(x_2)=m$ and $F(y_1)=v$

$$=\int_0^1\int_0^v\int_0^m\left(\begin{array}{l}1-m^{\frac{2}{k_0}}-2m^{\frac{2}{k_0}}v^{k_0}+2m^{\frac{2}{k_0}}v^{\frac{1}{k_0}}-2m^{\frac{1}{k_0}}v^{\frac{2}{k_0}}+2m^{\frac{1}{k_0}}v^{\frac{1}{k_0}+k_0}+2u^{\frac{1}{k_0}}v^{\frac{2}{k_0}}-2u^{\frac{1}{k_0}}v^{\frac{1}{k_0}+k_0}+2u^{\frac{1}{k_0}}m^{\frac{1}{k_0}}v^{k_0}\\ -2u^{\frac{1}{k_0}}m^{\frac{1}{k_0}}v^{\frac{1}{k_0}}+\frac{2m^{\frac{2}{k_0}+k_0}}{(k_0^{\,2}+2)}-\frac{2u^{\frac{2}{k_0}+k_0}}{(k_0^{\,2}+2)}\end{array}\right)^2 k_0u^{k_0-1}k_0m^{k_0-1}\frac{1}{k_0}v^{\frac{1}{k_0}-1}\,\mathrm{du\,dmdv}$$

$$=\frac{1}{3}\left[\frac{\left(\begin{array}{l}33842580k_0^{\,4}+7692600k_0^{\,2}+86039078k_0^{\,6}+30983346k_0^{\,16}+9744k_0^{\,24}+756000\\ +141987009k_0^{\,8}+8903892k_0^{\,18}+1693712k_0^{\,20}+161294441k_0^{\,10}+129975109k_0^{\,12}\\ +75125809k_0^{\,14}+191560k_0^{\,22}\end{array}\right)}{(2k_0^{\,2}+7)(4k_0^{\,2}+5)(2k_0^{\,2}+5)(k_0^{\,2}+2)^3(2k_0^{\,2}+1)(k_0^{\,2}+1)^3(3k_0^{\,2}+4)(2k_0^{\,2}+3)(3k_0^{\,2}+2)(3k_0^{\,2}+5)(k_0^{\,2}+3)}\right]$$

after simplification.

Now consider the second term of (2.4.2.2J).

Interchanging x_1 and x_2 in the above case and simplifying we have

$$\iiint_{x_2<x_1<y_1}P^2[Med(x_1,x_2,X_3)<Med(y_1,Y_2,Y_3)]\mathrm{dF(x_2)dF(x_1)dG(y_1)}$$

$$=\frac{1}{3}\left[\frac{\left(\begin{array}{l}33842580k_0^{\,4}+7692600k_0^{\,2}+86039078k_0^{\,6}+30983346k_0^{\,16}+9744k_0^{\,24}+756000\\ +141987009k_0^{\,8}+8903892k_0^{\,18}+1693712k_0^{\,20}+161294441k_0^{\,10}+129975109k_0^{\,12}\\ +75125809k_0^{\,14}+191560k_0^{\,22}\end{array}\right)}{(2k_0^{\,2}+7)(4k_0^{\,2}+5)(2k_0^{\,2}+5)(k_0^{\,2}+2)^3(2k_0^{\,2}+1)(k_0^{\,2}+1)^3(3k_0^{\,2}+4)(2k_0^{\,2}+3)(3k_0^{\,2}+2)(3k_0^{\,2}+5)(k_0^{\,2}+3)}\right].$$

Now consider the third term of (2.4.2.2J).

$$\iiint_{x_1<y_1<x_2} P^2[Med(x_1,x_2,X_3)<Med(y_1,Y_2,Y_3)]\,dF(x_1)\,dG(y_1)\,dF(x_2).$$

We have

$$P[Med(x_1,x_2,X_3)<Med(y_1,Y_2,Y_3)]=2J_1+2J_2+\ldots+2J_{30}$$

where

$$J_1=P[x_1<y_1<x_2<X_3<Y_2<Y_3]$$

$$=P[x_2<X_3<Y_2<Y_3]$$

$$=\int_{x_2}^{\infty}\int_{x_3}^{\infty}P(y_2<Y_3)\,dG(y_2)\,dF(x_3)$$

$$=\int_{x_2}^{\infty}\int_{x_3}^{\infty}\left[1-r^{\frac{1}{k_0}}\right]\frac{1}{k_0}r^{\frac{1}{k_0}-1}k_0n^{k_0-1}\,dr\,dn$$

$$=\left(1-m^{k_0}\right)-\frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)}\left(1-m^{\frac{1}{k_0}+k_0}\right)-\frac{1}{2}\left(1-m^{k_0}\right)+\frac{k_0^{\,2}}{2\left(k_0^{\,2}+2\right)}\left(1-m^{\frac{2}{k_0}+k_0}\right)$$

$$J_2=P[x_1<y_1<x_2<Y_2<X_3<Y_3]$$

$$=P[x_2<Y_2<X_3<Y_3]$$

$$=\left(1-m^{\frac{1}{k_0}}\right)-\frac{1}{\left(k_0^{\,2}+1\right)}\left(1-m^{\frac{1}{k_0}+k_0}\right)-\frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)}\left(1-m^{\frac{1}{k_0}}\right)+\frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)\left(k_0^{\,2}+2\right)}\left(1-m^{\frac{2}{k_0}+k_0}\right)$$

$$J_3=P[x_1<y_1<x_2<Y_2<Y_3<X_3]$$

$$=P[x_2<Y_2<Y_3<X_3]$$

$$=\left(1-m^{\frac{1}{k_0}}\right)-\frac{1}{2}\left(1-m^{\frac{2}{k_0}}\right)-\frac{1}{\left(k_0^{\,2}+1\right)}\left(1-m^{\frac{1}{k_0}}\right)+\frac{1}{\left(k_0^{\,2}+1\right)\left(k_0^{\,2}+2\right)}\left(1-m^{\frac{2}{k_0}+k_0}\right)$$

$$J_4=P[x_1<y_1<X_3<x_2<Y_2<Y_3]$$

$$=P[y_1<X_3<x_2]P[x_2<Y_2<Y_3]$$

$$= \left(m^{k_0} - v^{k_0}\right)\left[\left(1 - m^{\frac{1}{k_0}}\right) - \frac{1}{2}\left(1 - m^{\frac{2}{k_0}}\right)\right]$$

$$J_5 = P[x_1 < y_1 < X_3 < Y_2 < x_2 < Y_3]$$

$$= P[y_1 < X_3 < Y_2 < x_2]P[x_2 < Y_3]$$

$$= \left[\frac{1}{\left(k_0^{\,2}+1\right)}\left(m^{\frac{1}{k_0}+k_0} - v^{\frac{1}{k_0}+k_0}\right) - v^{k_0}\left(m^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right)\right]\left[1 - m^{\frac{1}{k_0}}\right]$$

$$J_6 = P[x_1 < y_1 < X_3 < Y_2 < Y_3 < x_2]$$

$$= P[y_1 < X_3 < Y_2 < Y_3 < x_2]$$

$$= \left[\frac{1}{\left(k_0^{\,2}+1\right)\left(k_0^{\,2}+2\right)}\left(m^{\frac{2}{k_0}+k_0} - v^{\frac{2}{k_0}+k_0}\right) - \frac{v^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}\left(m^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right) - \frac{v^{k_0}}{2}\left(m^{\frac{2}{k_0}} - v^{\frac{2}{k_0}}\right) + v^{\frac{1}{k_0}+k_0}\left(m^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right)\right]$$

$$J_7 = P[x_1 < X_3 < y_1 < x_2 < Y_2 < Y_3]$$

$$= P[x_1 < X_3 < y_1]P[x_2 < Y_2 < Y_3]$$

$$= \left(v^{k_0} - u^{k_0}\right)\left[\left(1 - m^{\frac{1}{k_0}}\right) - \frac{1}{2}\left(1 - m^{\frac{2}{k_0}}\right)\right]$$

$$J_8 = P[x_1 < X_3 < y_1 < Y_2 < x_2 < Y_3]$$

$$= P[x_1 < X_3 < y_1]P[y_1 < Y_2 < x_2]P[x_2 < Y_3]$$

$$= \left(v^{k_0} - u^{k_0}\right)\left(m^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right)\left[1 - m^{\frac{1}{k_0}}\right]$$

$$J_9 = P[x_1 < X_3 < y_1 < Y_2 < Y_3 < x_2]$$

$$= P[x_1 < X_3 < y_1]P[y_1 < Y_2 < Y_3 < x_2]$$

$$= \left(v^{k_0} - u^{k_0}\right)\left[\frac{1}{2}\left(m^{\frac{2}{k_0}} - v^{\frac{2}{k_0}}\right) - v^{\frac{1}{k_0}}\left(m^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right)\right]$$

$$J_{10} = P[x_1 < X_3 < Y_2 < y_1 < x_2 < Y_3]$$

$$= P[x_1 < X_3 < Y_2 < y_1]P[x_2 < Y_3]$$

$$= \left[\frac{1}{(k_0^{\,2}+1)}\left(v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right) - u^{k_0}\left(v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\right]\left[1 - m^{\frac{1}{k_0}}\right]$$

$$J_{11} = P[x_1 < Y_2 < X_3 < y_1 < x_2 < Y_3]$$

$$= P[x_1 < Y_2 < X_3 < y_1]P[x_2 < Y_3]$$

$$= \left[\frac{k_0^{\,2}}{(k_0^{\,2}+1)}\left(v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right) - u^{\frac{1}{k_0}}\left(v^{k_0} - u^{k_0}\right)\right]\left[1 - m^{\frac{1}{k_0}}\right]$$

$$J_{12} = P[x_1 < X_3 < Y_2 < y_1 < Y_3 < x_2]$$

$$= P[x_1 < X_3 < Y_2 < y_1]P[y_1 < Y_3 < x_2]$$

$$= \left[\frac{1}{(k_0^{\,2}+1)}\left(v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right) - u^{k_0}\left(v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\right]\left[m^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right]$$

$$J_{13} = P[x_1 < Y_2 < X_3 < y_1 < Y_3 < x_2]$$

$$= P[x_1 < Y_2 < X_3 < y_1]P[y_1 < Y_3 < x_2]$$

$$= \left[\frac{k_0^{\,2}}{(k_0^{\,2}+1)}\left(v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right) - u^{\frac{1}{k_0}}\left(v^{k_0} - u^{k_0}\right)\right]\left[m^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right]$$

$$J_{14} = P[x_1 < X_3 < Y_2 < Y_3 < y_1 < x_2]$$

$$= P[x_1 < X_3 < Y_2 < Y_3 < y_1]$$

$$= \left[\frac{1}{(k_0^{\,2}+1)(k_0^{\,2}+2)}\left(v^{\frac{2}{k_0}+k_0} - u^{\frac{2}{k_0}+k_0}\right) - \frac{u^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)}\left(v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right) - \frac{u^{k_0}}{2}\left[v^{\frac{2}{k_0}} - u^{\frac{2}{k_0}}\right] + u^{\frac{1}{k_0}+k_0}\left(v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\right]$$

$$J_{15} = P[x_1 < Y_2 < X_3 < Y_3 < y_1 < x_2]$$

$$= P[x_1 < Y_2 < X_3 < Y_3 < y_1]$$

$$= \left[\frac{k_0^{\,2}}{(k_0^{\,2}+1)(k_0^{\,2}+2)} \left(v^{\frac{2}{k_0}+k_0} - u^{\frac{2}{k_0}+k_0} \right) - \frac{k_0^{\,2} u^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)} \left(v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}} \right) - \frac{u^{\frac{1}{k_0}}}{(k_0^{\,2}+1)} \left[v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0} \right] + u^{\frac{1}{k_0}+k_0} \left(v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}} \right) \right]$$

$$J_{16} = P[X_3 < x_1 < y_1 < x_2 < Y_2 < Y_3]$$

$$= P[X_3 < x_1] P[x_2 < Y_2 < Y_3]$$

$$= u^{k_0} \left[\left(1 - m^{\frac{1}{k_0}} \right) - \frac{1}{2} \left(1 - m^{\frac{2}{k_0}} \right) \right]$$

$$J_{17} = P[X_3 < x_1 < y_1 < Y_2 < x_2 < Y_3]$$

$$= P[X_3 < x_1] P[y_1 < Y_2 < x_2] P[x_2 < Y_3]$$

$$= u^{k_0} \left(m^{\frac{1}{k_0}} - v^{\frac{1}{k_0}} \right) \left(1 - m^{\frac{1}{k_0}} \right)$$

$$J_{18} = P[X_3 < x_1 < y_1 < Y_2 < Y_3 < x_2]$$

$$= P[X_3 < x_1] P[y_1 < Y_2 < Y_3 < x_2]$$

$$= u^{k_0} \left[\frac{1}{2} \left(m^{\frac{2}{k_0}} - v^{\frac{2}{k_0}} \right) - v^{\frac{1}{k_0}} \left(m^{\frac{1}{k_0}} - v^{\frac{1}{k_0}} \right) \right]$$

$$J_{19} = P[X_3 < x_1 < Y_2 < y_1 < x_2 < Y_3]$$

$$= P[X_3 < x_1] P[x_1 < Y_2 < y_1] P[x_2 < Y_3]$$

$$= u^{k_0} \left(v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}} \right) \left(1 - m^{\frac{1}{k_0}} \right)$$

$$J_{20} = P[Y_2 < x_1 < X_3 < y_1 < x_2 < Y_3]$$

$$= P[Y_2 < x_1] P[x_1 < X_3 < y_1] P[x_2 < Y_3]$$

$$= u^{\frac{1}{k_0}} \left(v^{k_0} - u^{k_0} \right) \left(1 - m^{\frac{1}{k_0}} \right)$$

$$J_{21} = P[X_3 < x_1 < Y_2 < y_1 < Y_3 < x_2]$$

$$= P[X_3 < x_1]P[x_1 < Y_2 < y_1]P[y_1 < Y_3 < x_2]$$

$$= u^{k_0}\left(v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\left(1 - m^{\frac{1}{k_0}}\right)$$

$$J_{22} = P[Y_2 < x_1 < X_3 < y_1 < Y_3 < x_2]$$

$$= P[Y_2 < x_1]P[x_1 < X_3 < y_1]P[y_1 < Y_3 < x_2]$$

$$= u^{\frac{1}{k_0}}\left(v^{k_0} - u^{k_0}\right)\left(m^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right)$$

$$J_{23} = P[X_3 < x_1 < Y_2 < Y_3 < y_1 < x_2]$$

$$= P[X_3 < x_1]P[x_1 < Y_2 < Y_3 < y_1]$$

$$= u^{k_0}\left[\frac{1}{2}\left(v^{\frac{2}{k_0}} - u^{\frac{2}{k_0}}\right) - u^{\frac{1}{k_0}}\left(v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\right]$$

$$J_{24} = P[Y_2 < x_1 < X_3 < Y_3 < y_1 < x_2]$$

$$= P[Y_2 < x_1]P[x_1 < X_3 < Y_3 < y_1]$$

$$= u^{\frac{1}{k_0}}\left[\frac{1}{\left(k_0^{\,2}+1\right)}\left(v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right) - u^{k_0}\left(v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\right]$$

$$J_{25} = P[X_3 < Y_2 < x_1 < y_1 < x_2 < Y_3]$$

$$= P[X_3 < Y_2 < x_1]P[x_2 < Y_3]$$

$$= \frac{u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}\left[1 - m^{\frac{1}{k_0}}\right]$$

$$J_{26} = P[Y_2 < X_3 < x_1 < y_1 < x_2 < Y_3]$$

$$= P[Y_2 < X_3 < x_1]P[x_2 < Y_3]$$

$$= \frac{k_0^{\,2} u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}\left[1-m^{\frac{1}{k_0}}\right]$$

$$J_{27} = P[X_3 < Y_2 < x_1 < y_1 < Y_3 < x_2]$$

$$= P[X_3 < Y_2 < x_1]P[y_1 < Y_3 < x_2]$$

$$= \frac{u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}\left[m^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right]$$

$$J_{28} = P[Y_2 < X_3 < x_1 < y_1 < Y_3 < x_2]$$

$$= P[Y_2 < X_3 < x_1]P[y_1 < Y_3 < x_2]$$

$$= \frac{k_0^{\,2} u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}\left[m^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right]$$

$$J_{29} = P[X_3 < Y_2 < x_1 < Y_3 < y_1 < x_2]$$

$$= P[X_3 < Y_2 < x_1]P[x_1 < Y_3 < y_1]$$

$$= \frac{u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right]$$

$$J_{30} = P[Y_2 < X_3 < x_1 < Y_3 < y_1 < x_2]$$

$$= P[Y_2 < X_3 < x_1]P[x_1 < Y_3 < y_1]$$

$$= \frac{k_0^{\,2} u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right]$$

where $F(x_1) = u$ $F(x_2) = m$ and $F(y_1) = v$

Now adding all the 30 terms and simplifying we have

$$P[Med(x_1, x_2, X_3) < Med(y_1, Y_2, Y_3)] = 2J_1 + 2J_2 + \ldots + 2J_{30}$$

$$=1-2m^{\frac{1}{k_0}}+\frac{2m^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}+m^{\frac{2}{k_0}}-\frac{2m^{\frac{2}{k_0}+k_0}}{\left(k_0^{\,2}+2\right)}+\frac{2k_0^{\,2}v^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}-\frac{2k_0^{\,2}v^{\frac{2}{k_0}+k_0}}{\left(k_0^{\,2}+2\right)}-\frac{2u^{\frac{2}{k_0}+k_0}}{\left(k_0^{\,2}+2\right)}.$$

Thus integrating we have

$$\int_{-\infty-\infty-\infty}^{\infty\ x_2\ y_1}\int\int P^2[Med(x_1,x_2,X_3)<Med(y_1,Y_2,Y_3)]\mathrm{dF(x_1)dG(y_1)dF(x_2)}$$

$$=\int_{-\infty-\infty-\infty}^{\infty\ x_2\ y_1}\int\int\left(1-2m^{\frac{1}{k_0}}+\frac{2m^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}+m^{\frac{2}{k_0}}-\frac{2m^{\frac{2}{k_0}+k_0}}{\left(k_0^{\,2}+2\right)}+\frac{2k_0^{\,2}v^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}-\frac{2k_0^{\,2}v^{\frac{2}{k_0}+k_0}}{\left(k_0^{\,2}+2\right)}-\frac{2u^{\frac{2}{k_0}+k_0}}{\left(k_0^{\,2}+2\right)}\right)\mathrm{dF(x_1)dG(y_1)dF(x_2)}$$

$$=\int_0^1\int_0^m\int_0^v\left(1-2m^{\frac{1}{k_0}}+\frac{2m^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}+m^{\frac{2}{k_0}}-\frac{2m^{\frac{2}{k_0}+k_0}}{\left(k_0^{\,2}+2\right)}+\frac{2k_0^{\,2}v^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}-\frac{2k_0^{\,2}v^{\frac{2}{k_0}+k_0}}{\left(k_0^{\,2}+2\right)}-\frac{2u^{\frac{2}{k_0}+k_0}}{\left(k_0^{\,2}+2\right)}\right)\mathrm{k_0u^{k_0-1}}\frac{1}{\mathrm{k_0}}v^{\frac{1}{k_0}-1}\,\mathrm{k_0m^{k_0-1}du\,dvdm}$$

$$=\frac{1}{3}\frac{k_0^{\,2}\left(\begin{array}{l}4944k_0^{\,16}+57392k_0^{\,14}+292824k_0^{\,12}+841576k_0^{\,10}+1472215k_0^{\,8}+1592439k_0^{\,6}+1033730k_0^{\,4}\\+366240k_0^{\,2}+54000\end{array}\right)}{\left[\left(3k_0^{\,2}+2\right)\left(3k_0^{\,2}+4\right)\left(3k_0^{\,2}+5\right)\left(4k_0^{\,2}+5\right)\left(k_0^{\,2}+2\right)\left(2k_0^{\,2}+5\right)\left(4k_0^{\,2}+3\right)\left(2k_0^{\,2}+3\right)\left(2k_0^{\,2}+1\right)\left(k_0^{\,2}+1\right)^3\right]}$$

after letting $F(x_1)=u$ $F(x_2)=m$ and $F(y_1)=v$ and simplifying

Now consider the fourth term of (2.4.2.2I).

Interchanging x_1 and x_2 in the above and simplifying we get,

$$\iiint_{x_2<y_1<x_1}P^2[Med(x_1,x_2,X_3)<Med(y_1,Y_2,Y_3)]\mathrm{dF(x_2)dG(y_1)dF(x_1)}$$

$$=\frac{1}{3}\frac{k_0^{\,2}\left(\begin{array}{l}4944k_0^{\,16}+57392k_0^{\,14}+292824k_0^{\,12}+841576k_0^{\,10}+1472215k_0^{\,8}+1592439k_0^{\,6}+1033730k_0^{\,4}\\+366240k_0^{\,2}+54000\end{array}\right)}{\left[\left(3k_0^{\,2}+2\right)\left(3k_0^{\,2}+4\right)\left(3k_0^{\,2}+5\right)\left(4k_0^{\,2}+5\right)\left(k_0^{\,2}+2\right)\left(2k_0^{\,2}+5\right)\left(4k_0^{\,2}+3\right)\left(2k_0^{\,2}+3\right)\left(2k_0^{\,2}+1\right)\left(k_0^{\,2}+1\right)^3\right]}$$

Now consider the fifth term of (2.4.2.2I).

$$\iiint_{y_1<x_1<x_2}P^2[Med(x_1,x_2,X_3)<Med(y_1,Y_2,Y_3)]\mathrm{dG(y_1)dF(x_1)dF(x_2)}$$

We have

$$P[Med(x_1,x_2,X_3)<Med(y_1,Y_2,Y_3)]=2K_1+2K_2+\ldots+2K_{12}$$

where

$$K_1=P[y_1<x_1<x_2<X_3<Y_2<Y_3]$$

$$=P[x_2<X_3<Y_2<Y_3]$$

$$=(1-m^{k_0})-\frac{k_0^{\,2}}{(k_0^{\,2}+1)}\left(1-m^{\frac{1}{k_0}+k_0}\right)-\frac{1}{2}(1-m^{k_0})+\frac{k_0^{\,2}}{2(k_0^{\,2}+2)}\left(1-m^{\frac{2}{k_0}+k_0}\right)$$

$$K_2=P[y_1<x_1<x_2<Y_2<X_3<Y_3]$$

$$=P[x_2<Y_2<X_3<Y_3]$$

$$=\left(1-m^{\frac{1}{k_0}}\right)-\frac{1}{(k_0^{\,2}+1)}\left(1-m^{\frac{1}{k_0}+k_0}\right)-\frac{k_0^{\,2}}{(k_0^{\,2}+1)}\left(1-m^{\frac{1}{k_0}}\right)+\frac{k_0^{\,2}}{(k_0^{\,2}+1)(k_0^{\,2}+2)}\left(1-m^{\frac{2}{k_0}+k_0}\right)$$

$$K_3=P[y_1<x_1<x_2<Y_2<Y_3<X_3]$$

$$=P[x_2<Y_2<Y_3<X_3]$$

$$=\left(1-m^{\frac{1}{k_0}}\right)-\frac{1}{2}\left(1-m^{\frac{2}{k_0}}\right)-\frac{1}{(k_0^{\,2}+1)}\left(1-m^{\frac{1}{k_0}}\right)+\frac{1}{(k_0^{\,2}+1)(k_0^{\,2}+2)}\left(1-m^{\frac{2}{k_0}+k_0}\right)$$

$$K_4=P[y_1<x_1<X_3<x_2<Y_2<Y_3]$$

$$=P[x_1<X_3<x_2]P[x_2<Y_2<Y_3]$$

$$=(m^{k_0}-u^{k_0})\left[\left(1-m^{\frac{1}{k_0}}\right)-\frac{1}{2}\left(1-m^{\frac{2}{k_0}}\right)\right]$$

$$K_5=P[y_1<x_1<X_3<Y_2<x_2<Y_3]$$

$$=P[x_1<X_3<Y_2<x_2]P[x_2<Y_3]$$

$$=\left[\frac{1}{(k_0^{\,2}+1)}\left(m^{\frac{1}{k_0}+k_0}-u^{\frac{1}{k_0}+k_0}\right)-u^{k_0}\left(m^{\frac{1}{k_0}}-u^{\frac{1}{k_0}}\right)\right]\left[1-m^{\frac{1}{k_0}}\right]$$

$$K_6 = P[y_1 < x_1 < X_3 < Y_2 < Y_3 < x_2]$$

$$= P[x_1 < X_3 < Y_2 < Y_3 < x_2]$$

$$= \left[\frac{1}{(k_0^2+1)(k_0^2+2)}\left(m^{\frac{2}{k_0}+k_0} - u^{\frac{2}{k_0}+k_0}\right) - \frac{u^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}\left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right) - \frac{u^{k_0}}{2}\left(m^{\frac{2}{k_0}} - u^{\frac{2}{k_0}}\right) + u^{\frac{1}{k_0}+k_0}\left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\right]$$

$$K_7 = P[y_1 < X_3 < x_1 < x_2 < Y_2 < Y_3]$$

$$= P[y_1 < X_3 < x_1]P[x_2 < Y_2 < Y_3]$$

$$= \left(u^{k_0} - v^{k_0}\right)\left[\left(1 - m^{\frac{1}{k_0}}\right) - \frac{1}{2}\left(1 - m^{\frac{2}{k_0}}\right)\right]$$

$$K_8 = P[y_1 < X_3 < x_1 < Y_2 < x_2 < Y_3]$$

$$= P[y_1 < X_3 < x_1]P[x_1 < Y_2 < x_2]P[x_2 < Y_3]$$

$$= \left(u^{k_0} - v^{k_0}\right)\left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\left(1 - m^{\frac{1}{k_0}}\right)$$

$$K_9 = P[y_1 < X_3 < x_1 < Y_2 < Y_3 < x_2]$$

$$= P[y_1 < X_3 < x_1]P[x_1 < Y_2 < Y_3 < x_2]$$

$$= \left(u^{k_0} - v^{k_0}\right)\left[\frac{1}{2}\left(m^{\frac{2}{k_0}} - u^{\frac{2}{k_0}}\right) - u^{\frac{1}{k_0}}\left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\right]$$

$$K_{10} = P[X_3 < y_1 < x_1 < x_2 < Y_2 < Y_3]$$

$$= P[X_3 < y_1]P[x_2 < Y_2 < Y_3]$$

$$= \left(v^{k_0}\right)\left[\left(1 - m^{\frac{1}{k_0}}\right) - \frac{1}{2}\left(1 - m^{\frac{2}{k_0}}\right)\right]$$

$$K_{11} = P[X_3 < y_1 < x_1 < Y_2 < x_2 < Y_3]$$

$$= P[X_3 < y_1]P[x_1 < Y_2 < x_2]P[x_2 < Y_3]$$

$$= \left(v^{k_0}\right)\left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\left(1 - m^{\frac{1}{k_0}}\right)$$

$$K_{12} = P[X_3 < y_1 < x_1 < Y_2 < Y_3 < x_2]$$

$$= P[X_3 < y_1]P[x_1 < Y_2 < Y_3 < x_2]$$

$$= \left(v^{k_0}\right)\left[\frac{1}{2}\left(m^{\frac{2}{k_0}} - u^{\frac{2}{k_0}}\right) - u^{\frac{1}{k_0}}\left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\right]$$

where $F(x_1) = u \quad F(x_2) = m$ and $F(y_1) = v$

Now adding all the 12 terms and simplifying we have

$$P[Med(x_1, x_2, X_3) < Med(y_1, Y_2, Y_3)] = 2k_1 + 2k_2 + \ldots + 2k_{12}$$

$$= 1 + m^{\frac{2}{k_0}} + \frac{2m^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)} - 2m^{\frac{1}{k_0}} - \frac{2m^{\frac{2}{k_0}+k_0}}{\left(k_0^{\,2}+2\right)} + \frac{2u^{\frac{2}{k_0}+k_0}}{\left(k_0^{\,2}+2\right)} - \frac{2u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}.$$

Thus integrating we have

$$\int_{-\infty}^{\infty}\int_{-\infty}^{x_2}\int_{-\infty}^{x_1} P^2[Med(x_1, x_2, X_3) < Med(y_1, Y_2, Y_3)]\,dG(y_1)\,dF(x_1)\,dF(x_2)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{x_2}\int_{-\infty}^{x_1}\left(1 + m^{\frac{2}{k_0}} + \frac{2m^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)} - 2m^{\frac{1}{k_0}} - \frac{2m^{\frac{2}{k_0}+k_0}}{\left(k_0^{\,2}+2\right)} + \frac{2u^{\frac{2}{k_0}+k_0}}{\left(k_0^{\,2}+2\right)} - \frac{2u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}\right)^2 dG(y_1)\,dF(x_1)\,dF(x_2)$$

$$= \int_0^{\infty}\int_0^{m}\int_0^{u}\left(1 + m^{\frac{2}{k_0}} + \frac{2m^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)} - 2m^{\frac{1}{k_0}} - \frac{2m^{\frac{2}{k_0}+k_0}}{\left(k_0^{\,2}+2\right)} + \frac{2u^{\frac{2}{k_0}+k_0}}{\left(k_0^{\,2}+2\right)} - \frac{2u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}\right)^2 k_0 v^{\frac{1}{k_0}-1} u^{k_0-1} m^{k_0-1}\,dv\,du\,dm$$

$$= \frac{1}{3}\left[\frac{k_0^{\,4}\left(12864k_0^{\,12} + 93532k_0^{\,10} + 276154k_0^{\,8} + 421131k_0^{\,6} + 347855k_0^{\,4} + 146804k_0^{\,2} + 24660\right)}{\left(3k_0^{\,2}+4\right)\left(3k_0^{\,2}+2\right)\left(3k_0^{\,2}+5\right)\left(4k_0^{\,2}+5\right)\left(k_0^{\,2}+2\right)\left(2k_0^{\,2}+3\right)\left(4k_0^{\,2}+3\right)\left(2k_0^{\,2}+5\right)\left(2k_0^{\,2}+1\right)\left(k_0^{\,2}+1\right)^3}\right]$$

after letting $F(x_1) = u \quad F(x_2) = m$ and $F(y_1) = v$ and simplifying

Now consider the sixth term of (2.4.2.2I).

Interchanging x_1 and x_2 in the above case and simplifying we get,

$$\iiint_{y_1<x_2<x_1} P^2[Med(x_1,x_2,X_3)<Med(y_1,Y_2,Y_3)]dG(y_1)dF(x_2)dF(x_1)$$

$$=\frac{1}{3}\frac{k_0^4\left(12864k_0^{12}+93532k_0^{10}+276154k_0^8+421131k_0^6+347855k_0^4+146804k_0^2+24660\right)}{\left[\left(3k_0^2+4\right)\left(3k_0^2+2\right)\left(3k_0^2+5\right)\left(4k_0^2+5\right)\left(k_0^2+2\right)\left(2k_0^2+3\right)\left(4k_0^2+3\right)\left(2k_0^2+5\right)\left(2k_0^2+1\right)\left(k_0^2+1\right)^3\right]}$$

Thus finally adding the six terms we have (2.4.2.2I).

$$P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_2,X_4)<Med(Y_1,Y_4,Y_5)]$$

$$=\frac{2}{3}\frac{\left(\begin{array}{l}24432k_0^{24}+458824k_0^{22}+3851980k_0^{20}+19074430k_0^{18}+61898440k_0^{16}+138291989k_0^{14}\\+217375685k_0^{12}+241172935k_0^{10}+86413943k_0^8+97190922k_0^6+32117100k_0^4+5946120k_0^2+453600\end{array}\right)}{\left[\left(4k_0^2+3\right)\left(k_0^2+3\right)\left(3k_0^2+5\right)\left(3k_0^2+2\right)\left(2k_0^2+3\right)\left(4k_0^2+4\right)\left(k_0^2+1\right)^3\left(2k_0^2+1\right)\left(k_0^2+2\right)^3\left(4k_0^2+5\right)\left(2k_0^2+7\right)\right]}$$

Thus finally we have

$$\xi^*_{2,1}=P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_2,X_4)<Med(Y_1,Y_4,Y_5)]-[E(U)]^2$$

$$=\left[\frac{2}{3}\frac{\left(\begin{array}{l}24432k_0^{24}+458824k_0^{22}+3851980k_0^{20}+19074430k_0^{18}+61898440k_0^{16}+138291989k_0^{14}\\+217375685k_0^{12}+241172935k_0^{10}+86413943k_0^8+97190922k_0^6+32117100k_0^4+5946120k_0^2+453600\end{array}\right)}{\left[\left(4k_0^2+3\right)\left(k_0^2+3\right)\left(3k_0^2+5\right)\left(3k_0^2+2\right)\left(2k_0^2+3\right)\left(4k_0^2+4\right)\left(k_0^2+1\right)^3\left(2k_0^2+1\right)\left(k_0^2+2\right)^3\left(4k_0^2+5\right)\left(2k_0^2+7\right)\right]}\right]-\left[\frac{\left(19k_0^2+6\right)}{\left(k_0^2+1\right)\left(3k_0^2+2\right)\left(2k_0^2+3\right)}\right]^2$$

$$=\frac{1}{3}k_0^2\frac{\left(\begin{array}{l}413958312k_0^6+1130835144k_0^8+2112561233k_0^{10}+2737868719k_0^{12}+2496254551k_0^{14}\\+1610166389k_0^{16}+729979784k_0^{18}+227249660k_0^{20}+46200032k_0^{22}+18172944k_0^2\\+103933776k_0^4+18489560+5517312k_0^{24}+293184k_0^{26}\end{array}\right)}{\left[\left(4k_0^2+3\right)\left(k_0^2+3\right)\left(3k_0^2+5\right)\left(3k_0^2+2\right)^2\left(2k_0^2+3\right)^2\left(3k_0^2+4\right)\left(k_0^2+1\right)^3\left(2k_0^2+1\right)\left(k_0^2+2\right)^3\left(4k_0^2+5\right)\left(2k_0^2+7\right)\right]}.$$

Now consider

$$\xi^*_{1,3}=Cov[h(X_1,X_2,X_3;Y_1,Y_2,Y_3),h(X_1,X_4,X_5;Y_1,Y_2,Y_3)]$$

$$=E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)h(X_1,X_4,X_5;Y_1,Y_2,Y_3)]-\{E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)]\}^2$$

$$=P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_4,X_5)<Med(Y_1,Y_2,Y_3)]-[E(U)]^2$$

Now

$$P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_4,X_5)<Med(Y_1,Y_2,Y_3)]$$

$$= \iint_{-\infty<x<y<\infty} P^2[Med(x,X_2,X_3)<y]\,G(y)\overline{G}(y)\,dG(y)\,dF(x)$$

$$= \iint_{-\infty<x<y<\infty} P^2[Med(x,X_2,X_3)<y]\,G(y)\overline{G}(y)\,dG(y)\,dF(x) +$$

$$\iint_{-\infty<y<x<\infty} P^2[Med(x,X_2,X_3)<y]\,G(y)\overline{G}(y)\,dG(y)\,dF(x) \qquad \text{(2.4.2.2k)}$$

Now consider the first term of (2.4.2.2k)

$$\iint_{-\infty<x<y<\infty} P^2[Med(x,X_2,X_3)<y]\,G(y)\overline{G}(y)\,dG(y)\,dF(x)$$

We have

$$P[Med(x,X_2,X_3)<y] = 2L_1 + 2L_2 + \ldots + 2L_5$$

where

$$L_1 = P[X_2 < x < X_3 < y]$$

$$= P[X_2 < x]P[x < X_3 < y]$$

$$= [F^{k_0}(x)][F^{k_0}(y) - F^{k_0}(x)]$$

$$= [u^{k_0}][v^{k_0} - u^{k_0}]$$

$$L_2 = P[X_2 < x < y < X_3]$$

$$= P[X_2 < x]P[y < X_3]$$

$$= [F^{k_0}(x)][1 - F^{k_0}(y)]$$

$$= [u^{k_0}][1 - v^{k_0}]$$

$$L_3 = P[x < X_2 < X_3 < y]$$

$$= \frac{[F^{k_0}(y) - F^{k_0}(x)]^2}{2}$$

$$= \frac{\left[v^{k_0} - u^{k_0}\right]^2}{2}$$

$$L_4 = P[x < X_2 < y < X_3]$$

$$= P[x < X_2 < y]P[y < X_3]$$

$$= \left[F^{k_0}(y) - F^{k_0}(x)\right]\left[1 - F^{k_0}(y)\right]$$

$$= \left[v^{k_0} - u^{k_0}\right]\left[1 - v^{k_0}\right]$$

$$L_5 = P[X_3 < X_2 < x < y]$$

$$= P[X_3 < X_2 < x]]$$

$$= \frac{F^{2k_0}(x)}{2}$$

$$= \frac{u^{2k_0}}{2}$$

where $x = u$, $y = v$.

Now adding all the 5 terms we have

$$P[Med(x, X_2, X_3) < y] = 2L_1 + 2L_2 + \ldots + 2L_5$$

$$= \left(2v^{k_0} - v^{2k_0}\right).$$

Thus integrating we have

$$\int_{-\infty}^{\infty}\int_{x}^{\infty} P^2[Med(x, X_2, X_3) < y]6G(y)\overline{G}(y)dG(y)dF(x)$$

$$= \int_{-\infty}^{\infty}\int_{x}^{\infty}\left(\left(2v^{k_0} - v^{2k_0}\right)\right)^2 6G(y)\overline{G}(y)dG(y)dF(x)$$

$$= \int_{0}^{1}\int_{u}^{1}\left(\left(2v^{k_0} - v^{2k_0}\right)\right)^2 6v^{\frac{1}{k_0}}\left(1 - v^{\frac{1}{k_0}}\right)\frac{1}{k_0}v^{\frac{1}{k_0}-1}k_0u^{k_0-1}dvdu \text{ where } F(x) = u,\ F(y) = v$$

$$= 2\frac{\left(422k_0^{\,8} + 810k_0^{\,6} + 559k_0^{\,4} + 165k_0^{\,2} + 18\right)}{\left[\left(k_0^{\,2} + 1\right)\left(3k_0^{\,2} + 2\right)\left(2k_0^{\,2} + 1\right)\left(5k_0^{\,2} + 2\right)\left(4k_0^{\,2} + 3\right)\left(5k_0^{\,2} + 3\right)\right]}.$$

Now consider the second term of (2.4.2.2k)

$$\iint_{-\infty<y<x<\infty} P^2[Med(x,X_2,X_3)<y]\,6G(y)\overline{G}(y)\,dG(y)\,dF(x)$$

we have

$$P[Med(x,X_2,X_3)<y] = 2P[X_3 < X_2 < y < x]$$

$$= 2P[X_3 < X_2 < y]I_{(y<x)}$$

$$= 2\frac{F^2(y)}{2}$$

$$= F^{2k_0}(y)$$

$$= v^{2k_0} \qquad \text{where } F(y) = v\,.$$

Thus integrating we have,

$$\int_{-\infty}^{\infty}\int_{-\infty}^{x} P^2[Med(x,X_2,X_3)<y]\,6G(y)\overline{G}(y)\,dG(y)\,dF(x)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{x} v^{4k_0}\,6F^{\frac{1}{k_0}}(y)\left(1-F^{\frac{1}{k_0}}(y)\right)dF^{\frac{1}{k_0}}(y)\,dF^{k_0}(x)$$

$$= \int_0^1\int_0^v v^{4k_0}\,6v^{\frac{1}{k_0}}\left(1-v^{\frac{1}{k_0}}\right)\frac{1}{k_0}v^{\frac{1}{k_0}-1}k_0u^{k_0-1}\,dvdu \qquad \text{where } F(y) = v$$

$$= 3k_0^{\,2}\left[\frac{\left(9k_0^{\,2}+5\right)}{\left(2k_0^{\,2}+1\right)\left(5k_0^{\,2}+2\right)\left(4k_0^{\,2}+3\right)\left(5k_0^{\,2}+3\right)}\right].$$

Thus

$$P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_4,X_5)<Med(Y_1,Y_2,Y_3)] =$$

$$= \left[\frac{\left(37k_0^{\,4}+35k_0^{\,2}+6\right)}{\left(k_0^{\,2}+1\right)\left(4k_0^{\,2}+3\right)\left(3k_0^{\,2}+2\right)\left(2k_0^{\,2}+1\right)}\right].$$

Thus, finally we have

$$\xi^*_{1,3} = P[Med(X_1,X_2,X_3) < Med(Y_1,Y_2,Y_3), Med(X_1,X_4,X_5) < Med(Y_1,Y_2,Y_3)] - [E(U)]^2$$

$$= \left[\left[\frac{(37k_0^4 + 35k_0^2 + 6)}{(k_0^2+1)(4k_0^2+3)(3k_0^2+2)(2k_0^2+1)}\right]\right] - \left[\frac{(19k_0^2+6)}{(k_0^2+1)(3k_0^2+2)(2k_0^2+3)}\right]^2$$

Simplifying we get,

$$\xi^*_{1,3} = k_0^6 \left[\frac{(2492k_0^4 + 2659k_0^2 + 780 + 444k_0^6)}{(k_0^2+1)^2(4k_0^2+3)(3k_0^2+2)^2(2k_0^2+1)(2k_0^2+3)^2}\right].$$

Now consider

$$\xi^*_{3,1} = Cov[h(X_1,X_2,X_3;Y_1,Y_2,Y_3), h(X_1,X_2,X_3;Y_1,Y_4,Y_5)]$$

$$= E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)h(X_1,X_2,X_3;Y_1,Y_4,Y_5)] - \{E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)]\}^2$$

$$= P[Med(X_1,X_2,X_3) < Med(Y_1,Y_2,Y_3), Med(X_1,X_2,X_3) < Med(Y_1,Y_4,Y_5)] - [E(U)]^2$$

Now

$$P[Med(X_1,X_2,X_3) < Med(Y_1,Y_2,Y_3), Med(X_1,X_2,X_3) < Med(Y_1,Y_4,Y_5)]$$

$$= \iint_{-\infty<x<y<\infty} P^2[x < Med(y,Y_2,Y_3)]6F(x)\bar{F}(x)dF(x)dG(y)$$

$$= \iint_{-\infty<x<y<\infty} P^2[x < Med(y,Y_2,Y_3)]6F(x)\bar{F}(x)dF(x)dG(y) +$$

$$\iint_{-\infty<y<x<\infty} P^2[x < Med(y,Y_2,Y_3)]6F(x)\bar{F}(x)dF(x)dG(y) \qquad (2.4.2.2l)$$

Now consider the first term of (2.4.2.2l) corresponding to the case $x < y$.

$$\iint_{-\infty<x<y<\infty} P^2[x < Med(y,Y_2,Y_3)]6F(x)\bar{F}(x)dF(x)dG(y).$$

We have

$$P[x < Med(y, Y_2, Y_3)] = 2M_1 + 2M_2 + \ldots + 2M_5$$

where

$$M_1 = P[x < Y_2 < y < Y_3]$$

$$= P[x < Y_2 < y]P[y < Y_3]$$

$$= \left[F^{\frac{1}{k_0}}(y) - F^{\frac{1}{k_0}}(x)\right]\left[1 - F^{\frac{1}{k_0}}(y)\right]$$

$$= \left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right]\left[1 - v^{\frac{1}{k_0}}\right]$$

$$M_2 = P[Y_2 < x < y < Y_3]$$

$$= P[Y_2 < x]P[y < Y_3]$$

$$= \left[F^{\frac{1}{k_0}}(x)\right]\left[1 - F^{\frac{1}{k_0}}(y)\right]$$

$$= \left[u^{\frac{1}{k_0}}\right]\left[1 - v^{\frac{1}{k_0}}\right]$$

$$M_3 = P[x < y < Y_2 < Y_3]$$

$$= P[y < Y_2 < Y_3]]$$

$$= \frac{[\overline{F}^2(y)]}{2}$$

$$= \frac{\left[1 - F^{\frac{1}{k_0}}(y)\right]^2}{2}$$

$$= \frac{\left[1 - v^{\frac{1}{k_0}}\right]^2}{2}$$

$$M_4 = P[x < Y_3 < Y_2 < y]$$

$$= \frac{[G(y) - F(x)]^2}{2}$$

$$= \frac{\left[F^{\frac{1}{k_0}}(y) - F^{\frac{1}{k_0}}(x)\right]^2}{2}$$

$$= \frac{\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right]^2}{2}$$

$$M_5 = P[Y_3 < x < Y_2 < y]$$

$$= P[Y_3 < x]P[x < Y_2 < y]$$

$$= F(x)[G(y) - F(x)]$$

$$= F^{\frac{1}{k_0}}(x)\left[F^{\frac{1}{k_0}}(y) - F^{\frac{1}{k_0}}(x)\right]$$

$$= u^{\frac{1}{k_0}}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right]$$

where $F(x) = u$ and $F(y) = v$.

Now adding all the 5 terms we have

$$P[x < Med(y, Y_2, Y_3)] = 2M_1 + 2M_2 + \ldots + 2M_5$$

$$= \left[1 - u^{\frac{2}{k_0}}\right]$$

Thus integrating we have

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y} P^2[x < Med(y, Y_2, Y_3)]6F(x)\overline{F}(x)dF(x)dG(y)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{y}\left(1 - u^{\frac{2}{k_0}}\right)^2 6F(x)\overline{F}(x)dF(x)dG(y)$$

$$= \int_0^1 \int_0^v \left(u^{\frac{4}{k_0}} - 2u^{\frac{2}{k_0}} + 1 \right) \left(u^{k_0} - u^{2k_0}\right) k_0 u^{k_0 - 1} \frac{1}{k_0} v^{\frac{1}{k_0} - 1} \, du dv$$

$$= 10 \frac{\left(1250k_0^{\,2} + 120 + 4525k_0^{\,4} + 7750k_0^{\,6} + 6725k_0^{\,8} + 2850k_0^{\,10} + 468k_0^{\,12}\right)}{\left[\left(k_0^{\,2} + 2\right)\left(2k_0^{\,2} + 5\right)\left(3k_0^{\,2} + 4\right)\left(3k_0^{\,2} + 5\right)\left(k_0^{\,2} + 1\right)\left(2k_0^{\,2} + 3\right)\left(3k_0^{\,2} + 2\right)\left(2k_0^{\,2} + 1\right)\left(3k_0^{\,2} + 1\right)\right]}.$$

after letting $F(x) = u$ and $F(y) = v$ and simplifying.

Now consider the second term of (2.4.2.2l) corresponding to the case $y < x$.

$$\iint_{-\infty < y < x < \infty} P^2[x < Med(y, Y_2, Y_3)] \, 6F(x)\overline{F}(x) \, dF(x) \, dG(y)$$

We have

$$P[x < Med(y, Y_2, Y_3)] = 2P[y < x < Y_2 < Y_3]$$

$$= 2P[x < Y_2 < Y_3] I_{(y<x)}$$

$$= 2\frac{\overline{F}^2(x)}{2}$$

$$= \overline{F}^2(x)$$

$$= \left[1 - F^{\frac{1}{k_0}}(x)\right]^2$$

$$= \left[1 - F^{\frac{1}{k_0}}(x)\right]^2 .$$

Thus integrating we have

$$\int_{-\infty}^{\infty} \int_{y}^{\infty} P^2[x < Med(y, Y_2, Y_3)] \, 6F(x)\overline{F}(x) \, dF(x) \, dG(y)$$

$$= \int_{-\infty}^{\infty}\int_{y}^{\infty}\left[\left(1-u^{\frac{1}{k_0}}\right)^2\right]^2 6F^{k_0}(x)\left[1-F^{k_0}(x)\right]dF^{k_0}(x)dF^{\frac{1}{k_0}}(y)$$

$$= \int_{0}^{1}\int_{v}^{1}\left[\left(1-2u^{\frac{1}{k_0}}+u^{\frac{2}{k_0}}\right)\right]^2 6u^{k_0}\left[1-u^{k_0}\right]k_0u^{k_0-1}\frac{1}{k_0}v^{\frac{1}{k_0}-1}dudv$$

$$= 12k_0^{\,4}\left[\frac{\left(851k_0^{\,2}+211k_0^{\,6}+274+764k_0^{\,4}\right)}{\left(3k_0^{\,2}+5\right)\left(3k_0^{\,2}+1\right)\left(2k_0^{\,2}+1\right)\left(k_0^{\,2}+1\right)\left(2k_0^{\,2}+3\right)\left(k_0^{\,2}+2\right)\left(2k_0^{\,2}+5\right)\left(3k_0^{\,2}+2\right)\left(3k_0^{\,2}+4\right)}\right]$$

after letting $F(x)=u$ and $F(y)=v$ and Simplifying.

Thus finally adding the two terms we have (2.4.2.2l) i.e.

$$P\left[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_2,X_3)<Med(Y_1,Y_4,Y_5)\right]$$

$$= 2\left[\frac{\left(65k_0^{\,4}+106k_0^{\,2}+24\right)}{\left(3k_0^{\,2}+4\right)\left(3k_0^{\,2}+2\right)\left(k_0^{\,2}+2\right)\left(2k_0^{\,2}+3\right)\left(k_0^{\,2}+1\right)}\right]$$

Thus, finally we have

$$\xi^{*}_{3,1} = P\left[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_2,X_3)<Med(Y_1,Y_4,Y_5)\right]-\left[E(U)\right]^2$$

$$= \left[2\left[\frac{\left(65k_0^{\,4}+106k_0^{\,2}+24\right)}{\left(3k_0^{\,2}+4\right)\left(3k_0^{\,2}+2\right)\left(k_0^{\,2}+2\right)\left(2k_0^{\,2}+3\right)\left(k_0^{\,2}+1\right)}\right]\right]-\left[\frac{\left(19k_0^{\,2}+6\right)}{\left(k_0^{\,2}+1\right)\left(3k_0^{\,2}+2\right)\left(2k_0^{\,2}+3\right)}\right]^2$$

$$= k_0^{\,4}\left[\frac{\left(2659k_0^{\,4}+780k_0^{\,6}+2492k_0^{\,2}+444\right)}{\left(3k_0^{\,2}+4\right)\left(3k_0^{\,2}+2\right)^2\left(k_0^{\,2}+2\right)\left(2k_0^{\,2}+3\right)^2\left(k_0^{\,2}+1\right)^2}\right].$$

Now consider

$$\xi^*_{2,3} = Cov[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_2, X_4; Y_1, Y_2, Y_3)]$$

$$= E[h(X_1, X_2, X_3; Y_1, Y_2, Y_3)h(X_1, X_2, X_4; Y_1, Y_2, Y_3)] - \{E[h(X_1, X_2, X_3; Y_1, Y_2, Y_3)]\}^2$$

$$= P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_2, X_4) < Med(Y_1, Y_2, Y_3)] - [E(U)]^2$$

Now

$$P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_2, X_4) < Med(Y_1, Y_2, Y_3)]$$

$$= \iiint P^2[Med(x_1, x_2, X_3) < y]\,6G(y)\overline{G}(y)\,dG(y)\,dF(x_1)\,dF(x_2)$$

$$= \iiint_{-\infty < x_1 < x_2 < \infty} P^2[Med(x_1, x_2, X_3) < y]\,6G(y)\overline{G}(y)\,dG(y)\,dF(x_1)\,dF(x_2)$$
$$+ \iiint_{-\infty < x_2 < x_1 < \infty} P^2[Med(x_1, x_2, X_3) < y]\,6G(y)\overline{G}(y)\,dG(y)\,dF(x_1)\,dF(x_2) \qquad (2.4.2.2m)$$

Now consider the first term of (2.4.2.2m)

$$\iiint_{-\infty < x_1 < x_2 < \infty} P^2[Med(x_1, x_2, X_3) < y]\,6G(y)\overline{G}(y)\,dG(y)\,dF(x_1)\,dF(x_2)$$

Evaluating $P[Med(x_1, x_2, X_3) < y]$ and substituting, we have

$$= \int_{-\infty}^{\infty}\int_{x_1}^{\infty}\int_{x_2}^{\infty} (1 + w^{k_0})^2\,6F^{\frac{1}{k_0}}(y)\left(1 - F^{\frac{1}{k_0}}(y)\right)dF^{\frac{1}{k_0}}(y)\,dF^{k_0}(x_2)\,dF^{k_0}(x_1)$$

$$= \int_0^1\int_u^1\int_v^1 (1 + w^{k_0})^2\,6w^{\frac{1}{k_0}}\left(1 - w^{\frac{1}{k_0}}\right)\frac{1}{k_0}w^{\frac{1}{k_0}-1}k_0 v^{k_0-1}k_0 u^{k_0-1}\,dw\,dv\,du$$

letting $F(x_1) = u$ $F(x_2) = v$ and $F(y) = w$

$$= \frac{1}{2}\frac{\left(154k_0^{\,6} + 371k_0^{\,4} + 288k_0^{\,2} + 72\right)}{\left[\left(3k_0^{\,2} + 2\right)\left(k_0^{\,2} + 1\right)\left(2k_0^{\,2} + 3\right)\left(2k_0^{\,2} + 1\right)\left(4k_0^{\,2} + 3\right)\right]}.$$

Now consider the second term of (2.4.2.2m)

Interchanging x_1 and x_2 in the above and simplifying we have,

$$\iiint_{-\infty<x_2<x_1<\infty} P^2[Med(x_1,x_2,X_3)<y]\,6G(y)\overline{G}(y)\,dG(y)\,dF(x_1)\,dF(x_2)$$

$$=\frac{1}{2}\left[\frac{\left(154k_0^{\,6}+371k_0^{\,4}+288k_0^{\,2}+72\right)}{\left(3k_0^{\,2}+2\right)\left(k_0^{\,2}+1\right)\left(2k_0^{\,2}+3\right)\left(2k_0^{\,2}+1\right)\left(4k_0^{\,2}+3\right)}\right].$$

Thus finally adding the two terms we have (2.4.2.2m)

$$P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_2,X_4)<Med(Y_1,Y_2,Y_3)]$$

$$=\left[\frac{\left(154k_0^{\,6}+371k_0^{\,4}+288k_0^{\,2}+72\right)}{\left(3k_0^{\,2}+2\right)\left(k_0^{\,2}+1\right)\left(2k_0^{\,2}+3\right)\left(2k_0^{\,2}+1\right)\left(4k_0^{\,2}+3\right)}\right].$$

Thus, finally we have

$$\xi^*_{2,3}=P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_2,X_4)<Med(Y_1,Y_2,Y_3)]-[E(U)]^2$$

$$=\left[\frac{\left(154k_0^{\,6}+371k_0^{\,4}+288k_0^{\,2}+72\right)}{\left(3k_0^{\,2}+2\right)\left(k_0^{\,2}+1\right)\left(2k_0^{\,2}+3\right)\left(2k_0^{\,2}+1\right)\left(4k_0^{\,2}+3\right)}\right]-\left[\frac{\left(19k_0^{\,2}+6\right)}{\left(k_0^{\,2}+1\right)\left(3k_0^{\,2}+2\right)\left(2k_0^{\,2}+3\right)}\right]^2$$

$$=\frac{\left(924k_0^{\,12}+5152k_0^{\,10}+8815k_0^{\,8}+8443k_0^{\,6}+5415k_0^{\,4}+2052k_0^{\,2}+324\right)}{\left(3k_0^{\,2}+2\right)^2\left(k_0^{\,2}+1\right)^2\left(2k_0^{\,2}+3\right)^2\left(2k_0^{\,2}+1\right)\left(4k_0^{\,2}+3\right)}.$$

Now consider

$$\xi^*_{3,2}=Cov[h(X_1,X_2,X_3;Y_1,Y_2,Y_3),h(X_1,X_2,X_3;Y_1,Y_2,Y_4)]$$

$$=E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)h(X_1,X_2,X_3;Y_1,Y_2,Y_4)]-\{E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)]\}^2$$

$$=P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_4)]-[E(U)]^2$$

Now

$$P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_4)]$$

$$= \iiint_{\substack{-\infty<y_1<y_2<\infty \\ -\infty<x<\infty}} P^2[x<Med(y_1,y_2,Y_3)]6F(x)\overline{F}(x)dF(x)dG(y_1)dG(y_2)$$

$$= \iiint_{-\infty<y_1<y_2<\infty} P^2[x<Med(y_1,y_2,Y_3)]6F(x)\overline{F}(x)dF(x)dG(y_1)dG(y_2) + \iiint_{-\infty<y_2<y_1<\infty} P^2[x<Med(y_1,y_2,Y_3)]6F(x)\overline{F}(x)dF(x)dG(y_1)dG(y_2) \quad (2.4.2.2n)$$

Now consider the first term of (2.4.2.2n)

$$\iiint_{-\infty<y_1<y_2<\infty} P^2[x<Med(y_1,y_2,Y_3)]6F(x)\overline{F}(x)dF(x)dG(y_1)dG(y_2)$$

As before evaluating $P[x<Med(y_1,y_2,Y_3)]$ and substituting we have

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_2}\int_{-\infty}^{y_1} P^2[x<Med(y_1,y_2,Y_3)]6F(x)\overline{F}(x)dF(x)dG(y_1)dG(y_2)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{y_2}\int_{-\infty}^{y_1}\left(2-w^{\frac{1}{k_0}}\right)^2 6F^{k_0}(x)(1-F^{k_0}(x))dF^{k_0}(x)dF^{\frac{1}{k_0}}(y_1)dF^{\frac{1}{k_0}}(y_2)$$

$$= \int_0^1\int_0^v\int_0^u\left(4-4w^{\frac{1}{k_0}}+w^{\frac{2}{k_0}}\right)6w^{k_0}(1-w^{k_0})k_0w^{k_0-1}\frac{1}{k_0}u^{\frac{1}{k_0}-1}\frac{1}{k_0}v^{\frac{1}{k_0}-1}\,dwdudv$$

$$= \frac{1}{2}\frac{\left(192-384k_0-2208k_0^3+2192k_0^2+762k_0^{10}-4032k_0^5-648k_0^9-2808k_0^7+3839k_0^8+6346k_0^4+7369k_0^6\right)}{\left[(2k_0^2+1)(k_0^2+1)(3k_0^2+1)(3k_0^2+2)(2k_0^2+3)(k_0^2+2)(3k_0^2+4)\right]}$$

after letting $F(y_1)=u$ $F(y_2)=v$ and $F(x)=w$ and Simplifying.

Now consider the second term of (2.4.2.2n)

Interchanging y_1 and y_2 in the above and simplifying we have,

$$\iiint_{-\infty<x<y_2<y_1<\infty} P^2[x<Med(y_1,y_2,Y_3)]6F(x)\overline{F}(x)dF(x)dG(y_1)dG(y_2)$$

$$= \frac{1}{2}\frac{\left(192-384k_0-2208k_0^3+2192k_0^2+762k_0^{10}-4032k_0^5-648k_0^9-2808k_0^7+3839k_0^8+6346k_0^4+7369k_0^6\right)}{\left[(2k_0^2+1)(k_0^2+1)(3k_0^2+1)(3k_0^2+2)(2k_0^2+3)(k_0^2+2)(3k_0^2+4)\right]}$$

Thus finally adding the two terms we have (2.6.2.2n)

$$P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_4)]$$

$$=\frac{(192-384k_0-2208k_0^3+2192k_0^2+762k_0^{10}-4032k_0^5-648k_0^9-2808k_0^7+3839k_0^8+6346k_0^4+7369k_0^6)}{[(2k_0^2+1)(k_0^2+1)(3k_0^2+1)(3k_0^2+2)(2k_0^2+3)(k_0^2+2)(3k_0^2+4)]}$$

Thus, finally we have

$$\xi^*_{3,2}=P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_4)]-[E(U)]^2$$

$$=\left[\frac{(192-384k_0-2208k_0^3+2192k_0^2+762k_0^{10}-4032k_0^5-648k_0^9-2808k_0^7+3839k_0^8+6346k_0^4+7369k_0^6)}{[(2k_0^2+1)(k_0^2+1)(3k_0^2+1)(3k_0^2+2)(2k_0^2+3)(k_0^2+2)(3k_0^2+4)]}\right]-\left[\frac{(19k_0^2+6)}{(k_0^2+1)(3k_0^2+2)(2k_0^2+3)}\right]^2$$

$$=\frac{\begin{pmatrix}-2304k_0+163810k_0^6+65448k_0^4+242562k_0^8-137712k_0^7-147096k_0^9-73440k_0^5+224421k_0^{10}\\-20544k_0^3+13176k_0^2+864-29160k_0^{13}+37512k_0^{14}+125135k_0^{12}-89856k_0^{11}-3888k_0^{15}+4572k_0^{16}\end{pmatrix}}{[(2k_0^2+1)(k_0^2+1)^2(3k_0^2+1)(3k_0^2+2)^2(2k_0^2+3)^2(k_0^2+2)(3k_0^2+4)]}.$$

Now consider

$$\xi^*_{1,1}=Cov[h(X_1,X_2,X_3;Y_1,Y_2,Y_3),h(X_1,X_4,X_5;Y_1,Y_4,Y_5)]$$

$$=E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)h(X_1,X_4,X_5;Y_1,Y_4,Y_5)-\{E[h(X_1,X_2,X_3;Y_1,Y_2,Y_3)]\}^2]$$

$$=P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_4,X_5)<Med(Y_1,Y_4,Y_5)]-[E(U)]^2$$

Now,

$$P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_4,X_5)<Med(Y_1,Y_4,Y_5)]$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}P^2[Med(x,X_2,X_3)<Med(y,Y_2,Y_3)]dF(x)dG(y)$$

$$=N_1+N_2$$

where

$$N_1=\int_{-\infty}^{\infty}\int_{-\infty}^{y}P^2[Med(x,X_2,X_3)<Med(y,Y_2,Y_3)]dF(x)dG(y)$$

$$-\infty<x<y<\infty$$

$$N_2=\int_{-\infty}^{\infty}\int_{y}^{\infty}P^2[Med(x,X_2,X_3)<Med(y,Y_2,Y_3)]dF(x)dG(y).$$

$$-\infty<y<x<\infty$$

Now for x<y

$$P[Med(x,X_2,X_3)< Med(y,Y_2,Y_3)]= 4O_1 + 4O_2 + ... + 4O_{62}$$

where

$O_1 = P[x < y < X_2 < X_3 < Y_2 < Y_3]$ $O_2 = P[x < y < X_2 < Y_2 < X_3 < Y_3]$

$O_3 = P[x < y < X_2 < Y_2 < Y_3 < Y_3]$ $O_4 = P[x < X_2 < y < X_3 < Y_2 < Y_3]$

$O_5 = P[x < X_2 < y < Y_2 < X_3 < Y_3]$ $O_6 = P[x < X_2 < y < Y_2 < Y_3 < X_3]$

$O_7 = P[x < X_2 < X_3 < y < Y_2 < Y_3]$ $O_8 = P[x < X_2 < Y_2 < y < X_3 < Y_3]$

$O_9 = P[x < X_2 < Y_2 < y < Y_3 < X_3]$ $O_{10} = P[x < Y_2 < X_2 < y < X_3 < Y_3]$

$O_{11} = P[x < Y_2 < X_2 < y < Y_3 < X_3]$ $O_{12} = P[x < X_2 < X_3 < Y_2 < y < Y_3]$

$O_{13} = P[x < X_2 < Y_2 < X_3 < y < Y_3]$ $O_{14} = P[x < X_2 < Y_2 < Y_3 < y < X_3]$

$O_{15} = P[x < Y_2 < X_2 < X_3 < y < Y_3]$ $O_{16} = P[x < Y_2 < X_2 < Y_3 < y < X_3]$

$O_{17} = P[x < X_2 < X_3 < Y_2 < Y_3 < y]$ $O_{18} = P[x < X_2 < Y_2 < X_3 < Y_3 < y]$

$O_{19} = P[x < X_2 < Y_2 < Y_3 < X_3 < y]$ $O_{20} = P[x < Y_2 < X_2 < X_3 < Y_3 < y]$

$O_{21} = P[x < Y_2 < X_2 < Y_3 < X_3 < y]$ $O_{22} = PX_2 < x < y < X_3 < Y_2 < Y_3]$

$O_{23} = P[X_2 < x < y < Y_2 < X_3 < Y_3]$ $O_{24} = P[X_2 < x < y < Y_2 < Y_3 < X_3]$

$O_{25} = P[X_2 < x < X_3 < y < Y_2 < Y_3]$ $O_{26} = P[X_2 < x < Y_2 < y < X_3 < Y_3]$

$O_{27} = P[X_2 < x < Y_2 < y < Y_3 < X_3]$ $O_{28} = P[Y_2 < x < X_2 < y < X_3 < Y_3]$

$O_{29} = PY_2 < x < X_2 < y < Y_3 < X_3]$ $O_{30} = P[X_2 < x < X_3 < Y_2 < y < Y_3]$

$O_{31} = P[X_2 < x < Y_2 < X_3 < y < Y_3]$ $O_{32} = P[X_2 < x < Y_2 < Y_3 < y < X_3]$

$O_{33} = P[Y_2 < x < X_2 < X_3 < y < Y_3]$ $O_{34} = P[Y_2 < x < X_2 < Y_3 < y < X_3]$

$O_{35} = P[X_2 < x < X_3 < Y_2 < Y_3 < y]$ $O_{36} = P[X_2 < x < Y_2 < X_3 < Y_3 < y]$

$O_{37} = P[X_2 < x < Y_2 < Y_3 < X_3 < y]$ $O_{38} = P[Y_2 < x < X_2 < X_3 < Y_3 < y]$

$O_{39} = P[Y_2 < x < X_2 < Y_3 < X_3 < y]$ $O_{40} = P[X_2 < X_3 < x < y < Y_2 < Y_3]$

$O_{41} = P[X_2 < Y_2 < x < y < X_3 < Y_3]$ $O_{42} = P[X_2 < Y_2 < x < y < Y_3 < X_3]$

$O_{43} = P[Y_2 < X_2 < x < y < X_3 < Y_3]$ $O_{44} = P[Y_2 < X_2 < x < y < Y_3 < X_3]$

$O_{45} = P[X_2 < X_3 < x < Y_2 < y < Y_3]$ $O_{46} = P[X_2 < Y_2 < x < X_3 < y < Y_3]$

$O_{47} = P[X_2 < Y_2 < x < Y_3 < y < X_3]$ $O_{48} = P[Y_2 < X_2 < x < X_3 < y < Y_3]$

$O_{49} = P[Y_2 < X_2 < x < Y_3 < y < X_3]$ $O_{50} = P[X_2 < X_3 < x < Y_2 < Y_3 < y]$

$O_{51} = P[X_2 < Y_2 < x < X_3 < Y_3 < y]$ $O_{52} = P[X_2 < Y_2 < x < Y_3 < X_3 < y]$

$O_{53} = P[Y_2 < X_2 < x < X_3 < Y_3 < y]$ $O_{54} = P[Y_2 < X_2 < x < Y_3 < X_3 < y]$

$O_{55} = P[X_2 < X_3 < Y_2 < x < y < Y_3]$ $O_{56} = P[X_2 < Y_2 < X_3 < x < y < Y_3]$

$O_{57} = P[Y_2 < X_2 < X_3 < x < y < Y_3]$ $O_{58} = P[X_2 < X_3 < Y_2 < x < Y_3 < y]$

$O_{59} = P[X_2 < Y_2 < X_3 < x < Y_3 < y]$ $O_{60} = P[X_2 < X_3 < Y_2 < Y_3 < x < y]$

$O_{61} = P[X_2 < Y_2 < X_3 < Y_3 < x < y]$ $O_{62} = P[Y_2 < X_2 < X_3 < Y_3 < x < y]$

Let $F(x) = u$ $F(x_2) = m$ $F(x_3) = n$ $F(y) = v$ $F(y_2) = r$ $F(y_3) = s$ and consider

$$O_1 = P[x < y < X_2 < X_3 < Y_2 < Y_3]$$

$$= P[y < X_2 < X_3 < Y_2 < Y_3]$$

$$= \int_y^\infty P[x_2 < X_3 < Y_2 < Y_3]dF(x_2)$$

$$= \int_y^\infty \int_{x_2}^\infty P[x_3 < Y_2 < Y_3]dF(x_3)dF(x_2)$$

$$= \int_y^\infty \int_{x_2}^\infty \int_{x_3}^\infty P[y_2 < Y_3]dG(y_2)dF(x_3)dF(x_2)$$

$$= \int_v^1 \int_m^1 \int_n^1 \left[1 - F^{\frac{1}{k_0}}(y_2)\right] dF^{\frac{1}{k_0}}(y_2) dF^{\frac{1}{k_0}}(x_3) dF^{\frac{1}{k_0}}(x_2)$$

$$= \int_v^1 \int_m^1 \int_n^1 \left[1 - r^{\frac{1}{k_0}}\right] \frac{1}{k_0} r^{\frac{1}{k_0}-1} k_0 n^{k_0-1} k_0 m^{k_0-1} dr dn dm$$

$$= \frac{2\left(1-v^{k_0}\right)}{2\left(k_0^2+1\right)\left(k_0^2+2\right)} - \frac{\left(1-v^{2k_0}\right)}{4} + \frac{k_0^4\left(1-v^{\frac{1}{k_0}+2k_0}\right)}{\left(k_0^2+1\right)\left(2k_0^2+1\right)} - \frac{k_0^4\left(1-v^{\frac{2}{k_0}+2k_0}\right)}{2\left(k_0^2+2\right)\left(2k_0^2+2\right)}$$

$$O_2 = P[x < y < X_2 < Y_2 < X_3 < Y_3]$$

$$= P[y < X_2 < Y_2 < X_3 < Y_3]$$

$$= \int_y^\infty \int_{x_2}^\infty \int_{y_2}^\infty P[x_3 < Y_3] dF(x_3) dG(y_2) dF(x_2)$$

$$= \int_y^\infty \int_{x_2}^\infty \int_{y_2}^\infty \left[1 - F^{\frac{1}{k_0}}(x_3)\right] dF^{k_0}(x_3) dF^{\frac{1}{k_0}}(y_2) dF^{k_0}(x_2)$$

$$= \int_v^1 \int_m^1 \int_r^1 \left[1 - n^{\frac{1}{k_0}}\right] k_0 n^{k_0-1} \frac{1}{k_0} r^{\frac{1}{k_0}-1} k_0 m^{k_0-1} dn dr dm$$

$$= \frac{k_0^3\left(1-v^{k_0}\right)}{\left(k_0^2+1\right)\left(k_0^2+2\right)} - \frac{k_0^2\left(1-v^{\frac{1}{k_0}+k_0}\right)}{\left(k_0^2+1\right)^2} + \frac{k_0^2\left(1-v^{\frac{1}{k_0}+2k_0}\right)}{\left(k_0^2+1\right)\left(2k_0^2+1\right)} - \frac{k_0^4\left(1-v^{\frac{2}{k_0}+2k_0}\right)}{\left(k_0^2+1\right)\left(k_0^2+2\right)\left(2k_0^2+2\right)}$$

$$O_3 = P[x < y < X_2 < Y_2 < Y_3 < Y_3]$$

$$= P[y < X_2 < Y_2 < Y_3 < X_3]$$

$$= \int_y^\infty \int_{x_2}^\infty \int_{y_2}^\infty P[y_3 < X_3] dG(y_3) dG(y_2) dF(x_2)$$

$$= \int_y^\infty \int_{x_2}^\infty \int_{y_2}^\infty \left[1 - F^{k_0}(y_3)\right] dF^{\frac{1}{k_0}}(y_3) dF^{\frac{1}{k_0}}(y_2) dF^{k_0}(x_2)$$

$$= \int_v^1 \int_m^1 \int_r^1 \left[1 - s^{k_0}\right] \frac{1}{k_0} s^{\frac{1}{k_0}-1} \frac{1}{k_0} r^{\frac{1}{k_0}-1} k_0 m^{k_0-1} ds dr dm$$

$$= \frac{k_0^{\,2}\left(1-v^{k_0}\right)}{2\left(k_0^{\,2}+2\right)} - \frac{k_0^{\,4}\left(1-v^{\frac{1}{k_0}+k_0}\right)}{\left(k_0^{\,2}+1\right)^2} + \frac{k_0^{\,2}\left(1-v^{\frac{2}{k_0}+k_0}\right)}{2\left(k_0^{\,2}+2\right)} - \frac{k_0^{\,2}\left(1-v^{\frac{2}{k_0}+2k_0}\right)}{\left(k_0^{\,2}+1\right)\left(k_0^{\,2}+2\right)\left(2k_0^{\,2}+2\right)}$$

$$O_4 = P[x < X_2 < y < X_3 < Y_2 < Y_3]$$

$$= P[x < X_2 < y]P[y < X_3 < Y_2 < Y_3]$$

$$= [F(y)-F(x)]\int_y^{\infty}\int_{x_3}^{\infty}[1-G(y_2)]dG(y_2)dF(x_3)$$

$$= \left[v^{k_0}-u^{k_0}\right]\int_v^1\int_n^1\left[1-r^{\frac{1}{k_0}}\right]\frac{1}{k_0}r^{\frac{1}{k_0}-1}k_0n^{k_0-1}drdn$$

$$= \left[v^{k_0}-u^{k_0}\right]\left[\frac{\left(1-v^{\frac{1}{k_0}}\right)}{\left(k_0^{\,2}+1\right)} - \frac{\left(1-v^{\frac{1}{k_0}+k_0}\right)}{\left(k_0^{\,2}+1\right)} + \frac{k_0^{\,2}\left(1-v^{\frac{2}{k_0}+k_0}\right)}{2\left(k_0^{\,2}+2\right)}\right]$$

$$O_5 = P[x < X_2 < y < Y_2 < X_3 < Y_3]$$

$$= P[x < X_2 < y]P[y < Y_2 < X_3 < Y_3]$$

$$= [F(y)-F(x)]\int_y^{\infty}\int_{y_2}^{\infty}\left[1-F^{\frac{1}{k_0}}(x_3)\right]dF^{k_0}(x_3)dF^{\frac{1}{k_0}}(y_2)$$

$$= \left[v^{k_0}-u^{k_0}\right]\int_v^1\int_r^1\left[1-n^{\frac{1}{k_0}}\right]k_0n^{k_0-1}\frac{1}{k_0}r^{\frac{1}{k_0}-1}dndr$$

$$= \left[v^{k_0}-u^{k_0}\right]\left[\frac{\left(1-v^{\frac{1}{k_0}}\right)}{\left(k_0^{\,2}+1\right)} - \frac{\left(1-v^{\frac{1}{k_0}+k_0}\right)}{\left(k_0^{\,2}+1\right)} + \frac{k_0^{\,2}\left(1-v^{\frac{2}{k_0}+k_0}\right)}{\left(k_0^{\,2}+1\right)\left(k_0^{\,2}+2\right)}\right]$$

$$O_6 = P[x < X_2 < y < Y_2 < Y_3 < X_3]$$

$$= P[x < X_2 < y]P[y < Y_2 < Y_3 < X_3]$$

$$= [F(y) - F(x)] \int_y^{\infty} \int_{y_2}^{\infty} \left[1 - F^{k_0}(y_3)\right] dF^{\frac{1}{k_0}}(y_3) dF^{\frac{1}{k_0}}(y_2)$$

$$= \left[v^{k_0} - u^{k_0}\right] \int_v^1 \int_r^1 \left[1 - s^{k_0}\right] \frac{1}{k_0} s^{\frac{1}{k_0}-1} \frac{1}{k_0} r^{\frac{1}{k_0}-1} ds dr$$

$$= \left[v^{k_0} - u^{k_0}\right] \left[\frac{k_0^2 \left(1 - v^{\frac{1}{k_0}}\right)}{\left(k_0^2 + 1\right)} - \frac{1 \left(1 - v^{\frac{2}{k_0}}\right)}{2} + \frac{\left(1 - v^{\frac{2}{k_0}+k_0}\right)}{\left(k_0^2 + 1\right)\left(k_0^2 + 2\right)} \right]$$

$$O_7 = P[x < X_2 < X_3 < y < Y_2 < Y_3]$$

$$= P[x < X_2 < X_3 < y] P[y < Y_2 < Y_3]$$

$$= \int_x^y P(x_2 < X_3 < y) dF(x_2) \int_y^{\infty} P(y_2 < Y_3) dG(y_2)$$

$$= \int_x^y \left[F^{k_0}(y) - F^{k_0}(x_2)\right] dF^{k_0}(x_2) \int_y^{\infty} \left[1 - F^{\frac{1}{k_0}}(y_2)\right] dF^{\frac{1}{k_0}}(y_2)$$

$$= \int_u^v \left[v^{k_0} - m^{k_0}\right] k_0 m^{k_0-1} dm \int_v^1 \left[1 - r^{\frac{1}{k_0}}\right] \frac{1}{k_0} r^{\frac{1}{k_0}-1} dr$$

$$= \frac{1}{2} \left[v^{k_0} - u^{k_0}\right]^2 \left[\left(1 - v^{\frac{1}{k_0}}\right) - \frac{1}{2} \left(1 - v^{\frac{2}{k_0}}\right) \right]$$

$$O_8 = P[x < X_2 < Y_2 < y < X_3 < Y_3]$$

$$= P[x < X_2 < Y_2 < y] P[y < X_3 < Y_3]$$

$$= \int_x^y P(x_2 < Y_2 < y) dF(x_2) \int_y^{\infty} P(x_3 < Y_3) dF(x_3)$$

$$= \int_x^y \left[F^{\frac{1}{k_0}}(y) - F^{\frac{1}{k_0}}(x_2)\right] dF^{k_0}(x_2) \int_y^{\infty} \left[1 - F^{\frac{1}{k_0}}(x_3)\right] dF^{k_0}(x_3)$$

$$= \int_u^v \left[v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right] k_0 m^{k_0-1} dm \int_v^1 \left[1 - n^{\frac{1}{k_0}}\right] k_0 n^{k_0-1} dn$$

$$= \frac{1}{\left(k_0^{\,2}+1\right)}\left[v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right] - u^{k_0}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right]\left[\left(1-v^{k_0}\right) - \frac{k_0^{\,2}\left[1-v^{\frac{1}{k_0}+k_0}\right]}{\left(k_0^{\,2}+1\right)}\right]$$

$$O_9 = P[x < X_2 < Y_2 < y < Y_3 < X_3]$$

$$= P[x < X_2 < Y_2 < y]P[y < Y_3 < X_3]$$

$$= \int_x^y P(x_2 < Y_2 < y)dF(x_2)\int_y^\infty P(y_3 < X_3)dF(y_3)$$

$$= \int_x^y \left[F^{\frac{1}{k_0}}(y) - F^{\frac{1}{k_0}}(x_2)\right]dF^{k_0}(x_2)\int_y^\infty \left[1-F^{k_0}(y_3)\right]dF^{\frac{1}{k_0}}(y_3)$$

$$= \int_u^v \left[v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right]k_0 m^{k_0-1}dm\int_v^1 \left[1-s^{k_0}\right]\frac{1}{k_0}s^{\frac{1}{k_0}-1}ds$$

$$= \frac{1}{\left(k_0^{\,2}+1\right)}\left[v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right] - u^{k_0}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right]\left[\left(1-v^{\frac{1}{k_0}}\right) - \frac{\left[1-v^{\frac{1}{k_0}+k_0}\right]}{\left(k_0^{\,2}+1\right)}\right]$$

$$O_{10} = P[x < Y_2 < X_2 < y < X_3 < Y_3]$$

$$= P[x < Y_2 < X_2 < y]P[y < X_3 < Y_3]$$

$$= \int_x^y P(y_2 < X_2 < y)dG(y_2)\int_y^\infty P(x_3 < Y_3)dF(x_3)$$

$$= \int_x^y \left[F^{k_0}(y) - F^{k_0}(y_2)\right]dF^{\frac{1}{k_0}}(y_2)\int_y^\infty \left[1-F^{\frac{1}{k_0}}(x_3)\right]dF^{k_0}(x_3)$$

$$= \int_u^v \left[v^{k_0} - r^{k_0}\right]\frac{1}{k_0}r^{\frac{1}{k_0}-1}dr\int_v^1 \left[1-n^{\frac{1}{k_0}}\right]k_0 n^{k_0-1}dn$$

$$= \frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)}\left[v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right] - u^{\frac{1}{k_0}}\left[v^{k_0} - u^{k_0}\right]\left[\left(1-v^{k_0}\right) - \frac{k_0^{\,2}\left[1-v^{\frac{1}{k_0}+k_0}\right]}{\left(k_0^{\,2}+1\right)}\right]$$

$$O_{11} = P[x < Y_2 < X_2 < y < Y_3 < X_3]$$

$$= P[x < Y_2 < X_2 < y]P[y < Y_3 < X_3]$$

$$= \frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)}\left[v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right] - u^{\frac{1}{k_0}}\left[v^{k_0} - u^{k_0}\right]\left[\left(1-v^{\frac{1}{k_0}}\right) - \frac{\left[1-v^{\frac{1}{k_0}+k_0}\right]}{\left(k_0^{\,2}+1\right)}\right]$$

$$O_{12} = P[x < X_2 < X_3 < Y_2 < y < Y_3]$$

$$= P[x < X_2 < X_3 < Y_2 < y]P[y < Y_3]$$

$$= \int_x^y \int_x^{y_2} \int_x^{x_3} dF(x_2)dF(x_3)dG(y_2)(\overline{G}(y))$$

$$= \left[\int_u^v \int_u^r \int_u^n k_0 m^{k_0-1} k_0 n^{k_0-1} \frac{1}{k_0} r^{\frac{1}{k_0}-1} dmdndr\right]\left[1-v^{\frac{1}{k_0}}\right]$$

$$= \frac{1}{2\left(2k_0^{\,2}+1\right)}\left[v^{\frac{1}{k_0}+2k_0} - u^{\frac{1}{k_0}+2k_0}\right] + \frac{u^{2k_0}}{2}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right] - \frac{u^{k_0}}{\left(k_0^{\,2}+1\right)}\left[v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right]\left[1-v^{\frac{1}{k_0}}\right]$$

$$O_{13} = P[x < X_2 < Y_2 < X_3 < y < Y_3]$$

$$= P[x < X_2 < Y_2 < X_3 < y]P[y < Y_3]$$

$$= \int_x^y \int_x^{x_3} \int_x^{y_2} dF(x_2)dG(y_2)dF(x_3)(\overline{G}(y))$$

$$= \left[\int_u^v \int_u^n \int_u^r k_0 m^{k_0-1} \frac{1}{k_0} r^{\frac{1}{k_0}-1} k_0 n^{k_0-1} dmdrdn\right]\left[1-v^{\frac{1}{k_0}}\right]$$

$$= \frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)\left(2k_0^{\,2}+1\right)}\left[v^{\frac{1}{k_0}+2k_0} - u^{\frac{1}{k_0}+2k_0}\right] + \frac{k_0^{\,2}u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}\left[v^{k_0} - u^{k_0}\right] - \frac{k_0^{\,2}u^{k_0}}{\left(k_0^{\,2}+1\right)}\left[v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right]\left[1-v^{\frac{1}{k_0}}\right]$$

$$O_{14} = P[x < X_2 < Y_2 < Y_3 < y < X_3]$$

$$= P[x < X_2 < Y_2 < Y_3 < y]P[y < X_3]$$

$$= \int_x^y \int_x^{y_3} \int_x^{y_2} dF(x_2)dG(y_2)dG(y_3)\left[1 - F^{k_0}(y)\right]$$

$$= \left[\int_u^v \int_u^s \int_u^r k_0 m^{k_0-1} \frac{1}{k_0} r^{\frac{1}{k_0}-1} k_0 n^{k_0-1} dmdrdn\right]\left[1 - v^{\frac{1}{k_0}}\right]$$

$$= \left[\frac{1}{(k_0^2+1)(k_0^2+2)}\left[v^{\frac{2}{k_0}+k_0} - u^{\frac{2}{k_0}+k_0}\right] - \frac{u^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right] - \frac{u^{k_0}}{2}\left[v^{\frac{2}{k_0}} - u^{\frac{2}{k_0}}\right] + u^{\frac{1}{k_0}+k_0}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right]\right]\left[1 - v^{k_0}\right]$$

$$O_{15} = P[x < Y_2 < X_2 < X_3 < y < Y_3]$$

$$= P[x < Y_2 < X_2 < X_3 < y]P[y < Y_3]$$

$$= \int_x^y \int_x^{x_3} \int_x^{x_2} dG(y_2)dF(x_2)dF(x_3)\left[1 - F^{\frac{1}{k_0}}(y)\right]$$

$$= \left[\int_u^v \int_u^n \int_u^m \frac{1}{k_0} r^{\frac{1}{k_0}-1} k_0 m^{k_0-1} k_0 n^{k_0-1} drdmdn\right]\left[1 - v^{\frac{1}{k_0}}\right]$$

$$= \left[\frac{k_0^4}{(k_0^2+1)(2k_0^2+1)}\left[v^{\frac{1}{k_0}+2k_0} - u^{\frac{1}{k_0}+2k_0}\right] - \frac{k_0^2 u^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}\left[v^{k_0} - u^{k_0}\right] - \frac{u^{\frac{1}{k_0}}}{2}\left[v^{2k_0} - u^{2k_0}\right] + u^{\frac{1}{k_0}+k_0}\left[v^{k_0} - u^{k_0}\right]\right]\left[1 - v^{\frac{1}{k_0}}\right]$$

$$O_{16} = P[x < Y_2 < X_2 < Y_3 < y < X_3]$$

$$= P[x < Y_2 < X_2 < Y_3 < y]P[y < X_3]$$

$$= \int_x^y \int_x^{y_3} \int_x^{x_2} dG(y_2)dF(x_2)dG(y_3)\left[1 - F^{k_0}(y)\right]$$

$$= \left[\int_u^v \int_u^s \int_u^m \frac{1}{k_0} r^{\frac{1}{k_0}-1} k_0 m^{k_0-1} \frac{1}{k_0} s^{\frac{1}{k_0}-1} drdmds\right]\left[1 - v^{k_0}\right]$$

$$= \left[\frac{k_0^2}{(k_0^2+1)(k_0^2+2)}\left[v^{\frac{2}{k_0}+k_0} - u^{\frac{2}{k_0}+k_0}\right] - \frac{k_0^2 u^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right] - \frac{u^{\frac{1}{k_0}}}{(k_0^2+1)}\left[v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right] + u^{\frac{1}{k_0}+k_0}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right]\right]\left[1 - v^{k_0}\right]$$

$$O_{17} = P[x < X_2 < X_3 < Y_2 < Y_3 < y]$$

$$= \int_x^y \int_x^{y_3} \int_x^{y_2} \int_x^{x_3} dF(x_2)dF(x_3)dG(y_2)dG(y_3)$$

$$= \int_u^v \int_u^s \int_u^r \int_u^n k_0 m^{k_0-1} k_0 n^{k_0-1} \frac{1}{k_0} r^{\frac{1}{k_0}-1} \frac{1}{k_0} s^{\frac{1}{k_0}-1} dmdndrds$$

$$= \begin{bmatrix} \frac{1}{2(2k_0^2+1)(2k_0^2+2)}\left[v^{\frac{2}{k_0}+2k_0} - u^{\frac{2}{k_0}+2k_0}\right] - \frac{u^{\frac{1}{k_0}+2k_0}}{2(2k_0^2+1)}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right] - \frac{u^{2k_0}}{4}\left[v^{\frac{2}{k_0}} - u^{\frac{2}{k_0}}\right] + \frac{u^{\frac{1}{k_0}+2k_0}}{2}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right] \\ -\frac{u^{k_0}}{(k_0^2+1)(k_0^2+2)}\left[v^{\frac{2}{k_0}+k_0} - u^{\frac{2}{k_0}+k_0}\right] + \frac{u^{\frac{1}{k_0}+2k_0}}{(k_0^2+1)}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right] + \frac{u^{2k_0}}{2}\left[v^{\frac{2}{k_0}} - u^{\frac{2}{k_0}}\right] - u^{\frac{1}{k_0}+2k_0}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right] \end{bmatrix} \left[1 - v^{k_0}\right]$$

$$O_{18} = P[x < X_2 < Y_2 < X_3 < Y_3 < y]$$

$$= \int_x^y \int_x^{y_3} \int_x^{x_3} \int_x^{y_2} dF(x_2)dG(y_2)dF(x_3)dG(y_3)$$

$$= \int_u^v \int_u^s \int_u^n \int_u^r k_0 m^{k_0-1} \frac{1}{k_0} r^{\frac{1}{k_0}-1} k_0 n^{k_0-1} \frac{1}{k_0} s^{\frac{1}{k_0}-1} dmdrdnds$$

$$= \begin{bmatrix} \frac{k_0^2}{(k_0^2+1)(2k_0^2+1)(2k_0^2+2)}\left[v^{\frac{2}{k_0}+2k_0} - u^{\frac{2}{k_0}+2k_0}\right] - \frac{k_0^2 u^{\frac{1}{k_0}+2k_0}}{(k_0^2+1)(2k_0^2+1)}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right] - \frac{u^{\frac{1}{k_0}+k_0}}{(k_0^2+1)^2}\left[v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right] + \frac{u^{\frac{1}{k_0}+2k_0}}{(k_0^2+1)}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right] \\ -\frac{k_0^2}{(k_0^2+1)(k_0^2+2)}\left[v^{\frac{2}{k_0}+k_0} - u^{\frac{2}{k_0}+k_0}\right] + \frac{k_0^2 u^{\frac{1}{k_0}+2k_0}}{(k_0^2+1)}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right] + \frac{u^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}\left[v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right] - u^{\frac{1}{k_0}+2k_0}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right] \end{bmatrix}$$

$$O_{19} = P[x < X_2 < Y_2 < Y_3 < X_3 < y]$$

$$= \int_x^y \int_x^{y_3} \int_x^{x_3} \int_x^{y_2} dF(x_2)dG(y_2)dF(x_3)dG(y_3)$$

$$= \int_u^v \int_u^n \int_u^s \int_u^r k_0 m^{k_0-1} \frac{1}{k_0} r^{\frac{1}{k_0}-1} \frac{1}{k_0} s^{\frac{1}{k_0}-1} k_0 n^{k_0-1} dmdrdsdn$$

$$= \begin{bmatrix} \frac{k_0^2}{(k_0^2+1)(k_0^2+2)(2k_0^2+2)}\left[v^{\frac{2}{k_0}+2k_0} - u^{\frac{2}{k_0}+2k_0}\right] - \frac{u^{\frac{2}{k_0}+k_0}}{(k_0^2+1)(k_0^2+2)}\left[v^{k_0} - u^{k_0}\right] - \frac{k_0^2 u^{\frac{1}{k_0}+k_0}}{(k_0^2+1)^2}\left[v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right] + \frac{u^{\frac{2}{k_0}+k_0}}{(k_0^2+1)}\left[v^{k_0} - u^{k_0}\right] \\ -\frac{k_0^2 u^{k_0}}{2(k_0^2+2)}\left[v^{\frac{2}{k_0}+k_0} - u^{\frac{2}{k_0}+k_0}\right] + \frac{u^{\frac{2}{k_0}+k_0}}{2}\left[v^{k_0} - u^{k_0}\right] + \frac{k_0^2 u^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}\left[v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right] - u^{\frac{2}{k_0}+k_0}\left[v^{k_0} - u^{k_0}\right] \end{bmatrix}$$

$$O_{20} = P[x < Y_2 < X_2 < X_3 < Y_3 < y]$$

$$= \int_x^y \int_x^{y_3} \int_x^{x_3} \int_x^{x_2} dG(y_2)dF(x_2)dF(x_3)dG(y_3)$$

$$= \int_u^v \int_u^s \int_u^n \int_u^m \frac{1}{k_0} r^{\frac{1}{k_0}-1} k_0 m^{k_0-1} k_0 n^{k_0-1} \frac{1}{k_0} s^{\frac{1}{k_0}-1} drdmdnds$$

$$= \begin{bmatrix} \frac{k_0^4}{(k_0^2+1)(2k_0^2+1)(2k_0^2+2)}\left[v^{\frac{2}{k_0}+2k_0} - u^{\frac{2}{k_0}+2k_0}\right] - \frac{k_0^4 u^{\frac{1}{k_0}+2k_0}}{(k_0^2+1)(2k_0^2+1)}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right] - \frac{k_0^2 u^{\frac{1}{k_0}+k_0}}{(k_0^2+1)^2}\left[v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right] + \frac{k_0^2 u^{\frac{1}{k_0}+2k_0}}{(k_0^2+1)}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right] \\ -\frac{u^{\frac{1}{k_0}}}{2(2k_0^2+1)}\left[v^{\frac{1}{k_0}+2k_0} - u^{\frac{1}{k_0}+2k_0}\right] + \frac{u^{\frac{1}{k_0}+2k_0}}{2}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right] + \frac{u^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}\left[v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right] - u^{\frac{1}{k_0}+2k_0}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right] \end{bmatrix}$$

$$O_{21} = P[x < Y_2 < X_2 < Y_3 < X_3 < y]$$

$$= \int_x^y \int_x^{x_3} \int_x^{y_3} \int_x^{x_2} dG(y_2)dF(x_2)dG(y_3)dF(x_3)$$

$$= \int_u^v \int_u^n \int_u^s \int_u^m \frac{1}{k_0} r^{\frac{1}{k_0}-1} k_0 m^{k_0-1} \frac{1}{k_0} s^{\frac{1}{k_0}-1} k_0 n^{k_0-1} drdmdsdn$$

$$= \begin{bmatrix} \frac{k_0^4}{(k_0^2+1)(k_0^2+2)(2k_0^2+2)}\left[v^{\frac{2}{k_0}+2k_0} - u^{\frac{2}{k_0}+2k_0}\right] - \frac{k_0^2 u^{\frac{2}{k_0}+k_0}}{(k_0^2+1)(k_0^2+2)}\left[v^{k_0} - u^{k_0}\right] - \frac{k_0^4 u^{\frac{1}{k_0}+k_0}}{(k_0^2+1)^2}\left[v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right] + \frac{k_0^2 u^{\frac{2}{k_0}+k_0}}{(k_0^2+1)}\left[v^{k_0} - u^{k_0}\right] \\ -\frac{k_0^2 u^{\frac{1}{k_0}}}{(k_0^2+1)(2k_0^2+1)}\left[v^{\frac{1}{k_0}+2k_0} - u^{\frac{1}{k_0}+2k_0}\right] + \frac{u^{\frac{2}{k_0}+k_0}}{(k_0^2+1)}\left[v^{k_0} - u^{k_0}\right] + \frac{k_0^2 u^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}\left[v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right] - u^{\frac{2}{k_0}+k_0}\left[v^{k_0} - u^{k_0}\right] \end{bmatrix}$$

$$O_{22} = P[X_2 < x < y < X_3 < Y_2 < Y_3]$$

$$= P[X_2 < x]P[y < X_3 < Y_2 < Y_3]$$

$$= u^{k_0}\left[\frac{1}{2}\left(1 - v^{k_0}\right) - \frac{k_0^2}{\left(k_0^2+1\right)}\left(1 - v^{\frac{1}{k_0}+k_0}\right) + \frac{k_0^2}{2\left(k_0^2+2\right)}\left(1 - v^{\frac{2}{k_0}+k_0}\right)\right]$$

$$O_{23} = P[X_2 < x < y < Y_2 < X_3 < Y_3]$$

$$= P[X_2 < x]P[y < Y_2 < X_3 < Y_3]$$

$$= u^{k_0}\left[\frac{\left(1 - v^{\frac{1}{k_0}}\right)}{\left(k_0^2+1\right)} - \frac{\left(1 - v^{\frac{1}{k_0}+k_0}\right)}{\left(k_0^2+1\right)} + \frac{k_0^2\left(1 - v^{\frac{2}{k_0}+k_0}\right)}{\left(k_0^2+1\right)\left(k_0^2+2\right)}\right]$$

$$O_{24} = P[X_2 < x < y < Y_2 < Y_3 < X_3]$$

$$= P[X_2 < x]P[y < Y_2 < Y_3 < X_3]$$

$$= u^{k_0}\left[\frac{k_0^{\,2}\left(1-v^{\frac{1}{k_0}}\right)}{\left(k_0^{\,2}+1\right)} - \frac{\left(1-v^{\frac{2}{k_0}}\right)}{2} + \frac{\left(1-v^{\frac{2}{k_0}+k_0}\right)}{\left(k_0^{\,2}+1\right)\left(k_0^{\,2}+2\right)}\right]$$

$$O_{25} = P[X_2 < x < X_3 < y < Y_2 < Y_3]$$

$$= P[X_2 < x]P[x < X_3 < y]P[y < Y_2 < Y_3]$$

$$= u^{k_0}\left[v^{k_0} - u^{k_0}\right]\left[\left(1-v^{\frac{1}{k_0}}\right) - \frac{1}{2}\left(1-v^{\frac{2}{k_0}}\right)\right]$$

$$O_{26} = P[X_2 < x < Y_2 < y < X_3 < Y_3]$$

$$= P[X_2 < x]P[x < Y_2 < y]P[y < X_3 < Y_3]$$

$$= u^{k_0}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right]\left[\left(1-v^{k_0}\right) - \frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)}\left(1-v^{\frac{1}{k_0}+k_0}\right)\right]$$

$$O_{27} = P[X_2 < x < Y_2 < y < Y_3 < X_3]$$

$$= P[X_2 < x]P[x < Y_2 < y]P[y < Y_3 < X_3]$$

$$= u^{k_0}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right]\left[\left(1-v^{\frac{1}{k_0}}\right) - \frac{1}{\left(k_0^{\,2}+1\right)}\left(1-v^{\frac{1}{k_0}+k_0}\right)\right]$$

$$O_{28} = P[Y_2 < x < X_2 < y < X_3 < Y_3]$$

$$= P[Y_2 < x]P[x < X_2 < y]P[y < X_3 < Y_3]$$

$$= u^{k_0}\left[v^{k_0} - u^{k_0}\right]\left[\left(1-v^{k_0}\right) - \frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)}\left(1-v^{\frac{1}{k_0}+k_0}\right)\right]$$

$$O_{29} = P[Y_2 < x < X_2 < y < Y_3 < X_3]$$

$$= P[Y_2 < x]P[x < X_2 < y]P[y < Y_3 < X_3]$$

$$= u^{k_0}\left[v^{k_0} - u^{k_0}\left[\left(1 - v^{\frac{1}{k_0}}\right) - \frac{1}{\left(k_0^{\,2}+1\right)}\left(1 - v^{\frac{1}{k_0}+k_0}\right)\right]\right]$$

$$O_{30} = P[X_2 < x < X_3 < Y_2 < y < Y_3]$$

$$= P[X_2 < x]P[x < X_3 < Y_2 < y]P[y < Y_3]$$

$$= u^{k_0}\left[\frac{1}{\left(k_0^{\,2}+1\right)}\left(v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right) - u^{k_0}\left(v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\right]\left[1 - v^{\frac{1}{k_0}}\right]$$

$$O_{31} = P[X_2 < x < Y_2 < X_3 < y < Y_3]$$

$$= P[X_2 < x]P[x < Y_2 < X_3 < y]P[y < Y_3]$$

$$= u^{k_0}\left[\frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)}\left(v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right) - u^{\frac{1}{k_0}}\left(v^{k_0} - u^{k_0}\right)\right]\left[1 - v^{\frac{1}{k_0}}\right]$$

$$O_{32} = P[X_2 < x < Y_2 < Y_3 < y < X_3]$$

$$= P[X_2 < x]P[x < Y_2 < Y_3 < y]P[y < X_3]$$

$$= u^{k_0}\left[\frac{1}{2}\left(v^{\frac{2}{k_0}} - u^{\frac{2}{k_0}}\right) - u^{\frac{1}{k_0}}\left(v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\right]\left[1 - v^{k_0}\right]$$

$$O_{33} = P[Y_2 < x < X_2 < X_3 < y < Y_3]$$

$$= P[Y_2 < x]P[x < X_2 < X_3 < y]P[y < Y_3]$$

$$= u^{\frac{1}{k_0}}\left[\frac{v^{2k_0}}{2} - \frac{u^{2k_0}}{2} - u^{k_0}v^{k_0} + u^{2k_0}\right]\left[1 - v^{\frac{1}{k_0}}\right]$$

$$O_{34} = P[Y_2 < x < X_2 < Y_3 < y < X_3]$$

$$= P[Y_2 < x]P[x < X_2 < Y_3 < y]P[y < X_3]$$

$$= u^{\frac{1}{k_0}}\left[\frac{1}{\left(k_0^{\,2}+1\right)}\left(v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right) - u^{k_0}\left(v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\right]\left[1 - v^{k_0}\right]$$

$$O_{35} = P[X_2 < x < X_3 < Y_2 < Y_3 < y]$$

$$= P[X_2 < x]P[x < X_3 < Y_2 < Y_3 < y]$$

$$= u^{k_0}\left[\frac{1}{(k_0^2+1)(k_0^2+2)}\left[v^{\frac{2}{k_0}+k_0} - u^{\frac{2}{k_0}+k_0}\right] - \frac{u^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right] - \frac{u^{k_0}}{2}\left[v^{\frac{2}{k_0}} - u^{\frac{2}{k_0}}\right] + u^{\frac{1}{k_0}+k_0}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right]\right]$$

$$O_{36} = P[X_2 < x < Y_2 < X_3 < Y_3 < y]$$

$$= P[X_2 < x]P[x < Y_2 < X_3 < Y_3 < y]$$

$$= u^{k_0}\left[\frac{k_0^2}{(k_0^2+1)(k_0^2+2)}\left[v^{\frac{2}{k_0}+k_0} - u^{\frac{2}{k_0}+k_0}\right] - \frac{k_0^2 u^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right] - \frac{u^{\frac{1}{k_0}}}{(k_0^2+1)}\left[v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right] + u^{\frac{1}{k_0}+k_0}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right]\right]$$

$$O_{37} = P[X_2 < x < Y_2 < Y_3 < X_3 < y]$$

$$= P[X_2 < x]P[x < Y_2 < Y_3 < X_3 < y]$$

$$= u^{k_0}\left[\frac{k_0^2}{2(k_0^2+2)}\left[v^{\frac{2}{k_0}+k_0} - u^{\frac{2}{k_0}+k_0}\right] - \frac{u^{\frac{2}{k_0}}}{2}\left[v^{k_0} - u^{k_0}\right] - \frac{k_0^2 u^{\frac{1}{k_0}}}{(k_0^2+1)}\left[v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right] + u^{\frac{2}{k_0}}\left[v^{k_0} - u^{k_0}\right]\right]$$

$$O_{38} = P[Y_2 < x < X_2 < X_3 < Y_3 < y]$$

$$= P[Y_2 < x]P[x < X_2 < X_3 < Y_3 < y]$$

$$= u^{\frac{1}{k_0}}\left[\frac{1}{2(2k_0^2+1)}\left[v^{\frac{1}{k_0}+2k_0} - u^{\frac{1}{k_0}+2k_0}\right] + \frac{u^{2k_0}}{2}\left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right] - \frac{u^{k_0}}{(k_0^2+1)}\left[v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right]\right]$$

$$O_{39} = P[Y_2 < x < X_2 < Y_3 < X_3 < y]$$

$$= P[Y_2 < x]P[x < X_2 < Y_3 < X_3 < y]$$

$$= u^{\frac{1}{k_0}}\left[\frac{k_0^2}{(k_0^2+1)(2k_0^2+1)}\left[v^{\frac{1}{k_0}+2k_0} - u^{\frac{1}{k_0}+2k_0}\right] + \frac{k_0^2 u^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}\left[v^{k_0} - u^{k_0}\right] - \frac{k_0^2 u^{k_0}}{(k_0^2+1)}\left[v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right]\right]$$

$$O_{40} = P[X_2 < X_3 < x < y < Y_2 < Y_3]$$

$$= P[X_2 < X_3 < x]P[y < Y_2 < Y_3]$$

$$= \int_{-\infty}^{x} P[X_2 < x_3] dF(x_3) \frac{\overline{G}^2(y)}{2}$$

$$= \int_{-\infty}^{x} F^{k_0}(x_3) dF^{k_0}(x_3) \left[\left(1 - v^{\frac{1}{k_0}} \right) - \frac{1}{2} \left(1 - v^{\frac{2}{k_0}} \right) \right]$$

$$= \frac{u^{2k_0}}{2} \left[\left(1 - v^{\frac{1}{k_0}} \right) - \frac{1}{2} \left(1 - v^{\frac{2}{k_0}} \right) \right]$$

$$O_{41} = P[X_2 < Y_2 < x < y < X_3 < Y_3]$$

$$= P[X_2 < Y_2 < x] P[y < X_3 < Y_3]$$

$$= \int_{-\infty}^{x} P[X_2 < y_2] dG(y_2) \frac{\overline{G}^2(y)}{2}$$

$$= \int_{-\infty}^{x} F^{k_0}(y_2) dF^{\frac{1}{k_0}}(y_2) \left[\left(1 - v^{k_0} \right) - \frac{1}{2} \left(1 - v^{2k_0} \right) \right]$$

$$= \frac{u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2} + 1 \right)} \left[\left(1 - v^{k_0} \right) - \frac{k_0^{\,2}}{\left(k_0^{\,2} + 1 \right)} \left(1 - v^{\frac{1}{k_0}+k_0} \right) \right]$$

$$O_{42} = P[X_2 < Y_2 < x < y < Y_3 < X_3]$$

$$= P[X_2 < Y_2 < x] P[y < Y_3 < X_3]$$

$$= \frac{u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2} + 1 \right)} \left[\left(1 - v^{\frac{1}{k_0}} \right) - \frac{1}{\left(k_0^{\,2} + 1 \right)} \left(1 - v^{\frac{1}{k_0}+k_0} \right) \right]$$

$$O_{43} = P[Y_2 < X_2 < x < y < X_3 < Y_3]$$

$$= P[Y_2 < X_2 < x] P[y < X_3 < Y_3]$$

$$= \frac{k_0^{\,2} u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2} + 1 \right)} \left[\left(1 - v^{k_0} \right) - \frac{k_0^{\,2}}{\left(k_0^{\,2} + 1 \right)} \left(1 - v^{\frac{1}{k_0}+k_0} \right) \right]$$

$$O_{44} = P[Y_2 < X_2 < x < y < Y_3 < X_3]$$

$$= P[Y_2 < X_2 < x]P[y < Y_3 < X_3]$$

$$= \frac{k_0^{\,2} u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}\left[\left(1-v^{\frac{1}{k_0}}\right)-\frac{1}{\left(k_0^{\,2}+1\right)}\left(1-v^{\frac{1}{k_0}+k_0}\right)\right]$$

$$O_{45} = P[X_2 < X_3 < x < Y_2 < y < Y_3]$$

$$= P[X_2 < X_3 < x]P[x < Y_2 < y < Y_3]$$

$$= P[X_2 < X_3 < x]P[x < Y_2 < y]P[y < Y_3]$$

$$= \frac{u^{2k_0}}{2}\left(v^{\frac{1}{k_0}}-u^{\frac{1}{k_0}}\right)\left(1-v^{\frac{1}{k_0}}\right)$$

$$O_{46} = P[X_2 < Y_2 < x < X_3 < y < Y_3]$$

$$= P[X_2 < Y_2 < x]P[x < X_3 < y]P[y < Y_3]$$

$$= \frac{u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}\left(v^{k_0}-u^{k_0}\right)\left(1-v^{\frac{1}{k_0}}\right)$$

$$O_{47} = P[X_2 < Y_2 < x < Y_3 < y < X_3]$$

$$= P[X_2 < Y_2 < x]P[x < Y_3 < y]P[y < X_3]$$

$$= \frac{u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}\left(v^{\frac{1}{k_0}}-u^{\frac{1}{k_0}}\right)\left(1-v^{k_0}\right)$$

$$O_{48} = P[Y_2 < X_2 < x < X_3 < y < Y_3]$$

$$= P[Y_2 < X_2 < x]P[x < X_3 < y]P[y < Y_3]$$

$$= \frac{k_0^{\,2} u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}\left(v^{k_0}-u^{k_0}\right)\left(1-v^{\frac{1}{k_0}}\right)$$

$$O_{49} = P[Y_2 < X_2 < x < Y_3 < y < X_3]$$

$$= P[Y_2 < X_2 < x]P[x < Y_3 < y]P[y < X_3]$$

$$= \frac{{k_0}^2 u^{\frac{1}{k_0}+k_0}}{\left({k_0}^2+1\right)}\left(v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\left(1 - v^{k_0}\right)$$

$$O_{50} = P[X_2 < X_3 < x < Y_2 < Y_3 < y]$$

$$= P[X_2 < X_3 < x]P[x < Y_2 < Y_3 < y]$$

$$= \frac{u^{2k_0}}{2}\left[\frac{1}{2}\left(v^{\frac{2}{k_0}} - u^{\frac{2}{k_0}}\right) - u^{\frac{1}{k_0}}\left(v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\right]$$

$$O_{51} = P[X_2 < Y_2 < x < X_3 < Y_3 < y]$$

$$= P[X_2 < Y_2 < x]P[x < X_3 < Y_3 < y]$$

$$= \frac{u^{\frac{1}{k_0}+k_0}}{\left({k_0}^2+1\right)}\left[\frac{1}{\left({k_0}^2+1\right)}\left(v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right) - u^{k_0}\left(v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\right]$$

$$O_{52} = P[X_2 < Y_2 < x < Y_3 < X_3 < y]$$

$$= P[X_2 < Y_2 < x]P[x < Y_3 < X_3 < y]$$

$$= \frac{u^{\frac{1}{k_0}+k_0}}{\left({k_0}^2+1\right)}\left[\frac{{k_0}^2}{\left({k_0}^2+1\right)}\left(v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right) - u^{\frac{1}{k_0}}\left(v^{k_0} - u^{k_0}\right)\right]$$

$$O_{53} = P[Y_2 < X_2 < x < X_3 < Y_3 < y]$$

$$= P[Y_2 < X_2 < x]P[x < X_3 < Y_3 < y]$$

$$= \frac{{k_0}^2 u^{\frac{1}{k_0}+k_0}}{\left({k_0}^2+1\right)}\left[\frac{1}{\left({k_0}^2+1\right)}\left(v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right) - u^{k_0}\left(v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\right]$$

$$O_{54} = P[Y_2 < X_2 < x < Y_3 < X_3 < y]$$

$$= P[Y_2 < X_2 < x]P[x < Y_3 < X_3 < y]$$

$$= \frac{{k_0}^2 u^{\frac{1}{k_0}+k_0}}{\left({k_0}^2+1\right)}\left[\frac{{k_0}^2}{\left({k_0}^2+1\right)}\left(v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right) - u^{\frac{1}{k_0}}\left(v^{k_0} - u^{k_0}\right)\right]$$

$$O_{55} = P[X_2 < X_3 < Y_2 < x < y < Y_3]$$

$$= P[X_2 < X_3 < Y_2 < x]P[y < Y_3]$$

$$= \left[\int_{-\infty}^{x}\int_{-\infty}^{y_2}\int_{-\infty}^{x_3} dF(x_2)dF(x_3)dG(y_2)\right]\left[1 - v^{\frac{1}{k_0}}\right]$$

$$= \left[\int_0^u\int_0^r\int_0^n k_0 m^{k_0-1} n^{k_0-1} r^{\frac{1}{k_0}-1} dmdndr\right]\left[1 - v^{\frac{1}{k_0}}\right]$$

$$= \frac{u^{\frac{1}{k_0}+2k_0}}{2\left(2{k_0}^2+1\right)}\left[1 - v^{\frac{1}{k_0}}\right]$$

$$O_{56} = P[X_2 < Y_2 < X_3 < x < y < Y_3]$$

$$= P[X_2 < Y_2 < X_3 < x]P[y < Y_3]$$

$$= \left[\int_{-\infty}^{x}\int_{-\infty}^{x_3}\int_{-\infty}^{y_2} dF(x_2)dG(y_2)dF(x_3)\right]\left[1 - v^{\frac{1}{k_0}}\right]$$

$$= \left[\int_0^u\int_0^n\int_0^r k_0 m^{k_0-1}\frac{1}{k_0} r^{\frac{1}{k_0}-1} k_0 n^{k_0-1} dmdrdn\right]\left[1 - v^{\frac{1}{k_0}}\right]$$

$$= \frac{{k_0}^2 u^{\frac{1}{k_0}+2k_0}}{\left({k_0}^2+1\right)\left(2{k_0}^2+1\right)}\left[1 - v^{\frac{1}{k_0}}\right]$$

$$O_{57} = P[Y_2 < X_2 < X_3 < x < y < Y_3]$$

$$= P[Y_2 < X_2 < X_3 < x]P[y < Y_3]$$

$$= \left[\int_{-\infty}^{x}\int_{-\infty}^{x_3}\int_{-\infty}^{x_2} dG(y_2)dF(x_2)dF(x_3)\right]\left[1 - v^{\frac{1}{k_0}}\right]$$

$$= \left[\int_0^u \int_0^n \int_0^m \frac{1}{k_0} r^{\frac{1}{k_0}-1} k_0 m^{k_0-1} k_0 n^{k_0-1} dr dm dn \right] \left[1 - v^{\frac{1}{k_0}} \right]$$

$$= \frac{k_0{}^4 u^{\frac{1}{k_0}+2k_0}}{\left(k_0{}^2+1\right)\left(2k_0{}^2+1\right)} \left[1 - v^{\frac{1}{k_0}} \right]$$

$$O_{58} = P[X_2 < X_3 < Y_2 < x < Y_3 < y]$$

$$= P[X_2 < X_3 < Y_2 < x] P[x < Y_3 < y]$$

$$= \frac{u^{\frac{1}{k_0}+2k_0}}{2\left(2k_0{}^2+1\right)} \left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}} \right]$$

$$O_{59} = P[X_2 < Y_2 < X_3 < x < Y_3 < y]$$

$$= P[X_2 < Y_2 < X_3 < x] P[x < Y_3 < y]$$

$$= \frac{k_0{}^2 u^{\frac{1}{k_0}+2k_0}}{\left(k_0{}^2+1\right)\left(2k_0{}^2+1\right)} \left[v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}} \right]$$

$$O_{60} = P[X_2 < X_3 < Y_2 < Y_3 < x < y]$$

$$= P[X_2 < X_3 < Y_2 < Y_3 < x]$$

$$= \int_{-\infty}^{x} \int_{-\infty}^{y_3} \int_{-\infty}^{y_2} \int_{-\infty}^{x_3} dF(x_2) dF(x_3) dG(y_2) dG(y_3)$$

$$= \int_0^u \int_0^s \int_0^r \int_0^n k_0 m^{k_0-1} k_0 n^{k_0-1} \frac{1}{k_0} r^{\frac{1}{k_0}-1} \frac{1}{k_0} s^{\frac{1}{k_0}-1} dm dn dr ds$$

$$= \frac{u^{\frac{2}{k_0}+2k_0}}{2\left(2k_0{}^2+1\right)\left(2k_0{}^2+2\right)}$$

$$O_{61} = P[X_2 < Y_2 < X_3 < Y_3 < x < y]$$

$$= P[X_2 < Y_2 < X_3 < Y_3 < x]$$

$$= \int_{-\infty}^{x}\int_{-\infty}^{y_3}\int_{-\infty}^{x_3}\int_{-\infty}^{y_2} dF(x_2)dG(y_2)dF(x_3)dG(y_3)$$

$$= \int_{0}^{u}\int_{0}^{s}\int_{0}^{n}\int_{0}^{r} k_0 m^{k_0-1}\frac{1}{k_0}r^{\frac{1}{k_0}-1}k_0 n^{k_0-1}\frac{1}{k_0}s^{\frac{1}{k_0}-1}\,dmdrdnds$$

$$= \frac{{k_0}^2 u^{\frac{2}{k_0}+2k_0}}{\left({k_0}^2+1\right)\left(2{k_0}^2+1\right)\left(2{k_0}^2+2\right)}$$

$$O_{62} = P[Y_2 < X_2 < X_3 < Y_3 < x < y]$$

$$= P[Y_2 < X_2 < X_3 < Y_3 < x]$$

$$= \int_{-\infty}^{x}\int_{-\infty}^{y_3}\int_{-\infty}^{x_3}\int_{-\infty}^{x_2} dG(y_2)dF(x_2)dF(x_3)dG(y_3)$$

$$= \int_{0}^{u}\int_{0}^{s}\int_{0}^{n}\int_{0}^{m}\frac{1}{k_0}r^{\frac{1}{k_0}-1}k_0 m^{k_0-1}k_0 n^{k_0-1}\frac{1}{k_0}s^{\frac{1}{k_0}-1}\,drdmdnds$$

$$= \frac{{k_0}^4 u^{\frac{2}{k_0}+2k_0}}{\left({k_0}^2+1\right)\left(2{k_0}^2+1\right)\left(2{k_0}^2+2\right)}$$

Now adding the 62 terms and multiplying by 4 we have

$$P[Med(x,X_2,X_3) < Med(y,Y_2,Y_3) = 4O_1 + 4O_2 + ... + 4O_{62}]$$

$$= 4u^{k_0}v^{k_0} - 4u^{k_0}v^{2k_0} + 4u^{2k_0}v^{k_0} + 4u^{2k_0}v^{\frac{1}{k_0}} - 4u^{2k_0} + 4u^{\frac{1}{k_0}}v^{\frac{1}{k_0}+k_0} - 4u^{\frac{1}{k_0}}v^{\frac{1}{k_0}+2k_0} + 4u^{\frac{1}{k_0}}v^{2k_0} - 4u^{\frac{1}{k_0}}v^{k_0}$$
$$- 4u^{\frac{1}{k_0}+k_0}v^{\frac{1}{k_0}} - 4u^{\frac{1}{k_0}+k_0}v^{k_0} + 4u^{\frac{1}{k_0}+k_0} - 4v^{\frac{1}{k_0}+k_0}u^{k_0} + 4u^{\frac{1}{k_0}+k_0}v^{\frac{1}{k_0}+k_0} + 4u^{k_0}v^{\frac{1}{k_0}+2k_0} - 4u^{2k_0}v^{\frac{1}{k_0}+k_0} - \frac{4u^{\frac{2}{k_0}+k_0}}{\left({k_0}^2+2\right)}$$
$$- \frac{4{k_0}^2 v^{\frac{2}{k_0}+k_0}}{\left({k_0}^2+2\right)} - \frac{4{k_0}^2 v^{\frac{1}{k_0}+2k_0}}{\left(2{k_0}^2+1\right)} + \frac{4{k_0}^2 v^{\frac{1}{k_0}+k_0}}{\left({k_0}^2+1\right)} + \frac{2{k_0}^2 v^{\frac{2}{k_0}+2k_0}}{\left({k_0}^2+1\right)} + \frac{2\left(2{k_0}^2+2{k_0}^4+1\right)u^{\frac{2}{k_0}+2k_0}}{\left({k_0}^2+1\right)\left(2{k_0}^2+1\right)} - \frac{4{k_0}^4 u^{\frac{1}{k_0}+2k_0}v^{\frac{1}{k_0}}}{\left({k_0}^2+1\right)\left(2{k_0}^2+1\right)} + \frac{\left(7{k_0}^2+2\right)}{\left({k_0}^2+1\right)\left({k_0}^2+2\right)\left(2{k_0}^2+1\right)}$$

where $F(x) = u$ and $F(y) = v$.

Thus

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{y} P^2[Med(x,X_2,X_3) < Med(y,Y_2,Y_3)]dF(x)dG(y)$$

$$-\infty < x < y < \infty$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{y} \begin{pmatrix} 4u^{k_0}v^{k_0} - 4u^{k_0}v^{2k_0} + 4u^{2k_0}v^{k_0} + 4u^{2k_0}v^{\frac{1}{k_0}} - 4u^{2k_0} + 4u^{\frac{1}{k_0}}v^{\frac{1}{k_0}+k_0} - 4u^{\frac{1}{k_0}}v^{\frac{1}{k_0}+2k_0} + 4u^{\frac{1}{k_0}}v^{2k_0} - 4u^{\frac{1}{k_0}}v^{k_0} \\ -4u^{\frac{1}{k_0}+k_0}v^{\frac{1}{k_0}} - 4u^{\frac{1}{k_0}+k_0}v^{k_0} + 4u^{\frac{1}{k_0}+k_0} - 4v^{\frac{1}{k_0}+k_0}u^{k_0} + 4u^{\frac{1}{k_0}+k_0}v^{\frac{1}{k_0}+k_0} + 4u^{k_0}v^{\frac{1}{k_0}+2k_0} - 4u^{2k_0}v^{\frac{1}{k_0}+k_0} - \frac{4u^{\frac{2}{k_0}+k_0}}{(k_0^2+2)} \\ -\frac{4k_0^2v^{\frac{2}{k_0}+k_0}}{(k_0^2+2)} - \frac{4k_0^2v^{\frac{1}{k_0}+2k_0}}{(2k_0^2+1)} + \frac{4k_0^2v^{\frac{1}{k_0}+k_0}}{(k_0^2+1)} + \frac{2k_0^2v^{\frac{2}{k_0}+2k_0}}{(k_0^2+1)} + \frac{2(2k_0^2+2k_0^4+1)u^{\frac{2}{k_0}+2k_0}}{(k_0^2+1)(2k_0^2+1)} - \frac{4k_0^4u^{\frac{1}{k_0}+2k_0}v^{\frac{1}{k_0}}}{(k_0^2+1)(2k_0^2+1)} + \frac{(7k_0^2+2)}{(k_0^2+1)(k_0^2+2)(2k_0^2+1)} \end{pmatrix}^2 dF(x)dG(y)$$

$$-\infty < x < y < \infty$$

$$= \int_0^1\int_0^v \begin{pmatrix} 4u^{k_0}v^{k_0} - 4u^{k_0}v^{2k_0} + 4u^{2k_0}v^{k_0} + 4u^{2k_0}v^{\frac{1}{k_0}} - 4u^{2k_0} + 4u^{\frac{1}{k_0}}v^{\frac{1}{k_0}+k_0} - 4u^{\frac{1}{k_0}}v^{\frac{1}{k_0}+2k_0} + 4u^{\frac{1}{k_0}}v^{2k_0} - 4u^{\frac{1}{k_0}}v^{k_0} \\ -4u^{\frac{1}{k_0}+k_0}v^{\frac{1}{k_0}} - 4u^{\frac{1}{k_0}+k_0}v^{k_0} + 4u^{\frac{1}{k_0}+k_0} - 4v^{\frac{1}{k_0}+k_0}u^{k_0} + 4u^{\frac{1}{k_0}+k_0}v^{\frac{1}{k_0}+k_0} + 4u^{k_0}v^{\frac{1}{k_0}+2k_0} - 4u^{2k_0}v^{\frac{1}{k_0}+k_0} - \frac{4u^{\frac{2}{k_0}+k_0}}{(k_0^2+2)} \\ -\frac{4k_0^2v^{\frac{2}{k_0}+k_0}}{(k_0^2+2)} - \frac{4k_0^2v^{\frac{1}{k_0}+2k_0}}{(2k_0^2+1)} + \frac{4k_0^2v^{\frac{1}{k_0}+k_0}}{(k_0^2+1)} + \frac{2k_0^2v^{\frac{2}{k_0}+2k_0}}{(k_0^2+1)} + \frac{2(2k_0^2+2k_0^4+1)u^{\frac{2}{k_0}+2k_0}}{(k_0^2+1)(2k_0^2+1)} - \frac{4k_0^4u^{\frac{1}{k_0}+2k_0}v^{\frac{1}{k_0}}}{(k_0^2+1)(2k_0^2+1)} + \frac{(7k_0^2+2)}{(k_0^2+1)(k_0^2+2)(2k_0^2+1)} \end{pmatrix}^2 k_0u^{k_0-1}\frac{1}{k_0}v^{\frac{1}{k_0}-1}dudv$$

where $F(x) = u$ and $F(y) = v$

$$= \frac{\frac{1}{45}\begin{bmatrix} 15362887680000k_0^{46} + 189073253376000k_0^{44} + 1093897560597120k_0^{42} + 4056538417079904k_0^{40} \\ +11219787085520592k_0^{38} + 25461258579791592k_0^{36} + 49582398663755148k_0^{34} \\ +82801028360209848k_0^{32} + 116119425989696054k_0^{30} + 134227087809727456k_0^{28} \\ 126643376225942279k_0^{26} + 97204194134608879k_0^{24} + 60665756491658519k_0^{22} \\ +30819176528010870k_0^{20} + 26812259131803330k_0^{18} + 4226861070553517k_0^{16} \\ +11324996413604470k_0^{14} + 2413529690935250k_0^{12} + 402618804260200k_0^{10} + 51317278987520k_0^{8} \\ +4814609834880k_0^{6} + 312704402400k_0^{4} + 12534912000k_0^{2} + 23328000 \end{bmatrix}}{\begin{bmatrix} (2k_0^2+3)(4k_0^2+5)(7k_0^2+2)(4k_0^2+3)(3k_0^2+1)^2(4k_0^2+1)(6k_0^2+1)(5k_0^2+2)(5k_0^2+4)(2k_0^2+1)^3 \\ (3k_0^2+4)(7k_0^2+3)(3k_0^2+2)(k_0^2+1)^3(3k_0^2+5)(k_0^2+2)(5k_0^2+3)(7k_0^2+1)(5k_0^2+1) \end{bmatrix}}$$

on simplification.

Now consider case ii) y<x.

$$P[Med(x, X_2, X_3) < Med(y, Y_2, Y_3)] = 4P_1 + 4P_2 + ... + 4P_{27}$$

where

$$P_1 = P[y < x < X_2 < X_3 < Y_2 < Y_3] \qquad P_2 = P[y < x < X_2 < Y_2 < X_3 < Y_3]$$

$$P_3 = P[y < x < X_2 < Y_2 < Y_3 < X_3] \qquad P_4 = P[y < X_2 < x < X_3 < Y_2 < Y_3]$$

$$P_5 = P[y < X_2 < x < Y_2 < X_3 < Y_3] \qquad P_6 = P[y < X_2 < x < Y_2 < Y_3 < X_3]$$

$$P_7 = P[y < X_2 < X_3 < x < Y_2 < Y_3] \qquad P_8 = P[y < X_2 < X_3 < Y_2 < x < Y_3]$$

$$P_9 = P[y < X_2 < X_3 < Y_2 < Y_3 < x] \qquad P_{10} = P[X_2 < y < x < X_3 < Y_2 < Y_3]$$

$P_{11} = P[X_2 < y < x < Y_2 < X_3 < Y_3]$ $P_{12} = P[X_2 < y < x < Y_2 < Y_3 < X_3]$

$P_{13} = P[X_2 < y < X_3 < x < Y_2 < Y_3]$ $P_{14} = P[X_2 < y < X_3 < Y_2 < x < Y_3]$

$P_{15} = P[X_2 < y < X_3 < Y_2 < Y_3 < x]$ $P_{16} = P[X_2 < X_3 < y < x < Y_2 < Y_3]$

$P_{17} = P[X_2 < X_3 < y < Y_2 < x < Y_3]$ $P_{18} = P[X_2 < X_3 < y < Y_2 < Y_3 < x]$

$P_{19} = P[X_2 < X_3 < Y_2 < y < x < Y_3]$ $P_{20} = P[X_2 < Y_2 < X_3 < y < x < Y_3]$

$P_{21} = P[Y_2 < X_2 < X_3 < y < x < Y_3]$ $P_{22} = P[X_2 < X_3 < Y_2 < y < Y_3 < x]$

$P_{23} = P[X_2 < Y_2 < X_3 < y < Y_3 < x]$ $P_{24} = P[Y_2 < X_2 < X_3 < y < Y_3 < x]$

$P_{25} = P[X_2 < X_3 < Y_2 < Y_3 < y < x]$ $P_{26} = P[X_2 < Y_2 < X_3 < Y_3 < y < x]$

$P_{27} = P[Y_2 < X_2 < X_3 < Y_3 < y < x]$.

Let $F(x) = u$ $F(x_2) = m$ $F(x_3) = n$ $F(y) = v$ $F(y_2) = r$ $F(y_3) = s$ and consider

$P_1 = P[y < x < X_2 < X_3 < Y_2 < Y_3]$

$= P[x < X_2 < X_3 < Y_2 < Y_3]$

$$= \int_x^\infty \int_{x_2}^\infty \int_{x_3}^\infty P(y_2 < Y_3) dG(y_2) dF(x_3) dF(x_2)$$

$$= \int_u^1 \int_m^1 \int_n^1 \left[1 - r^{\frac{1}{k_0}}\right] \frac{1}{k_0} r^{\frac{1}{k_0}-1} k_0 n^{k_0-1} k_0 m^{k_0-1} dr dn dm$$

$$= (1-u^{k_0}) - \frac{1}{2}(1-u^{2k_0}) - \frac{k_0^2(1-u^{k_0})}{(k_0^2+1)} + \frac{k_0^4\left(1-u^{\frac{1}{k_0}+2k_0}\right)}{(k_0^2+1)(2k_0^2+1)} - \frac{1}{2}(1-u^{k_0}) + \frac{1}{4}(1-u^{2k_0}) + \frac{k_0^2(1-u^{k_0})}{2(k_0^2+2)} - \frac{k_0^4\left(1-u^{\frac{2}{k_0}+2k_0}\right)}{2(k_0^2+2)(2k_0^2+2)}$$

$P_2 = P[y < x < X_2 < Y_2 < X_3 < Y_3]$

$= P[x < X_2 < Y_2 < X_3 < Y_3]$

$$= \int_x^\infty \int_{x_2}^\infty \int_{y_2}^\infty P(x_3 < Y_3) dF(x_3) dG(y_2) dF(x_2)$$

$$=\int_u^1\int_m^1\int_r^1\left[1-n^{\frac{1}{k_0}}\right]k_0n^{k_0-1}\frac{1}{k_0}r^{\frac{1}{k_0}-1}k_0m^{k_0-1}dndrdm$$

$$=\left(1-u^{k_0}\right)-\frac{k_0^{\,2}\left(1-u^{\frac{1}{k_0}+k_0}\right)}{\left(k_0^{\,2}+1\right)}-\frac{\left(1-u^{k_0}\right)}{\left(k_0^{\,2}+1\right)}+\frac{k_0^{\,2}\left(1-u^{\frac{1}{k_0}+2k_0}\right)}{\left(k_0^{\,2}+1\right)\left(2k_0^{\,2}+1\right)}-\frac{k_0^{\,2}\left(1-u^{k_0}\right)}{\left(k_0^{\,2}+1\right)}+\frac{k_0^{\,4}\left(1-u^{\frac{1}{k_0}+k_0}\right)}{\left(k_0^{\,2}+1\right)\left(2k_0^{\,2}+1\right)}+\frac{k_0^{\,2}\left(1-u^{k_0}\right)}{\left(k_0^{\,2}+1\right)\left(k_0^{\,2}+2\right)}-\frac{k_0^{\,4}\left(1-u^{\frac{2}{k_0}+2k_0}\right)}{\left(k_0^{\,2}+1\right)\left(k_0^{\,2}+2\right)\left(2k_0^{\,2}+2\right)}$$

$$P_3=P\left[y<x<X_2<Y_2<Y_3<X_3\right]$$

$$=P\left[x<X_2<Y_2<Y_3<X_3\right]$$

$$=\int_x^\infty\int_{x_2}^\infty\int_{y_2}^\infty P(y_3<X_3)dG(y_3)dG(y_2)dF(x_2)$$

$$=\int_u^1\int_m^1\int_r^1\left[1-s^{k_0}\right]\frac{1}{k_0}s^{\frac{1}{k_0}-1}\frac{1}{k_0}r^{\frac{1}{k_0}-1}k_0m^{k_0-1}dsdrdm$$

$$=\left(1-u^{k_0}\right)-\frac{k_0^{\,2}\left(1-u^{\frac{1}{k_0}+k_0}\right)}{\left(k_0^{\,2}+1\right)}-\frac{\left(1-u^{k_0}\right)}{2}+\frac{k_0^{\,2}\left(1-u^{\frac{2}{k_0}+k_0}\right)}{2\left(k_0^{\,2}+2\right)}-\frac{\left(1-u^{k_0}\right)}{\left(k_0^{\,2}+1\right)}+\frac{k_0^{\,2}\left(1-u^{\frac{1}{k_0}+k_0}\right)}{\left(k_0^{\,2}+1\right)\left(k_0^{\,2}+1\right)}+\frac{\left(1-u^{k_0}\right)}{\left(k_0^{\,2}+1\right)\left(k_0^{\,2}+2\right)}-\frac{k_0^{\,2}\left(1-u^{\frac{2}{k_0}+2k_0}\right)}{\left(k_0^{\,2}+1\right)\left(k_0^{\,2}+2\right)\left(2k_0^{\,2}+2\right)}$$

$$P_4=P\left[y<X_2<x<X_3<Y_2<Y_3\right]$$

$$=P\left[y<X_2<x\right]P\left[x<X_3<Y_2<Y_3\right]$$

$$=\left[u^{k_0}-v^{k_0}\right]\left[\left(1-u^{k_0}\right)-\frac{k_0^{\,2}\left(1-u^{\frac{1}{k_0}+k_0}\right)}{\left(k_0^{\,2}+1\right)}-\frac{\left(1-u^{k_0}\right)}{2}+\frac{k_0^{\,2}\left(1-u^{\frac{2}{k_0}+k_0}\right)}{2\left(k_0^{\,2}+2\right)}\right]$$

$$P_5=P\left[y<X_2<x<Y_2<X_3<Y_3\right]$$

$$=P\left[y<X_2<x\right]P\left[x<Y_2<X_3<Y_3\right]$$

$$=\left[u^{k_0}-v^{k_0}\right]\left[\left(1-u^{\frac{1}{k_0}}\right)-\frac{\left(1-u^{\frac{1}{k_0}+k_0}\right)}{\left(k_0^{\,2}+1\right)}-\frac{k_0^{\,2}\left(1-u^{\frac{1}{k_0}}\right)}{\left(k_0^{\,2}+1\right)}+\frac{k_0^{\,2}\left(1-u^{\frac{2}{k_0}+k_0}\right)}{\left(k_0^{\,2}+1\right)\left(k_0^{\,2}+2\right)}\right]$$

$$P_6=P\left[y<X_2<x<Y_2<Y_3<X_3\right]$$

$$= P[y < X_2 < x]P[x < Y_2 < Y_3 < X_3]$$

$$= \left[u^{k_0} - v^{k_0}\right]\left[\left(1-u^{\frac{1}{k_0}}\right) - \frac{\left(1-u^{\frac{2}{k_0}}\right)}{2} - \frac{\left(1-u^{\frac{1}{k_0}}\right)}{\left(k_0^{\,2}+1\right)} + \frac{\left(1-u^{\frac{2}{k_0}+k_0}\right)}{\left(k_0^{\,2}+1\right)\left(k_0^{\,2}+2\right)}\right]$$

$$P_7 = P[y < X_2 < X_3 < x < Y_2 < Y_3]$$

$$= P[y < X_2 < X_3 < x]P[x < Y_2 < Y_3]$$

$$= \left[\frac{\left(u^{2k_0} - v^{2k_0}\right)}{2} - v^{k_0}\left(u^{k_0} - v^{k_0}\right)\right]\left[\left(1-u^{\frac{1}{k_0}}\right) - \frac{\left(1-u^{\frac{2}{k_0}}\right)}{2}\right]$$

$$P_8 = P[y < X_2 < X_3 < Y_2 < x < Y_3]$$

$$= P[y < X_2 < X_3 < Y_2 < x]P[x < Y_3]$$

$$= \left[\frac{\left(u^{\frac{1}{k_0}+2k_0} - v^{\frac{1}{k_0}+2k_0}\right)}{2\left(2k_0^{\,2}+1\right)} - \frac{v^{2k_0}\left(u^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right)}{2} - \frac{v^{k_0}\left(u^{\frac{1}{k_0}+k_0} - v^{\frac{1}{k_0}+k_0}\right)}{\left(k_0^{\,2}+1\right)} + v^{2k_0}\left(u^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right)\right]\left[1-u^{\frac{1}{k_0}}\right]$$

$$P_9 = P[y < X_2 < X_3 < Y_2 < Y_3 < x]$$

$$= \int_y^x \int_y^{y_3} \int_y^{y_2} \int_y^{x_3} dF(x_2)dF(x_3)dF(y_2)dF(y_3)$$

$$= \int_v^u \int_v^s \int_v^r \int_v^n k_0 m^{k_0-1} k_0 n^{k_0-1} \frac{1}{k_0} r^{\frac{1}{k_0}-1} \frac{1}{k_0} s^{\frac{1}{k_0}-1} dmdndrds$$

$$= \frac{\left(u^{\frac{2}{k_0}+2k_0} - v^{\frac{2}{k_0}+2k_0}\right)}{4\left(k_0^{\,2}+1\right)\left(2k_0^{\,2}+1\right)} - \frac{v^{\frac{1}{k_0}+2k_0}\left(u^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right)}{2\left(2k_0^{\,2}+1\right)} - \frac{v^{2k_0}\left(u^{\frac{2}{k_0}} - v^{\frac{2}{k_0}}\right)}{4} + \frac{v^{\frac{1}{k_0}+2k_0}\left(u^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right)}{2} - \frac{v^{k_0}\left(u^{\frac{2}{k_0}+k_0} - v^{\frac{2}{k_0}+k_0}\right)}{\left(k_0^{\,2}+1\right)\left(k_0^{\,2}+2\right)}$$

$$+\frac{v^{\frac{1}{k_0}+2k_0}\left(u^{\frac{1}{k_0}}-v^{\frac{1}{k_0}}\right)}{\left(k_0^{\,2}+1\right)}+\frac{v^{2k_0}\left(u^{\frac{2}{k_0}}-v^{\frac{2}{k_0}}\right)}{2}-v^{\frac{1}{k_0}+2k_0}\left(u^{\frac{1}{k_0}}-v^{\frac{1}{k_0}}\right)$$

$$P_{10}=P[X_2<y<x<X_3<Y_2<Y_3]$$

$$=P[X_2<y]P[x<X_3<Y_2<Y_3]$$

$$=v^{k_0}\left[\left(1-u^{k_0}\right)-\frac{k_0^{\,2}\left(1-u^{\frac{1}{k_0}+k_0}\right)}{\left(k_0^{\,2}+1\right)}-\frac{\left(1-u^{k_0}\right)}{2}+\frac{k_0^{\,2}\left(1-u^{\frac{2}{k_0}+k_0}\right)}{2\left(k_0^{\,2}+2\right)}\right]$$

$$P_{11}=P[X_2<y<x<Y_2<X_3<Y_3]$$

$$=P[X_2<y]P[x<Y_2<X_3<Y_3]$$

$$=v^{k_0}\left[\left(1-u^{\frac{1}{k_0}}\right)-\frac{\left(1-u^{\frac{1}{k_0}+k_0}\right)}{\left(k_0^{\,2}+1\right)}-\frac{k_0^{\,2}\left(1-u^{\frac{1}{k_0}}\right)}{\left(k_0^{\,2}+1\right)}+\frac{k_0^{\,2}\left(1-u^{\frac{2}{k_0}+k_0}\right)}{\left(k_0^{\,2}+1\right)\left(k_0^{\,2}+2\right)}\right]$$

$$P_{12}=P[X_2<y<x<Y_2<Y_3<X_3]$$

$$=P[X_2<y]P[x<Y_2<Y_3<X_3]$$

$$=v^{k_0}\left[\left(1-u^{\frac{1}{k_0}}\right)-\frac{\left(1-u^{\frac{2}{k_0}}\right)}{2}-\frac{\left(1-u^{\frac{1}{k_0}}\right)}{\left(k_0^{\,2}+1\right)}+\frac{\left(1-u^{\frac{2}{k_0}+k_0}\right)}{\left(k_0^{\,2}+1\right)\left(k_0^{\,2}+2\right)}\right]$$

$$P_{13}=P[X_2<y<X_3<x<Y_2<Y_3]$$

$$=P[X_2<y]P[y<X_3<x]P[x<Y_2<Y_3]$$

$$= v^{k_0}\left(u^{k_0} - v^{k_0}\right)\left[\left(1-u^{\frac{1}{k_0}}\right) - \frac{\left(1-u^{\frac{2}{k_0}}\right)}{2}\right]$$

$$P_{14} = P[X_2 < y < X_3 < Y_2 < x < Y_3]$$

$$= P[X_2 < y]P[y < X_3 < Y_2 < x]P[x < Y_3]$$

$$= v^{k_0}\left[\frac{\left(u^{\frac{1}{k_0}+k_0} - v^{\frac{1}{k_0}+k_0}\right)}{\left(k_0{}^2+1\right)} - v^{k_0}\left(u^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right)\right]\left[1-u^{\frac{1}{k_0}}\right]$$

$$P_{15} = P[X_2 < y < X_3 < Y_2 < Y_3 < x]$$

$$= P[X_2 < y]P[y < X_3 < Y_2 < Y_3 < x]$$

$$= v^{k_0}\left[\frac{\left(u^{\frac{2}{k_0}+k_0} - v^{\frac{2}{k_0}+k_0}\right)}{\left(k_0{}^2+1\right)\left(k_0{}^2+2\right)} - \frac{v^{\frac{1}{k_0}+k_0}\left(u^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right)}{\left(k_0{}^2+1\right)} - \frac{v^{k_0}\left(u^{\frac{2}{k_0}} - v^{\frac{2}{k_0}}\right)}{2} + v^{\frac{1}{k_0}+k_0}\left(u^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right)\right]$$

$$P_{16} = P[X_2 < X_3 < y < x < Y_2 < Y_3]$$

$$= P[X_2 < X_3 < y_3]P[x < Y_2 < Y_3]$$

$$= \frac{v^{2k_0}}{2}\left[\left(1-u^{\frac{1}{k_0}}\right) - \frac{1}{2}\left(1-u^{\frac{2}{k_0}}\right)\right]$$

$$P_{17} = P[X_2 < X_3 < y < Y_2 < x < Y_3]$$

$$= P[X_2 < X_3 < y]P[y < Y_2 < x]P[x < Y_3]$$

$$= \frac{v^{2k_0}}{2}\left[\left(u^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right) - \left(1-u^{\frac{1}{k_0}}\right)\right]$$

$$P_{18} = P[X_2 < X_3 < y < Y_2 < Y_3 < x]$$

$$= P[X_2 < X_3 < y]P[y < Y_2 < Y_3 < x]$$

$$= \frac{v^{2k_0}}{2}\left[\frac{1}{2}\left(u^{\frac{2}{k_0}} - v^{\frac{2}{k_0}}\right) - v^{\frac{1}{k_0}}\left(u^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right)\right]$$

$$P_{19} = P[X_2 < X_3 < Y_2 < y < x < Y_3]$$

$$= P[X_2 < X_3 < Y_2 < y]P[x < Y_3]$$

$$= \frac{v^{\frac{1}{k_0}+2k_0}}{2\left(2k_0^{\,2}+1\right)}\left(1 - u^{\frac{1}{k_0}}\right)$$

$$P_{20} = P[X_2 < Y_2 < X_3 < y < x < Y_3]$$

$$= P[X_2 < Y_2 < X_3 < y]P[x < Y_3]$$

$$= \frac{k_0^{\,2} v^{\frac{1}{k_0}+2k_0}}{\left(k_0^{\,2}+1\right)\left(2k_0^{\,2}+1\right)}\left(1 - u^{\frac{1}{k_0}}\right)$$

$$P_{21} = P[Y_2 < X_2 < X_3 < y < x < Y_3]$$

$$= P[Y_2 < X_2 < X_3 < y]P[x < Y_3]$$

$$= \frac{k_0^{\,4} v^{\frac{1}{k_0}+2k_0}}{\left(k_0^{\,2}+1\right)\left(2k_0^{\,2}+1\right)}\left(1 - u^{\frac{1}{k_0}}\right)$$

$$P_{22} = P[X_2 < X_3 < Y_2 < y < Y_3 < x]$$

$$= P[X_2 < X_3 < Y_2 < y]P[y < Y_3 < x]$$

$$= \frac{v^{\frac{1}{k_0}+2k_0}}{2\left(2k_0^{\,2}+1\right)}\left(u^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right)$$

$$P_{23} = P[X_2 < Y_2 < X_3 < y < Y_3 < x]$$

$$= P[X_2 < Y_2 < X_3 < y]P[y < Y_3 < x]$$

$$= \frac{k_0^{\,2} v^{\frac{1}{k_0}+2k_0}}{\left(k_0^{\,2}+1\right)\left(2k_0^{\,2}+1\right)}\left(u^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right)$$

$$P_{24} = P[Y_2 < X_2 < X_3 < y < Y_3 < x]$$

$$= P[Y_2 < X_2 < X_3 < y]P[y < Y_3 < x]$$

$$= \frac{k_0^{\,4} v^{\frac{1}{k_0}+2k_0}}{\left(k_0^{\,2}+1\right)\left(2k_0^{\,2}+1\right)}\left(u^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right)$$

$$P_{25} = P[X_2 < X_3 < Y_2 < Y_3 < y < x]$$

$$= P[X_2 < X_3 < Y_2 < Y_3 < y]$$

$$= \int_{-\infty}^{y}\int_{-\infty}^{y_3}\int_{-\infty}^{y_2} P(X_2 < x_3)dF(x_3)dG(y_2)dG(y_3)$$

$$= \int_0^v\int_0^s\int_0^r n^{k_0} k_0 n^{k_0-1}\frac{1}{k_0} r^{\frac{1}{k_0}-1}\frac{1}{k_0} s^{\frac{1}{k_0}-1} dndrds$$

$$= \frac{v^{\frac{2}{k_0}+2k_0}}{2\left(2k_0^{\,2}+1\right)\left(2k_0^{\,2}+2\right)}$$

$$P_{26} = P[X_2 < Y_2 < X_3 < Y_3 < y < x]$$

$$= P[X_2 < Y_2 < X_3 < Y_3 < y]$$

$$= \int_{-\infty}^{y}\int_{-\infty}^{y_3}\int_{-\infty}^{x_3} P(X_2 < y_2)dG(y_2)dF(x_3)dG(y_3)$$

$$= \int_0^v\int_0^s\int_0^n r^{k_0}\frac{1}{k_0} r^{\frac{1}{k_0}-1} k_0 n^{k_0-1}\frac{1}{k_0} s^{\frac{1}{k_0}-1} drdnds$$

$$= \frac{k_o^{\,2} v^{\frac{2}{k_0}+2k_0}}{\left(k_0^{\,2}+1\right)\left(2k_0^{\,2}+1\right)\left(2k_0^{\,2}+2\right)}$$

$$P_{27} = P[Y_2 < X_2 < X_3 < Y_3 < y < x]$$

$$= P[Y_2 < X_2 < X_3 < Y_3 < y]$$

$$= \int_{-\infty}^{y}\int_{-\infty}^{y_3}\int_{-\infty}^{x_3} P(Y_2 < x_2)dF(x_2)dF(x_3)dG(y_3)$$

$$= \int_{0}^{v}\int_{0}^{s}\int_{0}^{n} m^{\frac{1}{k_0}} k_0 m^{k_0-1} k_0 n^{k_0-1} \frac{1}{k_0} s^{\frac{1}{k_0}-1}\, dmdnds$$

$$= \frac{k_o{}^4 v^{\frac{2}{k_0}+2k_0}}{\left(k_0{}^2+1\right)\left(2k_0{}^2+1\right)\left(2k_0{}^2+2\right)}$$

Now Adding the 27 terms and multiplying by 4 we have for y < x

$$P[Med(x,X_2,X_3) < Med(y,Y_2,Y_3)] = 4P_1 + 4P_2 + ... + 4P_{27}$$

$$= \frac{4u^{\frac{1}{k_0}+2k_0}}{\left(2k_0{}^2+1\right)} - \frac{4u^{\frac{1}{k_0}+k_0}}{\left(k_0{}^2+1\right)} + \frac{4u^{\frac{2}{k_0}+k_0}}{\left(k_0{}^2+2\right)} + \frac{4k_0{}^2v^{\frac{1}{k_0}+2k_0}}{\left(2k_0{}^2+1\right)} - \frac{2u^{\frac{2}{k_0}+2k_0}}{\left(k_0{}^2+1\right)} - \frac{2k_0{}^2v^{\frac{2}{k_0}+2k_0}}{\left(k_0{}^2+1\right)} + \frac{\left(7k_0{}^2+2\right)}{\left(k_0{}^2+1\right)\left(k_0{}^2+2\right)\left(2k_0{}^2+1\right)}$$

where $F(x) = u \quad F(y) = v$.

Thus,

$$\int_{-\infty}^{\infty}\int_{y}^{\infty} P^2[Med(x,X_2,X_3) < Med(y,Y_2,Y_3)]dF(x)dG(y)$$

$$-\infty < y < x < \infty$$

$$= \int_{-\infty}^{\infty}\int_{y}^{x}\left(\frac{4u^{\frac{1}{k_0}+2k_0}}{\left(2k_0{}^2+1\right)} - \frac{4u^{\frac{1}{k_0}+k_0}}{\left(k_0{}^2+1\right)} + \frac{4u^{\frac{2}{k_0}+k_0}}{\left(k_0{}^2+2\right)} + \frac{4k_0{}^2v^{\frac{1}{k_0}+2k_0}}{\left(2k_0{}^2+1\right)} - \frac{2u^{\frac{2}{k_0}+2k_0}}{\left(k_0{}^2+1\right)} - \frac{2k_0{}^2v^{\frac{2}{k_0}+2k_0}}{\left(k_0{}^2+1\right)} + \frac{\left(7k_0{}^2+2\right)}{\left(k_0{}^2+1\right)\left(k_0{}^2+2\right)\left(2k_0{}^2+1\right)}\right)^2 dF(x)dG(y)$$

$$-\infty < y < x < \infty$$

$$\int_{0}^{1}\int_{v}^{1}\left(\frac{4u^{\frac{1}{k_0}+2k_0}}{\left(2k_0{}^2+1\right)} - \frac{4u^{\frac{1}{k_0}+k_0}}{\left(k_0{}^2+1\right)} + \frac{4u^{\frac{2}{k_0}+k_0}}{\left(k_0{}^2+2\right)} + \frac{4k_0{}^2v^{\frac{1}{k_0}+2k_0}}{\left(2k_0{}^2+1\right)} - \frac{2u^{\frac{2}{k_0}+2k_0}}{\left(k_0{}^2+1\right)} - \frac{2k_0{}^2v^{\frac{2}{k_0}+2k_0}}{\left(k_0{}^2+1\right)} + \frac{\left(7k_0{}^2+2\right)}{\left(k_0{}^2+1\right)\left(k_0{}^2+2\right)\left(2k_0{}^2+1\right)}\right)^2 k_0 u^{k_0-1}\frac{1}{k_0}v^{\frac{1}{k_0}-1}\, dudv$$

$$-\infty < y < x < \infty$$

where $F(x) = u \quad F(y) = v$

$$= \frac{\frac{1}{15}k_0{}^2\left(21708k_0{}^{12} + 366506k_0{}^{10} + 1428131k_0{}^8 + 2448716k_0{}^6 + 2131563k_0{}^4 + 928800k_0{}^2 + 162000\right)}{\left[\left(3k_0{}^2+4\right)\left(2k_0{}^2+3\right)\left(5k_0{}^2+4\right)\left(4k_0{}^2+5\right)\left(4k_0{}^2+3\right)\left(3k_0{}^2+5\right)\left(k_0{}^2+1\right)\left(5k_0{}^2+3\right)\right]}$$

Thus adding the first and the second case, we have

$$P[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_4,X_5)<Med(Y_1,Y_4,Y_5)]$$

$$=\left(\frac{\frac{1}{45}\left[\begin{array}{l}153628876800000k_0^{46}+1890732533760000k_0^{44}+10938975605971120k_0^{42}+40565384170799040k_0^{40}\\+112197870855205920k_0^{38}+254612585797915920k_0^{36}+495823986637551480k_0^{34}\\+828010283602098480k_0^{32}+1161194259896960540k_0^{30}+1342270878097274560k_0^{28}\\+1266433762259422790k_0^{26}+972041941346088790k_0^{24}+606657564916585190k_0^{22}\\+308191765280108720k_0^{20}+126812259131803330k_0^{18}+42268610705535170k_0^{16}\\+11324996413604470k_0^{14}+2413529690935250k_0^{12}+402618804260200k_0^{10}+51311727898752k_0^{8}\\+4814609834886k_0^{6}+312704402400k_0^{4}+12534912000k_0^{2}+23328000\end{array}\right]}{\left[\begin{array}{l}(2k_0^2+3)(4k_0^2+5)(7k_0^2+2)(4k_0^2+3)(3k_0^2+1)^2(4k_0^2+1)(6k_0^2+1)(5k_0^2+2)(5k_0^2+4)(2k_0^2+1)^3\\(3k_0^2+4)(7k_0^2+3)(3k_0^2+2)(k_0^2+1)^3(3k_0^2+5)(k_0^2+2)(5k_0^2+3)(7k_0^2+1)(5k_0^2+1)\end{array}\right]}\right)$$

$$+\left(\frac{\frac{1}{15}k_0^2(21708k_0^{12}+366506k_0^{10}+1428131k_0^8+2448716k_0^6+2131563k_0^4+928800k_0^2+162000)}{\left[(3k_0^2+4)(2k_0^2+3)(5k_0^2+4)(4k_0^2+5)(4k_0^2+3)(3k_0^2+5)(k_0^2+1)^3(5k_0^2+3)\right]}\right)$$

$$=\frac{\frac{1}{45}\left[\begin{array}{l}55163648888848k_0^8+26847131597566k_0^{12}+12806555134066900k_0^{14}-153628876800000k_0^{46}\\-189073253376000k_0^{44}-72771156707375980k_0^{22}-48566214790261400k_0^{16}+364367407756122840k_0^{20}\\+147952444159059100k_0^{18}+99423655223297772k_0^{32}+16485571190180479k_0^{28}+15498739547067562k_0^{26}\\+11794763188377301412k_0^{24}+14193742896088904k_0^{30}+10967925074443200k_0^{42}+11794504485856488k_0^{38}\\+2811416927832536k_0^{36}+574993721567948920k_0^{36}+30978415131875282k_0^{34}+4124831874890240k_0^{40}\\+440348362647160k_0^{10}+32438395400k_0^4+5084942997500k_0^6+127681920k_0^2+23328000\end{array}\right]}{\left[\begin{array}{l}(2k_0^2+3)(4k_0^2+5)(7k_0^2+2)(4k_0^2+3)(3k_0^2+1)^2(4k_0^2+1)(6k_0^2+1)(5k_0^2+2)(5k_0^2+4)(2k_0^2+1)^3(3k_0^2+4)\\(7k_0^2+3)(3k_0^2+2)^2(k_0^2+1)^3(3k_0^2+5)(k_0^2+2)(5k_0^2+3)(7k_0^2+1)(5k_0^2+1)\end{array}\right]}.$$

Thus,

$$\xi^*_{1,1}=[Med(X_1,X_2,X_3)<Med(Y_1,Y_2,Y_3),Med(X_1,X_4,X_5)<Med(Y_1,Y_4,Y_5)]-[E(U)]^2$$

$$=\left[\frac{\frac{1}{45}\left[\begin{array}{l}55163648888848k_0^8+26847131597566k_0^{12}+12806555134066900k_0^{14}-153628876800000k_0^{46}\\-189073253376000k_0^{44}-72771156707375980k_0^{22}-48566214790261400k_0^{16}+364367407756122840k_0^{20}\\+147952444159059100k_0^{18}+99423655223297772k_0^{32}+16485571190180479k_0^{28}+15498739547067562k_0^{26}\\+11794763188377301412k_0^{24}+14193742896088904k_0^{30}+10967925074443200k_0^{42}+11794504485856488k_0^{38}\\+2811416927832536k_0^{36}+574993721567948920k_0^{36}+30978415131875282k_0^{34}+4124831874890240k_0^{40}\\+440348362647160k_0^{10}+32438395400k_0^4+5084942997500k_0^6+127681920k_0^2+23328000\end{array}\right]}{\left[\begin{array}{l}(2k_0^2+3)(4k_0^2+5)(7k_0^2+2)(4k_0^2+3)(3k_0^2+1)^2(4k_0^2+1)(6k_0^2+1)(5k_0^2+2)(5k_0^2+4)(2k_0^2+1)^3(3k_0^2+4)\\(7k_0^2+3)(3k_0^2+2)^2(k_0^2+1)^3(3k_0^2+5)(k_0^2+2)(5k_0^2+3)(7k_0^2+1)(5k_0^2+1)\end{array}\right]}\right]-\left[\frac{(19k_0^2+6)}{(k_0^2+1)(3k_0^2+2)(2k_0^2+3)}\right]^2$$

$$= \frac{\frac{2}{45}k_0^2 \begin{pmatrix} -74056000390525k_0^{14} + 667078530048000k_0^{46} + 4565442324216960k_0^{44} - 476830871860400k_0^{22} \\ -531858308053724k_0^{16} - 3508040908867240k_0^{20} - 1854439539223396k_0^{18} + 166479084767335769k_0^{28} \\ +1028419200k_0^2 + 46088663040000k_0^{48} + 1496467465720k_0^8 + 6612982666087k_0^{12} \\ +20178103372439618k_0^{24} + 74922467099098906k_0^{26} + 19637585223733152k_0^{42} \\ +13360723327173139 2k_0^{38} + 23051847793175810 4k_0^{36} + 309784151251875282k_0^{34} \\ +59385938598813781171856 0k_0^{40} + 5943867213656k_0^{10} + 19843121520k_0^4 + 266760378580714909k_0^{30} \\ +32583616685770902 6k_0^{32} + 219359561424k_0^6 + 23328000 \end{pmatrix}}{\begin{bmatrix} (2k_0^2+3)^3(4k_0^2+5)(7k_0^2+2)(4k_0^2+3)(3k_0^2+1)^3(4k_0^2+1)(6k_0^2+1)(5k_0^2+2)(5k_0^2+4)(2k_0^2+1)^3(3k_0^2+4) \\ (7k_0^2+3)(3k_0^2+2)^3(k_0^2+1)^3(3k_0^2+5)(k_0^2+2)(5k_0^2+3)(7k_0^2+1)(5k_0^2+1) \end{bmatrix}}.$$

$$\xi^*_{2,2} = Cov\left[h(X_1, X_2, X_3; Y_1, Y_2, Y_3), h(X_1, X_2 X_4, ; Y_1, Y_2, Y_4)\right]$$

$$= E\left[h(X_1, X_2, X_3; Y_1, Y_2, Y_3) h(X_1, X_2 X_4, ; Y_1, Y_2, Y_4) - \{E[h(X_1, X_2, X_3; Y_1, Y_2, Y_3)]\}^2\right]$$

$$= P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_2, X_4) < Med(Y_1, Y_2, Y_4)] - [E(U)]^2$$

Now,

$$P[Med(X_1, X_2, X_3) < Med(Y_1, Y_2, Y_3), Med(X_1, X_2, X_4) < Med(Y_1, Y_2, Y_4)]$$

$$= \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] dF(x_1) dF(x_2) dG(y_1) dG(y_2)$$

$$= Q_1 + Q_2 + ... + Q_{20}$$

where

$$Q_1 = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] dF(x_1) dF(x_2) dG(y_1) dG(y_2)$$

$x_1 < x_2 < y_1 < y_2$

$$Q_2 = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] dF(x_1) dF(x_2) dG(y_1) dG(y_2)$$

$x_1 < x_2 < y_2 < y_1$

$$Q_3 = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] dF(x_1) dF(x_2) dG(y_1) dG(y_2)$$

$x_1 < y_1 < x_2 < y_2$

$$Q_4 = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] dF(x_1) dF(x_2) dG(y_1) dG(y_2)$$

$x_1 < y_2 < x_2 < y_1$

$$Q_5 = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dG(y_1)dG(y_2)$$

$$x_1 < y_1 < y_2 < x_2$$

$$Q_6 = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dG(y_1)dG(y_2)$$

$$x_1 < y_2 < y_1 < x_2$$

$$Q_7 = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dG(y_1)dG(y_2)$$

$$x_2 < x_1 < y_1 < y_2$$

$$Q_8 = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dG(y_1)dG(y_2)$$

$$x_2 < x_1 < y_2 < y_1$$

$$Q_9 = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dG(y_1)dG(y_2)$$

$$x_2 < y_1 < x_1 < y_2$$

$$Q_{10} = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dG(y_1)dG(y_2)$$

$$x_2 < y_2 < x_1 < y_1$$

$$Q_{11} = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dG(y_1)dG(y_2)$$

$$x_2 < y_1 < y_2 < x_1$$

$$Q_{12} = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dG(y_1)dG(y_2)$$

$$x_2 < y_2 < y_1 < x_1$$

$$Q_{13} = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dG(y_1)dG(y_2)$$

$$y_1 < x_1 < y_2 < x_2$$

$$Q_{14} = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dG(y_1)dG(y_2)$$

$$y_1 < x_2 < y_2 < x_1$$

$$Q_{15} = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dG(y_1)dG(y_2)$$

$$y_1 < x_1 < x_2 < y_2$$

$$Q_{16} = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dG(y_1)dG(y_2)$$

$$y_1 < x_2 < x_1 < y_2$$

$$Q_{17} = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dG(y_1)dG(y_2)$$

$$y_2 < x_1 < y_1 < x_2$$

$$Q_{18} = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dG(y_1)dG(y_2)$$

$$y_2 < x_2 < y_1 < x_1$$

$$Q_{19} = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dG(y_1)dG(y_2)$$

$$y_2 < x_1 < x_2 < y_1$$

$$Q_{20} = \int\int\int\int P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]dF(x_1)dF(x_2)dG(y_1)dG(y_2)$$

$$y_2 < x_2 < x_1 < y_1$$

Now consider case 1) $x_1 < x_2 < y_1 < y_2$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = R_1 + R_2 + ... + R_{30}$$

where

$$R_1 = P[x_1 < x_2 < X_3 < y_1 < y_2 < Y_3] \qquad R_2 = P[x_1 < x_2 < y_1 < X_3 < y_2 < Y_3]$$

$$R_3 = P[x_1 < x_2 < y_1 < y_2 < X_3 < Y_3] \qquad R_4 = P[x_1 < x_2 < y_1 < y_2 < Y_3 < X_3]$$

$$R_5 = P[x_1 < x_2 < Y_3 < y_1 < y_2 < X_3] \qquad R_6 = P[x_1 < x_2 < y_1 < Y_3 < y_2 < X_3]$$

$$R_7 = P[x_1 < X_3 < x_2 < y_1 < y_2 < Y_3] \qquad R_8 = P[x_1 < X_3 < x_2 < y_1 < Y_3 < y_2]$$

$$R_9 = P[x_1 < X_3 < x_2 < Y_3 < y_1 < y_2] \qquad R_{10} = P[x_1 < Y_3 < x_2 < X_3 < y_1 < y_2]$$

$$R_{11} = P[x_1 < Y_3 < x_2 < y_1 < X_3 < y_2] \qquad R_{12} = P[x_1 < Y_3 < x_2 < y_1 < y_2 < X_3]$$

$$R_{13} = P[x_1 < X_3 < Y_3 < x_2 < y_1 < y_2] \qquad R_{14} = P[x_1 < Y_3 < X_3 < x_2 < y_1 < y_2]$$

$$R_{15} = P[X_3 < x_1 < x_2 < y_1 < y_2 < Y_3] \qquad R_{16} = P[X_3 < x_1 < x_2 < y_1 < Y_3 < y_2]$$

$$R_{17} = P[Y_3 < x_1 < x_2 < y_1 < y_2 < X_3] \qquad R_{18} = P[Y_3 < x_1 < x_2 < y_1 < X_3 < y_2]$$

$$R_{19} = P[X_3 < x_1 < x_2 < Y_3 < y_1 < y_2] \qquad R_{20} = P[Y_3 < x_1 < x_2 < X_3 < y_1 < y_2]$$

$$R_{21} = P[X_3 < x_1 < Y_3 < x_2 < y_1 < y_2] \qquad R_{22} = P[Y_3 < x_1 < X_3 < x_2 < y_1 < y_2]$$

$$R_{23} = P[X_3 < Y_3 < x_1 < x_2 < y_1 < y_2] \qquad R_{24} = P[Y_3 < X_3 < x_1 < x_2 < y_1 < y_2]$$

$$R_{25} = P[x_1 < x_2 < Y_3 < y_1 < X_3 < y_2] \qquad R_{26} = P[x_1 < x_2 < X_3 < y_1 < Y_3 < y_2]$$

$$R_{27} = P[x_1 < x_2 < X_3 < Y_3 < y_1 < y_2] \qquad R_{28} = P[x_1 < x_2 < Y_3 < X_3 < y_1 < y_2]$$

$$R_{29} = P[x_1 < x_2 < y_1 < X_3 < Y_3 < y_2] \qquad R_{30} = P[x_1 < x_2 < y_1 < Y_3 < X_3 < y_2].$$

Now consider

$$R_1 = P[x_1 < x_2 < X_3 < y_1 < y_2 < Y_3]$$

$$= P[x_2 < X_3 < y_1]P[y_2 < Y_3]$$

$$= \left[F^{k_0}(y_1) - F^{k_0}(x_1)\right]\left(1 - F^{\frac{1}{k_0}}(y_2)\right)$$

$$= \left(v^{k_0} - m^{k_0}\right)\left(1 - r^{\frac{1}{k_0}}\right)$$

$$R_2 = P[x_1 < x_2 < y_1 < X_3 < y_2 < Y_3]$$

$$= P[y_1 < X_3 < y_2 < Y_3]$$

$$= P[y_1 < X_3 < y_2]P[y_2 < Y_3]$$

$$= \left(r^{k_0} - v^{k_0}\right)\left(1 - r^{\frac{1}{k_0}}\right)$$

$$R_3 = P[x_1 < x_2 < y_1 < y_2 < X_3 < Y_3]$$

$$= P[y_2 < X_3 < Y_3]$$

$$= \left[\left(1 - r^{k_0}\right) - \frac{1}{2}\left(1 - r^{2k_0}\right)\right]$$

$$R_4 = P[x < x_2 < y_1 < y_2 < Y_3 < X_3]$$

$$= P[y_2 < Y_3 < X_3]$$

$$= \left[\left(1 - r^{\frac{1}{k_0}}\right) - \frac{1}{\left(k_0^{\,2} + 1\right)}\left(1 - r^{\frac{1}{k_0} + k_0}\right)\right]$$

$$R_5 = P[x_1 < x_2 < Y_3 < y_1 < y_2 < X_3]$$

$$= P[x_2 < Y_3 < y_1]P[y_2 < X_3]$$

$$= \left[v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right]\left[1 - r^{k_0}\right]$$

$$R_6 = P[x_1 < x_2 < y_1 < Y_3 < y_2 < X_3]$$

$$= P[y_1 < Y_3 < y_2]P[y_2 < X_3]$$

$$= \left[r^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right]\left[1 - r^{k_0}\right]$$

$$R_7 = P[x_1 < X_3 < x_2 < y_1 < y_2 < Y_3]$$

$$= P[x_1 < X_3 < x_2]P[y_2 < Y_3]$$

$$= \left[m^{k_0} - u^{k_0}\right]\left[1 - r^{\frac{1}{k_0}}\right]$$

$$R_8 = P[x_1 < X_3 < x_2 < y_1 < Y_3 < y_2]$$

$$= P[x_1 < X_3 < x_2]P[y_1 < Y_3 < y_2]$$

$$= \left[m^{k_0} - u^{k_0}\right]\left[r^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right]$$

$$R_9 = P[x_1 < X_3 < x_2 < Y_3 < y_1 < y_2]$$

$$= P[x_1 < X_3 < x_2 < Y_3 < y_1]$$

$$= P[x_1 < X_3 < x_2]P[x_2 < Y_3 < y_1]$$

$$= \left[m^{k_0} - u^{k_0}\right]\left[v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right]$$

$$R_{10} = P[x_1 < Y_3 < x_2 < X_3 < y_1 < y_2]$$

$$= P[x_1 < Y_3 < x_2]P[x_2 < X_3 < y_1]$$

$$= \left[m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right]\left[v^{k_0} - m^{k_0}\right]$$

$$R_{11} = P[x_1 < Y_3 < x_2 < y_1 < X_3 < y_2]$$

$$= P[x_1 < Y_3 < x_2]P[y_1 < X_3 < y_2]$$

$$= \left[m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right]\left[r^{k_0} - v^{k_0}\right]$$

$$R_{12} = P[x_1 < Y_3 < x_2 < y_1 < y_2 < X_3]$$

$$= P[x_1 < Y_3 < x_2]P[y_2 < X_3]$$

$$= \left[m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right]\left[1 - r^{k_0}\right]$$

$$R_{13} = P[x_1 < X_3 < Y_3 < x_2 < y_1 < y_2]$$

$$= P[x_1 < X_3 < Y_3 < x_2]$$

$$= \left[\frac{\left(m^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right)}{\left(k_0^{\,2}+1\right)} - u^{k_0}\left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)\right]$$

$$R_{14} = P[x_1 < Y_3 < X_3 < x_2 < y_1 < y_2]$$

$$= P[x_1 < Y_3 < X_3 < x_2]$$

$$= \left[\frac{k_0^2 \left(m^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0} \right)}{\left(k_0^2+1\right)} - u^{\frac{1}{k_0}} \left(m^{k_0} - u^{k_0}\right) \right]$$

$$R_{15} = P[X_3 < x_1 < x_2 < y_1 < y_2 < Y_3]$$

$$= P[X_3 < x_1]P[y_2 < Y_3]$$

$$= u^{k_0}\left(1 - r^{\frac{1}{k_0}}\right)$$

$$R_{16} = P[X_3 < x_1 < x_2 < y_1 < Y_3 < y_2]$$

$$= P[X_3 < x_1]P[y_1 < Y_3 < y_2]$$

$$= u^{k_0}\left(r^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right)$$

$$R_{17} = P[Y_3 < x_1 < x_2 < y_1 < y_2 < X_3]$$

$$= P[Y_3 < x_1]P[y_2 < X_3]$$

$$= u^{\frac{1}{k_0}}\left(1 - r^{k_0}\right)$$

$$R_{18} = P[Y_3 < x_1 < x_2 < y_1 < X_3 < y_2]$$

$$= P[Y_3 < x_1]P[y_1 < X_3 < y_2]$$

$$= u^{\frac{1}{k_0}}\left(r^{k_0} - v^{k_0}\right)$$

$$R_{19} = P[X_3 < x_1 < x_2 < Y_3 < y_1 < y_2]$$

$$= P[X_3 < x_1]P[x_2 < Y_3 < y_1]$$

$$= u^{k_0}\left(v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}} \right)$$

$$R_{20} = P[Y_3 < x_1 < x_2 < X_3 < y_1 < y_2]$$

$$= P[Y_3 < x_1]P[x_2 < X_3 < y_1]$$

$$= u^{\frac{1}{k_0}}\left(v^{k_0} - m^{k_0}\right)$$

$$R_{21} = P[X_3 < x_1 < Y_3 < x_2 < y_1 < y_2]$$

$$= P[X_3 < x_1 < Y_3 < x_2]$$

$$= P[X_3 < x_1]P[x_1 < Y_3 < x_2]$$

$$= u^{k_0}\left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}} \right)$$

$$R_{22} = P[Y_3 < x_1 < X_3 < x_2 < y_1 < y_2]$$

$$= P[Y_3 < x_1]P[x_1 < X_3 < x_2]$$

$$= u^{\frac{1}{k_0}}\left(m^{k_0} - u^{k_0}\right)$$

$$R_{23} = P[X_3 < Y_3 < x_1 < x_2 < y_1 < y_2]$$

$$= P[X_3 < Y_3 < x_1]$$

$$= \frac{u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}$$

$$R_{24} = P[Y_3 < X_3 < x_1 < x_2 < y_1 < y_2]$$

$$= P[Y_3 < X_3 < x_1]$$

$$= \frac{k_0^{\,2}u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}$$

$$R_{25} = P[x_1 < x_2 < Y_3 < y_1 < X_3 < y_2]$$

$$= P[x_2 < Y_3 < y_1]P[y_1 < X_3 < y_2]$$

$$= \left(v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right)\left(r^{k_0} - v^{k_0}\right)$$

$$R_{26} = P[x_1 < x_2 < X_3 < y_1 < Y_3 < y_2]$$

$$= P[x_2 < X_3 < y_1]P[y_1 < Y_3 < y_2]$$

$$= \left(v^{k_0} - m^{k_0}\right)\left(r^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right)$$

$$R_{27} = P[x_1 < x_2 < X_3 < Y_3 < y_1 < y_2]$$

$$= P[x_2 < X_3 < Y_3 < y_1]$$

$$= \left[v^{\frac{1}{k_0}}\left(v^{k_0} - m^{k_0}\right) - \frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)}\left(v^{\frac{1}{k_0}+k_0} - m^{\frac{1}{k_0}+k_0}\right)\right]$$

$$R_{28} = P[x_1 < x_2 < Y_3 < X_3 < y_1 < y_2]$$

$$= P[x_2 < Y_3 < X_3 < y_1]$$

$$= \left[v^{k_0}\left(v^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right) - \frac{1}{\left(k_0^{\,2}+1\right)}\left(v^{\frac{1}{k_0}+k_0} - m^{\frac{1}{k_0}+k_0}\right)\right]$$

$$R_{29} = P[x_1 < x_2 < y_1 < X_3 < Y_3 < y_2]$$

$$= P[y_1 < X_3 < Y_3 < y_2]$$

$$= \frac{1}{\left(k_0^{\,2}+1\right)}\left(r^{\frac{1}{k_0}+k_0} - v^{\frac{1}{k_0}+k_0}\right) - v^{k_0}\left(r^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right)$$

$$R_{30} = P[x_1 < x_2 < y_1 < Y_3 < X_3 < y_2]$$

$$= P[y_1 < Y_3 < X_3 < y_2]$$

$$= \frac{k_{0^2}}{\left(k_0^{\,2}+1\right)}\left(r^{\frac{1}{k_0}+k_0} - v^{\frac{1}{k_0}+k_0}\right) - v^{\frac{1}{k_0}}\left(r^{k_0} - v^{k_0}\right)$$

where $F(x_1)=u \quad F(x_2)=m \quad F(x_3)=n \quad F(y_1)=v \quad F(y_2)=r \quad F(y_3)=s$.

Adding the 30 terms we have for $x_1 < x_2 < y_1 < y_2$

$$P[Med(x_1,x_2,X_3)< Med(y_1,y_2,Y_3)]= R_1+R_2+.....+R_{30}$$

$$=\frac{3}{2}+\frac{r^{2k_0}}{2}-\frac{1}{(k_0^{\,2}+1)}-\frac{k_0^{\,2}r^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)}.$$

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_2}\int_{-\infty}^{y_1}\int_{-\infty}^{x_2} P^2[Med(x_1,x_2,X_3)< Med(y_1,y_2,Y_3)]dF^{k_0}(x_1)dF^{k_0}(x_2)dF^{\frac{1}{k_0}}(y_1)dF^{\frac{1}{k_0}}(y_2)$$

$$-\infty < x_1 < x_2 < y_1 < y_2 < \infty$$

$$=\int_0^1\int_0^r\int_0^v\int_0^m\left[\frac{3}{2}+\frac{r^{2k_0}}{2}-\frac{1}{(k_0^{\,2}+1)}-\frac{k_0^{\,2}r^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)}\right]^2 k_0u^{k_0-1}k_0m^{k_0-1}\frac{1}{k_0}v^{\frac{1}{k_0}-1}\frac{1}{k_0}r^{\frac{1}{k_0}-1}\,dudmdvdr$$

$$-\infty < x_1 < x_2 < y_1 < y_2 < \infty$$

$$=\frac{1}{24}\left[\frac{(18+213k_0^{\,10}+150k_0^{\,2}+642k_0^{\,8}+791k_0^{\,6}+490k_0^{\,4})}{(2k_0^{\,2}+1)^2(k_0^{\,2}+1)^3(3k_0^{\,2}+1)(5k_0^{\,2}+3)}\right].$$

after simplifying.

Consider case 2) $x_1 < x_2 < y_2 < y_1$

Interchanging y_1 and y_2 in case 1 and simplifying we get,

Thus,

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_1}\int_{-\infty}^{y_2}\int_{-\infty}^{x_2} P^2[Med(x_1,x_2,X_3)< Med(y_1,y_2,Y_3)]dF^{k_0}(x_1)dF^{k_0}(x_2)dF^{\frac{1}{k_0}}(y_2)dF^{\frac{1}{k_0}}(y_1)$$

$$-\infty < x_1 < x_2 < y_2 < y_1 < \infty$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{v}\int_{-\infty}^{r}\int_{-\infty}^{m}\left[\frac{3}{2}+\frac{v^{2k_0}}{2}-\frac{1}{\left(k_0^{\,2}+1\right)}-\frac{k_0^{\,2}v^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}\right]^2 k_0u^{k_0-1}k_0m^{k_0-1}\frac{1}{k_0}r^{\frac{1}{k_0}-1}\frac{1}{k_0}v^{\frac{1}{k_0}-1}\,dudmdrdv$$

$$-\infty < x_1 < x_2 < y_2 < y_1 < \infty$$

$$= \frac{1}{24}\left[\frac{\left(18+213k_0^{\,10}+150k_0^{\,2}+642k_0^{\,8}+791k_0^{\,6}+490k_0^{\,4}\right)}{\left(2k_0^{\,2}+1\right)^2\left(k_0^{\,2}+1\right)^3\left(3k_0^{\,2}+1\right)\left(5k_0^{\,2}+3\right)}\right].$$

Consider case 3) $x_1 < y_1 < x_2 < y_2$

$$P[Med(x_1,x_2,X_3) < Med(y_1,y_2,Y_3)] = S_1 + S_2 + \ldots\ldots + S_{21}$$

where

$S_1 = P[X_3 < x_1 < y_1 < x_2 < y_2 < Y_3]$ $\quad$ $S_2 = P[x_1 < X_3 < y_1 < x_2 < y_2 < Y_3]$

$S_3 = P[x_1 < y_1 < X_3 < x_2 < y_2 < Y_3]$ $\quad$ $S_4 = P[x_1 < y_1 < x_2 < X_3 < y_2 < Y_3]$

$S_5 = P[X_3 < x_1 < y_1 < x_2 < Y_3 < y_2]$ $\quad$ $S_6 = P[x_1 < X_3 < y_1 < x_2 < Y_3 < y_2]$

$S_7 = P[x_1 < y_1 < X_3 < x_2 < Y_3 < y_2]$ $\quad$ $S_8 = P[x_1 < y_1 < x_2 < Y_3 < y_2 < X_3]$

$S_9 = P[X_3 < x_1 < y_1 < Y_3 < x_2 < y_2]$ $\quad$ $S_{10} = P[x_1 < X_3 < y_1 < Y_3 < x_2 < y_2]$

$S_{11} = P[X_3 < x_1 < Y_3 < y_1 < x_2 < y_2]$ $\quad$ $S_{12} = P[Y_3 < x_1 < X_3 < y_1 < x_2 < y_2]$

$S_{13} = P[X_3 < Y_3 < x_1 < y_1 < x_2 < y_2]$ $\quad$ $S_{14} = P[Y_3 < X_3 < x_1 < y_1 < x_2 < y_2]$

$S_{15} = P[x_1 < X_3 < Y_3 < y_1 < x_2 < y_2]$ $\quad$ $S_{16} = P[x_1 < Y_3 < X_3 < y_1 < x_2 < y_2]$

$S_{17} = P[x_1 < y_1 < X_3 < Y_3 < x_2 < y_2]$ $\quad$ $S_{18} = P[x_1 < y_1 < x_2 < X_3 < Y_3 < y_2]$

$S_{19} = P[x_1 < y_1 < x_2 < Y_3 < X_3 < y_2]$ $\quad$ $S_{20} = P[x_1 < y_1 < x_2 < y_2 < X_3 < Y_3]$

$S_{21} = P[x_1 < y_1 < x_2 < y_2 < Y_3 < X_3]$.

Now Consider

$S_1 = P[X_3 < x_1 < y_1 < x_2 < y_2 < Y_3]$

$$= P[X_3 < x_1]P[y_2 < Y_3]$$

$$= u^{k_0}\left(1 - r^{\frac{1}{k_0}}\right)$$

$$S_2 = P[x_1 < X_3 < y_1 < x_2 < y_2 < Y_3]$$

$$= P[x_1 < X_3 < y_1]P[y_2 < Y_3]$$

$$= \left(v^{k_0} - u^{k_0}\right)\left(1 - r^{\frac{1}{k_0}}\right)$$

$$S_3 = P[x_1 < y_1 < X_3 < x_2 < y_2 < Y_3]$$

$$= P[y_1 < X_3 < x_2]P[y_2 < Y_3]$$

$$= \left(m^{k_0} - v^{k_0}\right)\left(1 - r^{\frac{1}{k_0}}\right)$$

$$S_4 = P[x_1 < y_1 < x_2 < X_3 < y_2 < Y_3]$$

$$= P[x_2 < X_3 < y_2 < Y_3]$$

$$= P[x_2 < X_3 < y_2]P[y_2 < Y_3]$$

$$= \left(r^{k_0} - m^{k_0}\right)\left(1 - r^{\frac{1}{k_0}}\right)$$

$$S_5 = P[X_3 < x_1 < y_1 < x_2 < Y_3 < y_2]$$

$$= P[X_3 < x_1]P[x_2 < Y_3 < y_2]$$

$$= u^{k_0}\left(r^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right)$$

$$S_6 = P[x_1 < X_3 < y_1 < x_2 < Y_3 < y_2]$$

$$= P[x_1 < X_3 < y_1]P[x_2 < Y_3 < y_2]$$

$$= \left(v^{k_0} - u^{k_0}\right)\left(r^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right)$$

$$S_7 = P[x_1 < y_1 < X_3 < x_2 < Y_3 < y_2]$$

$$= P[y_1 < X_3 < x_2]P[x_2 < Y_3 < y_2]$$

$$= \left(m^{k_0} - v^{k_0}\right)\left(r^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right)$$

$$S_8 = P[x_1 < y_1 < x_2 < Y_3 < y_2 < X_3]$$

$$= P[x_2 < Y_3 < y_2 < X_3]$$

$$= P[x_2 < Y_3 < y_2]P[y_2 < X_3]$$

$$= \left(r^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right)\left(1 - r^{k_0}\right)$$

$$S_9 = P[X_3 < x_1 < y_1 < Y_3 < x_2 < y_2]$$

$$= P[X_3 < x_1]P[y_1 < Y_3 < x_2]$$

$$= u^{k_0}\left(m^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right)$$

$$S_{10} = P[x_1 < X_3 < y_1 < Y_3 < x_2 < y_2]$$

$$= P[x_1 < X_3 < y_1 < Y_3 < x_2]$$

$$= P[x_1 < X_3 < y_1]P[y_1 < Y_3 < x_2]$$

$$= \left(v^{k_0} - u^{k_0}\right)\left(m^{\frac{1}{k_0}} - v^{\frac{1}{k_0}}\right)$$

$$S_{11} = P[X_3 < x_1 < Y_3 < y_1 < x_2 < y_2]$$

$$= P[X_3 < x_1]P[x_1 < Y_3 < y_1]$$

$$= u^{k_0}\left(v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}} \right)$$

$$S_{12} = P[Y_3 < x_1 < X_3 < y_1 < x_2 < y_2]$$

$$= P[Y_3 < x_1]P[x_1 < X_3 < y_1]$$

$$= u^{\frac{1}{k_0}}\left(v^{k_0} - u^{k_0} \right)$$

$$S_{13} = P[X_3 < Y_3 < x_1 < y_1 < x_2 < y_2]$$

$$= P[X_3 < Y_3 < x_1]$$

$$= \frac{u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}$$

$$S_{14} = P[Y_3 < X_3 < x_1 < y_1 < x_2 < y_2]$$

$$= P[Y_3 < X_3 < x_1]$$

$$= \frac{k_0^{\,2} u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}$$

$$S_{15} = P[x_1 < X_3 < Y_3 < y_1 < x_2 < y_2]$$

$$= P[x_1 < X_3 < Y_3 < y_1]$$

$$= v^{\frac{1}{k_0}}\left(v^{k_0} - u^{k_0} \right) - \frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)}\left(v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0} \right)$$

$$S_{16} = P[x_1 < Y_3 < X_3 < y_1 < x_2 < y_2]$$

$$= P[x_1 < Y_3 < X_3 < y_1]$$

$$= v^{k_0}\left(v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}} \right) - \frac{1}{\left(k_0^{\,2}+1\right)}\left(v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0} \right)$$

$$S_{17} = P[x_1 < y_1 < X_3 < Y_3 < x_2 < y_2]$$

$$= P[y_1 < X_3 < Y_3 < x_2]$$

$$= \frac{1}{(k_0^2+1)}\left(m^{\frac{1}{k_0}+k_0} - v^{\frac{1}{k_0}+k_0} \right) - v^{k_0}\left(m^{\frac{1}{k_0}} - v^{\frac{1}{k_0}} \right)$$

$$S_{18} = P[x_1 < y_1 < x_2 < X_3 < Y_3 < y_2]$$

$$= P[x_2 < X_3 < Y_3 < y_2]$$

$$= r^{\frac{1}{k_0}}\left(r^{k_0} - m^{k_0}\right) - \frac{k_0^2}{(k_0^2+1)}\left(r^{\frac{1}{k_0}+k_0} - m^{\frac{1}{k_0}+k_0} \right)$$

$$S_{19} = P[x_1 < y_1 < x_2 < Y_3 < X_3 < y_2]$$

$$= P[x_2 < Y_3 < X_3 < y_2]$$

$$= r^{k_0}\left(r^{\frac{1}{k_0}} - m^{\frac{1}{k_0}} \right) - \frac{1}{(k_0^2+1)}\left(r^{\frac{1}{k_0}+k_0} - m^{\frac{1}{k_0}+k_0} \right)$$

$$S_{20} = P[x_1 < y_1 < x_2 < y_2 < X_3 < Y_3]$$

$$= P[y_2 < X_3 < Y_3]$$

$$= \left(1 - r^{k_0}\right) - \frac{1}{2}\left(1 - r^{2k_0}\right)$$

$$S_{21} = P[x_1 < y_1 < x_2 < y_2 < Y_3 < X_3]$$

$$= P[y_2 < Y_3 < X_3]$$

$$= \left(1 - r^{\frac{1}{k_0}} \right) - \frac{1}{(k_0^2+1)}\left(1 - r^{\frac{1}{k_0}+1} \right)$$

Adding 21 terms we get for case 3) $x_1 < y_1 < x_2 < y_2$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = S_1 + S_2 + ... + S_{21}$$

$$= \frac{3}{2} + \frac{1}{2}r^{2k_0} + \frac{m^{\frac{1}{k_0}+k_0}}{(k_0^2+1)} - m^{\frac{1}{k_0}} - \frac{1}{(k_0^2+1)} - \frac{k_0^2 r^{\frac{1}{k_0}+k_0}}{(k_0^2+1)} + \frac{k_0^2 v^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}$$

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_2}\int_{-\infty}^{x_2}\int_{-\infty}^{y_1} P^2[Med(x_1,x_2,X_3)<Med(y_1,y_2,Y_3)]dF^{k_0}(x_1)dG^{\frac{1}{k_0}}(y_1)dF^{k_0}(x_2)dG^{\frac{1}{k_0}}(y_2)$$

$$-\infty<x_1<y_1<x_2<y_2<\infty$$

$$=\int_0^1\int_0^r\int_0^m\int_0^v\left(\frac{3}{2}+\frac{1}{2}r^{2k_0}+\frac{m^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}-m^{\frac{1}{k_0}}-\frac{1}{(k_0^2+1)}-\frac{k_0^2r^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}+\frac{k_0^2v^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}\right)^2 k_0u^{k_0-1}\frac{1}{k_0}v^{\frac{1}{k_0}-1}k_0m^{k_0-1}\frac{1}{k_0}r^{\frac{1}{k_0}-1}dudvdmdr$$

$$-\infty<x_1<y_1<x_2<y_2<\infty$$

$$=\frac{1}{12}k_0^2\left[\frac{\left(\begin{array}{l}133639k_0^{19}+441319k_0^{14}+1112038k_0^{10}+877199k_0^{12}+31608k_0^2+2592+21870k_0^{18}\\+920458k_0^8+1584k_0^{20}+494847k_0^6+166206k_0^4\end{array}\right)}{(5k_0^2+3)(k_0^2+2)(2k_0^2+3)(4k_0^2+3)(3k_0^2+1)(2k_0^2+1)^2(k_0^2+1)^4(3k_0^2+4)(3k_0^2+2)}\right].$$

Consider case 4) $x_1<y_2<x_2<y_1$

Interchanging y_1 and y_2 in case 3 and simplifying we get,

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_1}\int_{-\infty}^{x_2}\int_{-\infty}^{y_2} P^2[Med(x_1,x_2,X_3)<Med(y_1,y_2,Y_3)]dF^{k_0}(x_1)dF^{\frac{1}{k_0}}(y_2)dF^{k_0}(x_2)dF^{\frac{1}{k_0}}(y_1)$$

$$-\infty<x_1<y_2<x_2<y_1<\infty$$

$$=\int_0^1\int_0^v\int_0^m\int_0^r\left(\frac{3}{2}+\frac{1}{2}v^{2k_0}+\frac{m^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}-m^{\frac{1}{k_0}}-\frac{1}{(k_0^2+1)}-\frac{k_0^2v^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}+\frac{k_0^2r^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}\right)^2 k_0u^{k_0-1}\frac{1}{k_0}r^{\frac{1}{k_0}-1}k_0m^{k_0-1}\frac{1}{k_0}v^{\frac{1}{k_0}-1}dudrdmdv$$

$$-\infty<x_1<y_1<x_2<y_2<\infty$$

$$=\frac{1}{12}k_0^2\left[\frac{\left(\begin{array}{l}133639k_0^{19}+441319k_0^{14}+1112038k_0^{10}+877199k_0^{12}+31608k_0^2+2592+21870k_0^{18}\\+920458k_0^8+1584k_0^{20}+494847k_0^6+166206k_0^4\end{array}\right)}{(5k_0^2+3)(k_0^2+2)(2k_0^2+3)(4k_0^2+3)(3k_0^2+1)(2k_0^2+1)^2(k_0^2+1)^4(3k_0^2+4)(3k_0^2+2)}\right]$$

Consider case 5) $x_1<y_1<y_2<x_2$

$$P[Med(x_1,x_2,X_3)<Med(y_1,y_2,Y_3)]=T_1+T_2+.....+T_{15}$$

where

$T_1 = P[X_3 < x_1 < y_1 < y_2 < x_2 < Y_3]$

$T_2 = P[x_1 < X_3 < y_1 < y_2 < x_2 < Y_3]$

$T_3 = P[x_1 < y_1 < X_3 < y_2 < x_2 < Y_3]$

$T_4 = P[X_3 < x_1 < y_1 < y_2 < Y_3 < x_2]$

$T_5 = P[x_1 < X_3 < y_1 < y_2 < Y_3 < x_2]$

$T_6 = P[x_1 < y_1 < X_3 < y_2 < Y_3 < x_2]$

$T_7 = P[X_3 < x_1 < y_1 < Y_3 < y_2 < x_2]$

$T_8 = P[x_1 < X_3 < y_1 < Y_3 < y_2 < x_2]$

$T_9 = P[x_1 < y_1 < X_3 < Y_3 < y_2 < x_2]$

$T_{10} = P[X_3 < x_1 < Y_3 < y_1 < y_2 < x_2]$

$T_{11} = P[x_1 < X_3 < Y_3 < y_1 < y_2 < x_2]$

$T_{12} = P[x_1 < Y_3 < X_3 < y_1 < y_2 < x_2]$

$T_{13} = P[Y_3 < X_3 < x_1 < y_1 < y_2 < x_2]$

$T_{14} = P[Y_3 < x_1 < X_3 < y_1 < y_2 < x_2]$

$T_{15} = P[X_3 < Y_3 < x_1 < y_1 < y_2 < x_2]$.

Now Consider,

$$T_1 = P[X_3 < x_1 < y_1 < y_2 < x_2 < Y_3]$$

$$= P[X_3 < x_1]P[x_2 < Y_3]$$

$$= u^{k_0}\left(1 - m^{\frac{1}{k_0}}\right)$$

$$T_2 = P[x_1 < X_3 < y_1 < y_2 < x_2 < Y_3]$$

$$= P[x_1 < X_3 < y_1]P[x_2 < Y_3]$$

$$= \left(v^{k_0} - u^{k_0}\right)\left(1 - m^{\frac{1}{k_0}}\right)$$

$$T_3 = P[x_1 < y_1 < X_3 < y_2 < x_2 < Y_3]$$

$$= P[y_1 < X_3 < y_2]P[x_2 < Y_3]$$

$$= \left(r^{k_0} - v^{k_0}\right)\left(1 - m^{\frac{1}{k_0}}\right)$$

$$T_4 = P[X_3 < x_1 < y_1 < y_2 < Y_3 < x_2]$$

$$= P[X_3 < x_1]P[y_2 < Y_3 < x_2]$$

$$= u^{k_0}\left(m^{\frac{1}{k_0}} - r^{\frac{1}{k_0}} \right)$$

$$T_5 = P[x_1 < X_3 < y_1 < y_2 < Y_3 < x_2]$$

$$= P[x_1 < X_3 < y_1]P[y_2 < Y_3 < x_2]$$

$$= \left(v^{k_0} - u^{k_0}\right)\left(m^{\frac{1}{k_0}} - r^{\frac{1}{k_0}} \right)$$

$$T_6 = P[x_1 < y_1 < X_3 < y_2 < Y_3 < x_2]$$

$$= P[y_1 < X_3 < y_2]P[y_2 < Y_3 < x_2]$$

$$= \left(r^{k_0} - v^{k_0}\right)\left(m^{\frac{1}{k_0}} - r^{\frac{1}{k_0}} \right)$$

$$T_7 = P[X_3 < x_1 < y_1 < Y_3 < y_2 < x_2]$$

$$= P[X_3 < x_1]P[y_1 < Y_3 < y_2]$$

$$= u^{k_0}\left(r^{\frac{1}{k_0}} - v^{\frac{1}{k_0}} \right)$$

$$T_8 = P[x_1 < X_3 < y_1 < Y_3 < y_2 < x_2]$$

$$= P[x_1 < X_3 < y_1]P[y_1 < Y_3 < y_2]$$

$$= \left(v^{k_0} - u^{k_0}\right)\left(r^{\frac{1}{k_0}} - v^{\frac{1}{k_0}} \right)$$

$$T_9 = P[x_1 < y_1 < X_3 < Y_3 < y_2 < x_2]$$

$$= P[y_1 < X_3 < Y_3 < y_2]$$

$$= \frac{1}{\left(k_0^{\,2}+1\right)}\left(r^{\frac{1}{k_0}+k_0} - v^{\frac{1}{k_0}+k_0} \right) - v^{k_0}\left(r^{\frac{1}{k_0}} - v^{\frac{1}{k_0}} \right)$$

$$T_{10} = P[X_3 < x_1 < Y_3 < y_1 < y_2 < x_2]$$

$$= P[X_3 < x_1 < Y_3 < y_1]$$

$$= P[X_3 < x_1]P[x_1 < Y_3 < y_1]$$

$$= u^{k_0}\left(v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right)$$

$$T_{11} = P[x_1 < X_3 < Y_3 < y_1 < y_2 < x_2]$$

$$= P[x_1 < X_3 < Y_3 < y_1]$$

$$= v^{\frac{1}{k_0}}\left(v^{k_0} - u^{k_0}\right) - \frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)}\left(v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right)$$

$$T_{12} = P[x_1 < Y_3 < X_3 < y_1 < y_2 < x_2]$$

$$= P[x_1 < Y_3 < X_3 < y_1]$$

$$= v^{k_0}\left(v^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right) - \frac{1}{\left(k_0^{\,2}+1\right)}\left(v^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right)$$

$$T_{13} = P[Y_3 < X_3 < x_1 < y_1 < y_2 < x_2]$$

$$= P[Y_3 < X_3 < x_1]$$

$$= \frac{k_0^{\,2}u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}$$

$$T_{14} = P[Y_3 < x_1 < X_3 < y_1 < y_2 < x_2]$$

$$= P[Y_3 < x_1 < X_3 < y_1]$$

$$= P[Y_3 < x_1]P[x_1 < X_3 < y_1]$$

$$= u^{\frac{1}{k_0}}\left(v^{k_0} - u^{k_0}\right)$$

$$T_{15} = P[X_3 < Y_3 < x_1 < y_1 < y_2 < x_2]$$

$$= P[X_3 < Y_3 < x_1]$$

$$= \frac{u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}$$

Adding 15 terms we get for case 5) $x_1 < y_1 < y_2 < x_2$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = T_1 + T_2 + ... + T_{15}$$

$$= r^{k_0} - \frac{k_0^{\,2} r^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)} + \frac{k_0^{\,2} v^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}.$$

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{x_2}\int_{-\infty}^{y_2}\int_{-\infty}^{y_1} P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]\, dF^{k_0}(x_1)\, dF^{\frac{1}{k_0}}(y_1)\, dF^{\frac{1}{k_0}}(y_2)\, dF^{k_0}(x_2)$$

$$-\infty < x_1 < y_1 < y_2 < x_2 < \infty$$

$$= \int_0^1\int_0^m\int_0^r\int_0^v \left(r^{k_0} - \frac{k_0^{\,2} r^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)} + \frac{k_0^{\,2} v^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)} \right)^2 k_0 u^{k_0-1} \frac{1}{k_0} v^{\frac{1}{k_0}-1} \frac{1}{k_0} r^{\frac{1}{k_0}-1} k_0 m^{k_0-1}\, du dv dr dm$$

$$-\infty < x_1 < y_1 < y_2 < x_2 < \infty$$

$$= \frac{1}{12} k_0^{\,2} \frac{\left(24k_0^{\,10} + 160k_0^{\,8} + 451k_0^{\,6} + 576k_0^{\,4} + 334k_0^{\,2} + 72\right)}{\left[\left(k_0^{\,2}+1\right)^4 \left(3k_0^{\,2}+2\right)\left(2k_0^{\,2}+1\right)\left(3k_0^{\,2}+4\right)\left(4k_0^{\,2}+3\right)\right]}$$

Consider case 6) $x_1 < y_2 < y_1 < x_2$

Interchanging y_1 and y_2 in case 5 and simplifying we get,

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{x_2}\int_{-\infty}^{y_1}\int_{-\infty}^{y_2} P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]\, dF^{k_0}(x_1)\, dF^{\frac{1}{k_0}}(y_2)\, dF^{\frac{1}{k_0}}(y_1)\, dF^{k_0}(x_2)$$

$$-\infty < x_1 < y_2 < y_1 < x_2 < \infty$$

$$= \int_0^1 \int_0^m \int_0^v \int_0^r \left(v^{k_0} - \frac{k_0^{\,2} v^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)} + \frac{k_0^{\,2} r^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)} \right)^2 k_0 u^{k_0-1} \frac{1}{k_0} r^{\frac{1}{k_0}-1} \frac{1}{k_0} v^{\frac{1}{k_0}-1} k_0 m^{k_0-1} du dr dv dm$$

$$-\infty < x_1 < y_2 < y_1 < x_2 < \infty$$

$$= \frac{1}{12} k_0^{\,2} \frac{\left(24k_0^{\,10} + 160k_0^{\,8} + 451k_0^{\,6} + 576k_0^{\,4} + 334k_0^{\,2} + 72\right)}{\left[(k_0^{\,2}+1)^4 (3k_0^{\,2}+2)(2k_0^{\,2}+1)(3k_0^{\,2}+4)(4k_0^{\,2}+3)\right]}.$$

Consider case 7) $x_2 < x_1 < y_1 < y_2$

Interchanging x_1 and x_2 in case 1 and simplifying we get,

Thus

$$\int_{-\infty}^{\infty} \int_{-\infty}^{y_2} \int_{-\infty}^{y_1} \int_{-\infty}^{x_1} P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] dF^{k_0}(x_2) dF^{k_0}(x_1) dF^{\frac{1}{k_0}}(y_1) dF^{\frac{1}{k_0}}(y_2)$$

$$-\infty < x_2 < x_1 < y_1 < y_2 < \infty$$

$$= \int_0^1 \int_0^r \int_0^v \int_0^u \left[\frac{3}{2} + \frac{r^{2k_0}}{2} - \frac{1}{(k_0^{\,2}+1)} - \frac{k_0^{\,2} r^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)} \right]^2 k_0 m^{k_0-1} k_0 u^{k_0-1} \frac{1}{k_0} v^{\frac{1}{k_0}-1} \frac{1}{k_0} r^{\frac{1}{k_0}-1} dm du dv dr$$

$$-\infty < x_1 < x_2 < y_1 < y_2 < \infty$$

Simplifying we get,

$$= \frac{1}{24} \left[\frac{\left(18 + 213k_0^{\,10} + 150k_0^{\,2} + 642k_0^{\,8} + 791k_0^{\,6} + 490k_0^{\,4}\right)}{(2k_0^{\,2}+1)^2 (k_0^{\,2}+1)^3 (3k_0^{\,2}+1)(5k_0^{\,2}+3)} \right].$$

Consider case 8) $x_2 < x_1 < y_2 < y_1$

Interchanging x_1 and x_2 in case 2 and simplifying we get,

Thus,

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_1}\int_{-\infty}^{y_2}\int_{-\infty}^{x_1} P^2[Med(x_1,x_2,X_3)<Med(y_1,y_2,Y_3)]dF^{k_0}(x_2)dF^{k_0}(x_1)dF^{\frac{1}{k_0}}(y_2)dF^{\frac{1}{k_0}}(y_1)$$

$$-\infty < x_2 < x_1 < y_2 < y_1 < \infty$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{v}\int_{-\infty}^{r}\int_{-\infty}^{u}\left[\frac{3}{2}+\frac{v^{2k_0}}{2}-\frac{1}{(k_0^{\,2}+1)}-\frac{k_0^{\,2}v^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)}\right]^2 k_0 m^{k_0-1}k_0 u^{k_0-1}\frac{1}{k_0}r^{\frac{1}{k_0}-1}\frac{1}{k_0}v^{\frac{1}{k_0}-1}\,dmdudrdv$$

$$-\infty < x_2 < x_1 < y_2 < y_1 < \infty$$

$$= \frac{1}{24}\left[\frac{\left(18+213k_0^{\,10}+150k_0^{\,2}+642k_0^{\,8}+791k_0^{\,6}+490k_0^{\,4}\right)}{\left(2k_0^{\,2}+1\right)^2\left(k_0^{\,2}+1\right)^3\left(3k_0^{\,2}+1\right)\left(5k_0^{\,2}+3\right)}\right].$$

Consider case 9) $x_2 < y_1 < x_1 < y_2$

Interchanging x_1 and x_2 in case 3 and simplifying we get,

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_2}\int_{-\infty}^{x_1}\int_{-\infty}^{y_1} P^2[Med(x_1,x_2,X_3)<Med(y_1,y_2,Y_3)]dF^{k_0}(x_2)dF^{\frac{1}{k_0}}(y_2)dF^{k_0}(x_1)dF^{\frac{1}{k_0}}(y_1)$$

$$-\infty < x_2 < y_1 < x_1 < y_2 < \infty$$

$$= \int_{0}^{1}\int_{0}^{r}\int_{0}^{u}\int_{0}^{v}\left(\frac{3}{2}+\frac{1}{2}r^{2k_0}+\frac{u^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)}-u^{\frac{1}{k_0}}-\frac{1}{(k_0^{\,2}+1)}-\frac{k_0^{\,2}r^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)}+\frac{k_0^{\,2}v^{\frac{1}{k_0}+k_0}}{(k_0^{\,2}+1)}\right)^2 k_0 m^{k_0-1}\frac{1}{k_0}v^{\frac{1}{k_0}-1}k_0 u^{k_0-1}\frac{1}{k_0}r^{\frac{1}{k_0}-1}\,dmdvdudr$$

$$-\infty < x_1 < y_1 < x_2 < y_2 < \infty$$

$$= \frac{1}{12}k_0^{\,2}\left[\frac{\left(\begin{array}{l}133639k_0^{\,19}+441319k_0^{\,14}+1112038k_0^{\,10}+877199k_0^{\,12}+31608k_0^{\,2}+2592+21870k_0^{\,18}\\+920458k_0^{\,8}+1584k_0^{\,20}+494847k_0^{\,6}+166206k_0^{\,4}\end{array}\right)}{\left(5k_0^{\,2}+3\right)\left(k_0^{\,2}+2\right)\left(2k_0^{\,2}+3\right)\left(4k_0^{\,2}+3\right)\left(3k_0^{\,2}+1\right)\left(2k_0^{\,2}+1\right)^2\left(k_0^{\,2}+1\right)^4\left(3k_0^{\,2}+4\right)\left(3k_0^{\,2}+2\right)}\right].$$

Consider case 10) $x_2 < y_2 < x_1 < y_1$

Interchanging x_1 and x_2 in case 4 and simplifying we get,

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_1}\int_{-\infty}^{x_1}\int_{-\infty}^{y_2} P^2[Med(x_1,x_2,X_3)< Med(y_1,y_2,Y_3)]dF(x_2)dF(y_2)dF(x_1)dF(y_1)$$

$$-\infty < x_2 < y_2 < x_1 < y_1 < \infty$$

$$= \int_0^1\int_0^v\int_0^u\int_0^r \left(\frac{3}{2}+\frac{1}{2}v^{2k_0}+\frac{u^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}-u^{\frac{1}{k_0}}-\frac{1}{(k_0^2+1)}-\frac{k_0^2 v^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}+\frac{k_0^2 r^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}\right)^2 k_0 m^{k_0-1}\frac{1}{k_0}r^{\frac{1}{k_0}-1}k_0u^{k_0-1}\frac{1}{k_0}v^{\frac{1}{k_0}-1}dmdrdudv$$

$$-\infty < x_2 < y_2 < x_1 < y_1 < \infty$$

$$= \frac{1}{12}k_0^2\left[\frac{\left(\begin{array}{l}133639k_0^{19}+441319k_0^{14}+1112038k_0^{10}+877199k_0^{12}+31608k_0^2+2592+21870k_0^{18}\\+920458k_0^8+1584k_0^{20}+494847k_0^6+166206k_0^4\end{array}\right)}{(5k_0^2+3)(k_0^2+2)(2k_0^2+3)(4k_0^2+3)(3k_0^2+1)(2k_0^2+1)^2(k_0^2+1)^4(3k_0^2+4)(3k_0^2+2)}\right]$$

Consider case 11) $x_2 < y_1 < y_2 < x_1$

Interchanging x_1 and x_2 in case 5 and simplifying we get,

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{x_1}\int_{-\infty}^{y_2}\int_{-\infty}^{y_1} P^2[Med(x_1,x_2,X_3)< Med(y_1,y_2,Y_3)]dF^{k_0}(x_2)dF^{\frac{1}{k_0}}(y_1)dF^{\frac{1}{k_0}}(y_2)dF^{k_0}(x_1)$$

$$-\infty < x_2 < y_1 < y_2 < x_1 < \infty$$

$$= \int_0^1\int_0^u\int_0^r\int_0^v \left(r^{k_0}-\frac{k_0^2 r^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}+\frac{k_0^2 v^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}\right)^2 k_0 m^{k_0-1}\frac{1}{k_0}v^{\frac{1}{k_0}-1}\frac{1}{k_0}r^{\frac{1}{k_0}-1}k_0u^{k_0-1}dmdvdrdu$$

$$-\infty < x_1 < y_1 < y_2 < x_2 < \infty$$

$$= \frac{1}{12}k_0^2\frac{\left(24k_0^{10}+160k_0^8+451k_0^6+576k_0^4+334k_0^2+72\right)}{\left[(k_0^2+1)^4(3k_0^2+2)(2k_0^2+1)(3k_0^2+4)(4k_0^2+3)\right]}.$$

Consider case 12) $x_2 < y_2 < y_1 < x_1$

Interchanging x_1 and x_2 in case 6 and simplifying we get,

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{x_1}\int_{-\infty}^{y_1}\int_{-\infty}^{y_2} P^2[Med(x_1,x_2,X_3)<Med(y_1,y_2,Y_3)]dF(x_2)dF(y_2)dF(y_1)dF(x_1)$$

$$-\infty < x_2 < y_2 < y_1 < x_1 < \infty$$

$$= \int_0^1\int_0^u\int_0^v\int_0^r \left(v^{k_0} - \frac{{k_0}^2 v^{\frac{1}{k_0}+k_0}}{({k_0}^2+1)} + \frac{{k_0}^2 r^{\frac{1}{k_0}+k_0}}{({k_0}^2+1)} \right)^2 k_0 m^{k_0-1} \frac{1}{k_0} r^{\frac{1}{k_0}-1} \frac{1}{k_0} v^{\frac{1}{k_0}-1} k_0 u^{k_0-1} dmdrdvdu$$

$$-\infty < x_2 < y_2 < y_1 < x_1 < \infty$$

$$= \frac{1}{12} {k_0}^2 \frac{(24{k_0}^{10}+160{k_0}^8+451{k_0}^6+576{k_0}^4+334{k_0}^2+72)}{[({k_0}^2+1)^4(3{k_0}^2+2)(2{k_0}^2+1)(3{k_0}^2+4)(4{k_0}^2+3)]}.$$

Consider case 13) $y_1 < x_1 < y_2 < x_2$

$$P[Med(x_1,x_2,X_3)<Med(y_1,y_2,Y_3)] = U_1+U_2+.....+U_9$$

where

$U_1 = P[X_3 < y_1 < x_1 < y_2 < x_2 < Y_3]$ $\quad$ $U_2 = P[y_1 < X_3 < x_1 < y_2 < x_2 < Y_3]$

$U_3 = P[y_1 < x_1 < X_3 < y_2 < x_2 < Y_3]$ $\quad$ $U_4 = P[X_3 < y_1 < x_1 < y_2 < Y_3 < x_2]$

$U_5 = P[y_1 < X_3 < x_1 < y_2 < Y_3 < x_2]$ $\quad$ $U_6 = P[y_1 < x_1 < X_3 < y_2 < Y_3 < x_2]$

$U_7 = P[X_3 < y_1 < x_1 < Y_3 < y_2 < x_2]$ $\quad$ $U_8 = P[y_1 < X_3 < x_1 < Y_3 < y_2 < x_2]$

$U_9 = P[y_1 < x_1 < X_3 < Y_3 < y_2 < x_2]$.

Nw Consider,

$$U_1 = P[X_3 < y_1 < x_1 < y_2 < x_2 < Y_3]$$

$$= P[X_3 < y_1]P[x_2 < Y_3]$$

$$= v^{k_0}\left(1-m^{\frac{1}{k_0}}\right)$$

$$U_2 = P[y_1 < X_3 < x_1 < y_2 < x_2 < Y_3]$$

$$= P[y_1 < X_3 < x_1]P[x_2 < Y_3]$$

$$= \left(u^{k_0} - v^{k_0}\right)\left(1 - m^{\frac{1}{k_0}}\right)$$

$$U_3 = P[y_1 < x_1 < X_3 < y_2 < x_2 < Y_3]$$

$$= P[x_1 < X_3 < y_2]P[x_2 < Y_3]$$

$$= \left(r^{k_0} - u^{k_0}\right)\left(1 - m^{\frac{1}{k_0}}\right)$$

$$U_4 = P[X_3 < y_1 < x_1 < y_2 < Y_3 < x_2]$$

$$= P[X_3 < y_1]P[y_2 < Y_3 < x_2]$$

$$= v^{k_0}\left[m^{\frac{1}{k_0}} - r^{\frac{1}{k_0}}\right]$$

$$U_5 = P[y_1 < X_3 < x_1 < y_2 < Y_3 < x_2]$$

$$= P[y_1 < X_3 < x_1]P[y_2 < Y_3 < x_2]$$

$$= \left(u^{k_0} - v^{k_0}\right)\left[m^{\frac{1}{k_0}} - r^{\frac{1}{k_0}}\right]$$

$$U_6 = P[y_1 < x_1 < X_3 < y_2 < Y_3 < x_2]$$

$$= P[x_1 < X_3 < y_2]P[y_2 < Y_3 < x_2]$$

$$= \left(r^{k_0} - u^{k_0}\right)\left[m^{\frac{1}{k_0}} - r^{\frac{1}{k_0}}\right]$$

$$U_7 = P[X_3 < y_1 < x_1 < Y_3 < y_2 < x_2]$$

$$= P[X_3 < y_1]P[x_1 < Y_3 < y_2]$$

$$= v^{k_0}\left[r^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right]$$

$$U_8 = P[y_1 < X_3 < x_1 < Y_3 < y_2 < x_2]$$

$$= P[y_1 < X_3 < x_1]P[x_1 < Y_3 < y_2]$$

$$= \left(u^{k_0} - v^{k_0}\right)\left[r^{\frac{1}{k_0}} - u^{\frac{1}{k_0}}\right]$$

$$U_9 = P[y_1 < x_1 < X_3 < Y_3 < y_2 < x_2]$$

$$= P[x_1 < X_3 < Y_3 < y_2]$$

$$= r^{\frac{1}{k_0}}\left(r^{k_0} - u^{k_0}\right) - \frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)}\left(r^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0}\right)$$

Adding 9 terms we get for case 13) $y_1 < x_1 < y_2 < x_2$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = U_1 + U_2 + \ldots\ldots + U_9$$

$$= r^{k_0} - \frac{k_0^{\,2} r^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)} - \frac{u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}.$$

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{x_2}\int_{-\infty}^{y_2}\int_{-\infty}^{x_1} P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]\,dF^{\frac{1}{k_0}}(y_1)\,dF^{k_0}(x_1)\,dF^{\frac{1}{k_0}}(y_2)\,dF^{k_0}(x_2)$$

$$-\infty < y_1 < x_1 < y_2 < x_2 < \infty$$

$$= \int_0^1\int_0^m\int_0^r\int_0^u \left(r^{k_0} - \frac{k_0^{\,2} r^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)} - \frac{u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}\right)^2 \frac{1}{k_0} v^{\frac{1}{k_0}-1} k_0 u^{k_0-1} \frac{1}{k_0} r^{\frac{1}{k_0}-1} k_0 m^{k_0-1}\,dv\,du\,dr\,dm$$

$$-\infty < y_1 < x_1 < y_2 < x_2 < \infty$$

$$= \frac{1}{12} k_0^{\,4} \left[\frac{\left(181k_0^{\,2} + 103k_0^{\,6} + 237k_0^{\,4} + 46\right)}{\left(k_0^{\,2}+1\right)^4\left(3k_0^{\,2}+2\right)\left(2k_0^{\,2}+1\right)\left(3k_0^{\,2}+4\right)\left(4k_0^{\,2}+3\right)}\right].$$

Consider case 14) $y_1 < x_2 < y_2 < x_1$

Interchanging x_1 and x_2 in case 13 and simplifying we get,

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{x_1}\int_{-\infty}^{y_2}\int_{-\infty}^{x_2} P^2[Med(x_1,x_2,X_3)<Med(y_1,y_2,Y_3)]dF^{\frac{1}{k_0}}(y_1)dF^{k_0}(x_2)dF^{\frac{1}{k_0}}(y_2)dF^{k_0}(x_1)$$

$$-\infty < y_1 < x_2 < y_2 < x_1 < \infty$$

$$= \int_0^1\int_0^u\int_0^r\int_0^m \left(r^{k_0} - \frac{k_0{}^2 r^{\frac{1}{k_0}+k_0}}{(k_0{}^2+1)} + \frac{m^{\frac{1}{k_0}+k_0}}{(k_0{}^2+1)} \right)^2 \frac{1}{k_0} v^{\frac{1}{k_0}-1} k_0 m^{k_0-1} \frac{1}{k_0} r^{\frac{1}{k_0}-1} k_0 u^{k_0-1} dvdmdrdu$$

$$-\infty < x_2 < y_2 < y_1 < x_1 < \infty$$

$$= \frac{1}{12} k_0{}^4 \left[\frac{(181k_0{}^2 + 103k_0{}^6 + 237k_0{}^4 + 46)}{(k_0{}^2+1)^4 (3k_0{}^2+2)(2k_0{}^2+1)(3k_0{}^2+4)(4k_0{}^2+3)} \right].$$

Consider case 15) $y_1 < x_1 < x_2 < y_2$

$$P[Med(x_1,x_2,X_3)<Med(y_1,y_2,Y_3)] = V_1 + V_2 + ... + V_{15}$$

where

$V_1 = P[X_3 < y_1 < x_1 < x_2 < y_2 < Y_3]$ $\quad$ $V_2 = P[y_1 < X_3 < x_1 < x_2 < y_2 < Y_3]$

$V_3 = P[y_1 < x_1 < X_3 < x_2 < y_2 < Y_3]$ $\quad$ $V_4 = P[y_1 < x_1 < x_2 < X_3 < y_2 < Y_3]$

$V_5 = P[y_1 < x_1 < x_2 < y_2 < X_3 < Y_3]$ $\quad$ $V_6 = P[y_1 < x_1 < x_2 < y_2 < Y_3 < X_3]$

$V_7 = P[X_3 < y_1 < x_1 < x_2 < Y_3 < y_2]$ $\quad$ $V_8 = P[y_1 < X_3 < x_1 < x_2 < Y_3 < y_2]$

$V_9 = P[y_1 < x_1 < X_3 < x_2 < Y_3 < y_2]$ $\quad$ $V_{10} = P[y_1 < x_1 < x_2 < X_3 < Y_3 < y_2]$

$V_{11} = P[y_1 < x_1 < x_2 < Y_3 < X_3 < y_2]$ $\quad$ $V_{12} = P[y_1 < x_1 < x_2 < Y_3 < y_2 < X_3]$

$V_{13} = P[X_3 < y_1 < x_1 < Y_3 < x_2 < y_2]$ $\quad$ $V_{14} = P[y_1 < X_3 < x_1 < Y_3 < x_2 < y_2]$

$V_{15} = P[y_1 < x_1 < X_3 < Y_3 < x_2 < y_2]$.

Now Consider

$V_1 = P[X_3 < y_1 < x_1 < x_2 < y_2 < Y_3]$

$$= P[X_3 < y_1]P[y_2 < Y_3]$$

$$= v^{k_0}\left(1 - r^{\frac{1}{k_0}}\right)$$

$$V_2 = P[y_1 < X_3 < x_1 < x_2 < y_2 < Y_3]$$

$$= P[y_1 < X_3 < x_1]P[y_2 < Y_3]$$

$$= \left(u^{k_0} - v^{k_0}\right)\left(1 - r^{\frac{1}{k_0}}\right)$$

$$V_3 = P[y_1 < x_1 < X_3 < x_2 < y_2 < Y_3]$$

$$= P[x_1 < X_3 < x_2]P[y_2 < Y_3]$$

$$= P[y_1 < X_3 < x_1]P[y_2 < Y_3]$$

$$= \left(m^{k_0} - u^{k_0}\right)\left(1 - r^{\frac{1}{k_0}}\right)$$

$$V_4 = P[y_1 < x_1 < x_2 < X_3 < y_2 < Y_3]$$

$$= P[x_2 < X_3 < y_2]P[y_2 < Y_3]$$

$$= \left(r^{k_0} - m^{k_0}\right)\left(1 - r^{\frac{1}{k_0}}\right)$$

$$V_5 = P[y_1 < x_1 < x_2 < y_2 < X_3 < Y_3]$$

$$= P[y_2 < X_3 < Y_3]$$

$$= \left(1 - r^{k_0}\right) - \frac{1}{2}\left(1 - r^{2k_0}\right)$$

$$V_6 = P[y_1 < x_1 < x_2 < y_2 < Y_3 < X_3]$$

$$= P[y_2 < Y_3 < X_3]$$

$$= \left(1 - r^{\frac{1}{k_0}}\right) - \frac{1}{\left(k_0^{\,2}+1\right)}\left(1 - r^{\frac{1}{k_0}+k_0}\right)$$

$$V_7 = P[X_3 < y_1 < x_1 < x_2 < Y_3 < y_2]$$

$$= P[X_3 < y_1]P[x_2 < Y_3 < y_2]$$

$$= v^{k_0}\left(r^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right)$$

$$V_8 = P[y_1 < X_3 < x_1 < x_2 < Y_3 < y_2]$$

$$= P[y_1 < X_3 < x_1]P[x_2 < Y_3 < y_2]$$

$$= \left(u^{k_0} - v^{k_0}\right)\left(r^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right)$$

$$V_9 = P[y_1 < x_1 < X_3 < x_2 < Y_3 < y_2]$$

$$= P[x_1 < X_3 < x_2]P[x_2 < Y_3 < y_2]$$

$$= \left(m^{k_0} - u^{k_0}\right)\left(r^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right)$$

$$V_{10} = P[y_1 < x_1 < x_2 < X_3 < Y_3 < y_2]$$

$$= P[x_2 < X_3 < Y_3 < y_2]$$

$$= r^{\frac{1}{k_0}}\left(r^{k_0} - m^{k_0}\right) - \frac{k_0^{\,2}}{\left(k_0^{\,2}+1\right)}\left[r^{\frac{1}{k_0}+k_0} - m^{\frac{1}{k_0}+k_0}\right]$$

$$V_{11} = P[y_1 < x_1 < x_2 < Y_3 < X_3 < y_2]$$

$$= P[x_2 < Y_3 < X_3 < y_2]$$

$$= r^{k_0}\left(r^{\frac{1}{k_0}} - m^{\frac{1}{k_0}}\right) - \frac{1}{\left(k_0^{\,2}+1\right)}\left[r^{\frac{1}{k_0}+k_0} - m^{\frac{1}{k_0}+k_0}\right]$$

$$V_{12} = P[y_1 < x_1 < x_2 < Y_3 < y_2 < X_3]$$

$$= P[x_2 < Y_3 < y_2]P[y_2 < X_3]$$

$$= \left(r^{\frac{1}{k_0}} - m^{\frac{1}{k_0}} \right)\left(1 - r^{k_0}\right)$$

$$V_{13} = P[X_3 < y_1 < x_1 < Y_3 < x_2 < y_2]$$

$$= P[X_3 < y_1]P[x_1 < Y_3 < x_2]$$

$$= v^{k_0}\left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}} \right)$$

$$V_{14} = P[y_1 < X_3 < x_1 < Y_3 < x_2 < y_2]$$

$$= P[y_1 < X_3 < x_1]P[x_1 < Y_3 < x_2]$$

$$= \left(u^{k_0} - v^{k_0}\right)\left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}} \right)$$

$$V_{15} = P[y_1 < x_1 < X_3 < Y_3 < x_2 < y_2]$$

$$= P[x_1 < X_3 < Y_3 < x_2]$$

$$= \frac{\left(m^{\frac{1}{k_0}+k_0} - u^{\frac{1}{k_0}+k_0} \right)}{\left(k_0^{\,2}+1\right)} - u^{k_0}\left(m^{\frac{1}{k_0}} - u^{\frac{1}{k_0}} \right).$$

Adding 15 terms we get for 15) $y_1 < x_1 < x_2 < y_2$

$$P[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)] = V_1 + V_2 + \ldots\ldots + V_{15}$$

$$= \frac{3}{2} + \frac{1}{2}r^{2k_0} - \frac{1}{\left(k_0^{\,2}+1\right)} - m^{\frac{1}{k_0}} - \frac{u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)} - \frac{k_0^{\,2} r^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)} + \frac{m^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}$$

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_2}\int_{-\infty}^{x_2}\int_{-\infty}^{x_1} P^2[Med(x_1, x_2, X_3) < Med(y_1, y_2, Y_3)]\, dF^{\frac{1}{k_0}}(y_1)\, dF^{k_0}(x_1)\, dF^{k_0}(x_2)\, dF^{\frac{1}{k_0}}(y_2)$$

$$-\infty < y_1 < x_1 < x_2 < y_2 < \infty$$

$$=\int_0^1\int_0^r\int_0^m\int_0^u\left(\frac{3}{2}+\frac{1}{2}r^{2k_0}-\frac{1}{(k_0^2+1)}-m^{\frac{1}{k_0}}-\frac{u^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}-\frac{k_0^2r^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}+\frac{m^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}\right)^2\frac{1}{k_0}v^{\frac{1}{k_0}-1}k_0u^{k_0-1}k_0m^{k_0-1}\frac{1}{k_0}r^{\frac{1}{k_0}-1}dvdudmdr$$

$$-\infty<y_1<x_1<x_2<y_2<\infty$$

$$=\frac{1}{12}k_0^4\frac{\left(\begin{array}{l}1872+45957k_0^{16}+4770k_0^{18}+446753k_0^{12}+216k_0^2+193882k_0^{14}+625996k_0^{10}+22080k_0^2\\+557740k_0^8+317514k_0^6+111700k_0^4\end{array}\right)}{\left[(k_0^2+1)^4(2k_0^2+1)^2(3k_0^2+1)(2k_0^2+3)(k_0^2+2)(4k_0^2+3)(3k_0^2+2)(5k_0^2+3)(3k_0^2+4)\right]}.$$

Consider case 16) $y_1<x_2<x_1<y_2$

Interchanging x_1 and x_2 in case 15 and simplifying we get,

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_2}\int_{-\infty}^{x_1}\int_{-\infty}^{x_2}P^2[Med(x_1,x_2,X_3)<Med(y_1,y_2,Y_3)]dF^{\frac{1}{k_0}}(y_1)dF^{k_0}(x_2)dF^{k_0}(x_1)dF^{\frac{1}{k_0}}(y_2)$$

$$-\infty<y_1<x_2<x_1<y_2<\infty$$

$$=\int_0^1\int_0^r\int_0^u\int_0^m\left(\frac{3}{2}+\frac{1}{2}r^{2k_0}-\frac{1}{(k_0^2+1)}-u^{\frac{1}{k_0}}-\frac{m^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}-\frac{k_0^2r^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}+\frac{u^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}\right)^2\frac{1}{k_0}v^{\frac{1}{k_0}-1}k_0m^{k_0-1}k_0u^{k_0-1}\frac{1}{k_0}r^{\frac{1}{k_0}-1}dvdmdudr$$

$$-\infty<y_1<x_2<x_1<y_2<\infty$$

$$=\frac{1}{12}k_0^4\frac{\left(\begin{array}{l}1872+45957k_0^{16}+4770k_0^{18}+446753k_0^{12}+216k_0^2+193882k_0^{14}+625996k_0^{10}+22080k_0^2\\+557740k_0^8+317514k_0^6+111700k_0^4\end{array}\right)}{\left[(k_0^2+1)^4(2k_0^2+1)^2(3k_0^2+1)(2k_0^2+3)(k_0^2+2)(4k_0^2+3)(3k_0^2+2)(5k_0^2+3)(3k_0^2+4)\right]}$$

Consider case 17) $y_2<x_1<y_1<x_2$

Interchanging y_1 and y_2 in case 15 and simplifying we get,

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{x_2}\int_{-\infty}^{y_1}\int_{-\infty}^{x_1}P^2[Med(x_1,x_2,X_3)<Med(y_1,y_2,Y_3)]dF^{\frac{1}{k_0}}(y_2)dF^{k_0}(x_1)dF^{\frac{1}{k_0}}(y_1)dF^{k_0}(x_2)$$

$$-\infty<y_2<x_1<y_1<x_2<\infty$$

$$=\int_0^1\int_0^m\int_0^v\int_0^u\left(v^{k_0}-\frac{k_0^2v^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}-\frac{u^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}\right)^2\frac{1}{k_0}r^{\frac{1}{k_0}-1}k_0u^{k_0-1}\frac{1}{k_0}v^{\frac{1}{k_0}-1}k_0m^{k_0-1}drdudvdm$$

$$-\infty<y_2<x_1<y_1<x_2<\infty$$

$$=\frac{1}{12}k_0^{\,4}\left[\frac{\left(181k_0^{\,2}+103k_0^{\,6}+237k_0^{\,4}+46\right)}{\left(k_0^{\,2}+1\right)^4\left(3k_0^{\,2}+2\right)\left(2k_0^{\,2}+1\right)\left(3k_0^{\,2}+4\right)\left(4k_0^{\,2}+3\right)}\right].$$

Consider case 18) $y_2 < x_2 < y_1 < x_1$

Interchanging y_1 and y_2 in case 14 and simplifying we get,

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{x_1}\int_{-\infty}^{y_1}\int_{-\infty}^{x_2} P^2\left[Med(x_1,x_2,X_3)< Med(y_1,y_2,Y_3)\right]dF^{\frac{1}{k_0}}(y_2)\,dF^{k_0}(x_2)\,dF^{\frac{1}{k_0}}(y_1)\,dF^{k_0}(x_1)$$

$$-\infty < y_2 < x_2 < y_1 < x_1 < \infty$$

$$=\int_0^1\int_0^u\int_0^v\int_0^m\left(v^{k_0}-\frac{k_0^{\,2}v^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}-\frac{m^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}\right)^2\frac{1}{k_0}r^{\frac{1}{k_0}-1}k_0m^{k_0-1}\frac{1}{k_0}v^{\frac{1}{k_0}-1}k_0u^{k_0-1}drdmdvdu$$

$$-\infty < y_2 < x_2 < y_1 < x_1 < \infty$$

$$=\frac{1}{12}k_0^{\,4}\left[\frac{\left(181k_0^{\,2}+103k_0^{\,6}+237k_0^{\,4}+46\right)}{\left(k_0^{\,2}+1\right)^4\left(3k_0^{\,2}+2\right)\left(2k_0^{\,2}+1\right)\left(3k_0^{\,2}+4\right)\left(4k_0^{\,2}+3\right)}\right].$$

Consider case 19) $y_2 < x_1 < x_2 < y_1$

Interchanging y_1 and y_2 in case 15 and simplifying we get,

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_1}\int_{-\infty}^{x_2}\int_{-\infty}^{x_1} P^2\left[Med(x_1,x_2,X_3)< Med(y_1,y_2,Y_3)\right]dF^{\frac{1}{k_0}}(y_2)\,dF^{k_0}(x_1)\,dF^{k_0}(x_2)\,dF^{\frac{1}{k_0}}(y_1)$$

$$-\infty < y_2 < x_1 < x_2 < y_1 < \infty$$

$$=\int_0^1\int_0^v\int_0^m\int_0^n\left(\frac{3}{2}+\frac{1}{2}v^{2k_0}-\frac{1}{\left(k_0^{\,2}+1\right)}-m^{\frac{1}{k_0}}-\frac{u^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}-\frac{k_0^{\,2}v^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}+\frac{m^{\frac{1}{k_0}+k_0}}{\left(k_0^{\,2}+1\right)}\right)^2\frac{1}{k_0}r^{\frac{1}{k_0}-1}k_0u^{k_0-1}k_0m^{k_0-1}\frac{1}{k_0}v^{\frac{1}{k_0}-1}drdudmdv$$

$$-\infty < y_1 < x_1 < x_2 < y_2 < \infty$$

$$=\frac{1}{12}k_0^{\,4}\left[\frac{\begin{pmatrix}1872+45957k_0^{\,16}+4770k_0^{\,18}+446753k_0^{\,12}+216k_0^{\,2}+193882k_0^{\,14}+625996k_0^{\,10}+22080k_0^{\,2}\\+557740k_0^{\,8}+317514k_0^{\,6}+111700k_0^{\,4}\end{pmatrix}}{\left(k_0^{\,2}+1\right)^4\left(2k_0^{\,2}+1\right)^5\left(3k_0^{\,2}+1\right)\left(2k_0^{\,2}+3\right)\left(k_0^{\,2}+2\right)\left(4k_0^{\,2}+3\right)\left(3k_0^{\,2}+2\right)\left(5k_0^{\,2}+3\right)\left(3k_0^{\,2}+4\right)}\right].$$

Consider case 20) $y_2 < x_2 < x_1 < y_1$

Interchanging y_1 and y_2 in case 16 and simplifying we get,

Thus

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_1}\int_{-\infty}^{x_1}\int_{-\infty}^{x_2} P^2[Med(x_1,x_2,X_3) < Med(y_1,y_2,Y_3)]dF^{\frac{1}{k_0}}(y_2)dF^{k_0}(x_2)dF^{k_0}(x_1)dF^{\frac{1}{k_0}}(y_1)$$

$$-\infty < y_2 < x_2 < x_1 < y_1 < \infty$$

$$= \int_0^1\int_0^v\int_0^m\int_0^u \left(\frac{3}{2}+\frac{1}{2}v^{2k_0}-\frac{1}{(k_0^2+1)}-u^{\frac{1}{k_0}}-\frac{u^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}-\frac{k_0^2 v^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}+\frac{m^{\frac{1}{k_0}+k_0}}{(k_0^2+1)}\right)^2 \frac{1}{k_0} r^{\frac{1}{k_0}-1} k_0 m^{k_0-1} k_0 u^{k_0-1} \frac{1}{k_0} v^{\frac{1}{k_0}-1} drdudmdv$$

$$-\infty < y_2 < x_2 < x_1 < y_1 < \infty$$

$$= \frac{1}{12}k_0^4 \frac{\left(\begin{array}{l}1872+45957k_0^{16}+4770k_0^{18}+446753k_0^{12}+216k_0^2+193882k_0^{14}+625996k_0^{10}+22080k_0^2\\+557740k_0^8+317514k_0^6+111700k_0^4\end{array}\right)}{\left[(k_0^2+1)^4(2k_0^2+1)^2(3k_0^2+1)(2k_0^2+3)(k_0^2+2)(4k_0^2+3)(3k_0^2+2)(5k_0^2+3)(3k_0^2+4)\right]}$$

Finally adding the 20 cases we get

$$P[Med(X_1,X_2,X_3) < Med(Y_1,Y_2,Y_3), Med(X_1,X_2,X_4) < Med(Y_1,Y_2,Y_4)]$$

$$= \frac{1}{2}\frac{\left(\begin{array}{l}72k_0^{20}+5046k_0^{18}+48867k_0^{16}+196924k_0^{14}+429185k_0^{12}+560630k_0^{10}+457110k_0^8\\+233230k_0^6+71904k_0^4+12168k_0^2+864\end{array}\right)}{\left[(3k_0^2+4)(5k_0^2+3)(3k_0^2+2)(4k_0^2+3)(k_0^2+2)(3k_0^2+1)(2k_0^2+3)(2k_0^2+1)(k_0^2+1)^3\right]}$$

Thus,

$$\xi^*_{2,2} = P[Med(X_1,X_2,X_3) < Med(Y_1,Y_2,Y_3), Med(X_1,X_2,X_4) < Med(Y_1,Y_2,Y_4)] - [E(U)]^2$$

$$= \left[\frac{1}{2}\frac{\left(\begin{array}{l}72k_0^{20}+5046k_0^{18}+48867k_0^{16}+196924k_0^{14}+429185k_0^{12}+560630k_0^{10}+457110k_0^8\\+233230k_0^6+71904k_0^4+12168k_0^2+864\end{array}\right)}{\left[(3k_0^2+4)(5k_0^2+3)(3k_0^2+2)(4k_0^2+3)(k_0^2+2)(3k_0^2+1)(2k_0^2+3)(2k_0^2+1)(k_0^2+1)^3\right]}\right] - \left[\frac{(19k_0^2+6)}{(k_0^2+1)(3k_0^2+2)(2k_0^2+3)}\right]^2$$

$$= \frac{1}{2}k_0^2 \frac{\left(\begin{array}{l}31212k_0^{20}+359232k_0^{18}+432k_0^{22}+1587171k_0^{16}+3570352k_0^{14}+4557687k_0^{12}\\+3397342k_0^{10}+1409228k_0^8+248480k_0^6-23568k_0^4-15840k_0^2-1728\end{array}\right)}{\left[(3k_0^2+4)(5k_0^2+3)(3k_0^2+2)^2(4k_0^2+3)(k_0^2+2)(2k_0^2+3)^2(3k_0^2+1)(2k_0^2+1)(k_0^2+1)^3\right]}.$$

Thus finally we have the variance i.e. 2.4.2.1 is

$$\text{Var}(U)=\frac{1}{\binom{n}{3}\binom{m}{3}}\sum_{c=0}^{3}\sum_{d=0}^{3}\binom{3}{c}\binom{m-3}{3-c}\binom{3}{d}\binom{n-3}{3-d}\xi^{*}_{c,d}$$

$$=\frac{1}{\binom{n}{3}^{2}}\sum_{c=0}^{3}\sum_{d=0}^{3}\binom{3}{c}\binom{n-3}{3-c}\binom{3}{d}\binom{n-3}{3-d}\xi^{*}_{c,d}$$

$$=\frac{1}{\binom{n}{3}^{2}}\left\{\begin{array}{l}\left\{\frac{(n-3)^{2}(n-4)^{2}(n-5)}{2}\right\}(\xi^{*}_{1,0}+\xi^{*}_{0,1})+\{(n-3)^{2}(n-4)(n-5)\}(\xi^{*}_{2,0}+\xi^{*}_{0,2})\\+\left\{\frac{(n-3)(n-4)(n-5)}{3}\right\}(\xi^{*}_{3,0}+\xi^{*}_{0,3})+\left\{\frac{9(n-3)^{2}(n-4)^{2}}{4}\right\}(\xi^{*}_{1,1})\\+\{9(n-3)^{2}(n-4)\}(\xi^{*}_{1,2}+\xi^{*}_{2,1})+\{9(n-3)^{2}\}(\xi^{*}_{2,2})+\{3(n-3)(n-4)\}(\xi^{*}_{1,3}+\xi^{*}_{3,1})\\+\{3(n-3)\}(\xi^{*}_{2,3}+\xi^{*}_{3,2})+\xi^{*}_{3,3}\end{array}\right\} \quad (2.4.2.3)$$

Where

$\xi^{*}_{1,0},\xi^{*}_{0,1},\xi^{*}_{2,0},\xi^{*}_{0,2},\xi^{*}_{3,0},\xi^{*}_{0,3},\xi^{*}_{1,1},\xi^{*}_{1,2},\xi^{*}_{2,1},\xi^{*}_{2,2},\xi^{*}_{1,3},\xi^{*}_{3,1},\xi^{*}_{2,3},\xi^{*}_{3,2},\xi^{*}_{3,3}$

are as defined earlier.

CHAPTER III

THE DEVELOPMENT OF SPRT

3.1. INTRODUCTION

In this chapter we deal with the development of the SPRT based on the Normal approximation to the distribution of U-Statistics.

Let $X_1, X_2, X_3, \ldots$ and $Y_1, Y_2, Y_3, \ldots$ be two sequences of iid. Random variables with CDF's F(.) and G(.) respectively, where G(.)= $F^k(.)$.The test statistic proposed by Shetty and Govindarajalu is $U_n = \sum_{i=1}^{n}\sum_{j=1}^{n} I(X_i < Y_j)$

where we have imposed the extra condition that the number of X's and Y's be equal.We have obtained the null mean and the null variance and also the mean and the variance under the alternative hypothesis in the 2nd chapter.Here in this chapter we develop the SPRT based on the statistic U using the normal approximation to our statistic U when n is large for testing the simple hypothesis $H_0 : F(x) = F^{k_0}(x)$ Vs $H_1 : G(x) = F^{\frac{1}{k_0}}(x)$.This follows from the two sample U-statistic theorem that $\frac{U - E(U)}{\sqrt{Var(U)}}$ has standard normal distribution.

We have the Sequential test procedure as follows

We have our test statistic

$$U \approx AN\left(E_{H_0}(U), Var_{H_0}(U)\right) \quad \text{and} \quad U \approx AN\left(E_{H_1}(U), Var_{H_1}(U)\right)$$

Then the likelihood ratio is given by

$$\frac{f(x,\mu_1)}{f(x,\mu_0)} = \frac{\frac{1}{\sqrt{2\pi}\sqrt{Var_{H_1}(U)}} e^{-\frac{1}{2}\frac{(U-E_{H_1}(U))^2}{Var_{H_1}(U)}}}{\frac{1}{\sqrt{2\pi}\sqrt{Var_{H_0}(U)}} e^{-\frac{1}{2}\frac{(U-E_{H_0}(U))^2}{Var_{H_0}(U)}}}$$

$$= \frac{\sqrt{Var_{H_0}(U)}}{\sqrt{Var_{H_1}(U)}} e^{-\frac{1}{2}\frac{(U-E_{H_1}(U))^2}{Var_{H_1}(U)}} . e^{-\frac{1}{2}\frac{(U-E_{H_0}(U))^2}{Var_{H_0}(U)}}$$

Now taking log on both sides we have

$$\log\frac{f(x,\mu_1)}{f(x,\mu_0)} = \log\frac{\sqrt{Var_{H_0}(U)}}{\sqrt{Var_{H_1}(U)}} + \frac{1}{2}\frac{(U-E_{H_0}(U))^2}{Var_{H_0}(U)} - \frac{1}{2}\frac{(U-E_{H_1}(U))^2}{Var_{H_1}(U)}$$

Now we have the continuation region at stage n is given by

$$\log B < \log\frac{f(x,\mu_1)}{f(x,\mu_0)} < \log A$$

At stage n, we reject H_0 when

$$\log\frac{f_{H_1}(U)}{f_{H_0}(U)} \geq \log A$$

At stage n, we accept H_0 when

$$\log\frac{f_{H_1}(U)}{f_{H_0}(U)} \leq \log B$$

and sampling is continued as long as

$$\log B < \log\frac{f_{H_1}(U)}{f_{H_0}(U)} < \log A$$

In our case, the continuation region is

$$\log B < \log \frac{\sqrt{Var_{H_0}(U)}}{\sqrt{Var_{H_1}(U)}} + \frac{1}{2}\frac{(U - E_{H_0}(U))^2}{Var_{H_0}(U)} - \frac{1}{2}\frac{(U - E_{H_1}(U))^2}{Var_{H_1}(U)} < \log A$$

i.e. the continuation region at stage m is given by

$$\log B - \log \frac{\sqrt{Var_{H_0}(U)}}{\sqrt{Var_{H_1}(U)}} < \frac{1}{2}\frac{(U - E_{H_0}(U))^2}{Var_{H_0}(U)} - \frac{1}{2}\frac{(U - E_{H_1}(U))^2}{Var_{H_1}(U)} < \log A - \log \frac{\sqrt{Var_{H_0}(U)}}{\sqrt{Var_{H_1}(U)}}$$

where $A = \frac{1-\beta}{\alpha}$ and $B = \frac{\beta}{1-\alpha}$

3.2. OC FUNCTION

Now to obtain the operating characteristic(OC) function of the Sequential Probability Ratio Test(SPRT) based on our proposed statistic U. We use the Normal approximation to the distribution of U when n is large. This follows from the two-sample U-statistic theorem that $\frac{U - E(U)}{\sqrt{Var(U)}}$ has standard normal distribution.

For testing simple hypothesis $H_0 : \mu = \mu_0$ against $H_1 : \mu = \mu_1$ when the underlying pdf is $f_\mu(x)$, the OC function is given by

$$L(\mu) = \frac{A^{h(\mu)} - 1}{A^{h(\mu)} - B^{h(\mu)}}$$

where $A = \frac{1-\beta}{\alpha}$ and $B = \frac{\beta}{1-\alpha}$ and $\alpha = \beta = 0.05$

where $h(\mu)$ is obtained such that the inequality

$$E\left(\frac{f_{\mu_1}(X)}{f_{\mu_0}(X)}\right)^{h(\mu)}=1$$

In our case $f_{\mu_0}(x)$ is normal with mean $\frac{1}{2}$ and variance given by equation 2.3.2.3

and $f_{\mu_1}(x)$ is normal with mean $\mu=\frac{\left(19k_0^{\,2}+6\right)}{\left(k_0^{\,2}+1\right)\left(3k_0^{\,2}+2\right)\left(2k_0^{\,2}+3\right)}$

and variance is given by 2.4.2.3

Thus,

We have

$$E\left[\frac{\frac{1}{\sigma_1\sqrt{2\pi}}e^{-\frac{1}{2\sigma_1^{\,2}}(x-\mu_1)^2}}{\frac{1}{\sigma_0\sqrt{2\pi}}e^{-\frac{1}{2\sigma_0^{\,2}}(x-\mu_0)^2}}\right]^{h(\mu)}=1$$

$$\int_{-\infty}^{\infty}\left[\left(\frac{\sigma_0}{\sigma_1}\right)^{h(\mu)}e^{-\frac{h(\mu)}{2\sigma_1^{\,2}}(x-\mu_1)^2+\frac{h(\mu)}{2\sigma_0^{\,2}}(x-\mu_0)^2}\right]\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2\sigma^2}(x-\mu)^2}dx=1$$

$$\left(\frac{\sigma_0}{\sigma_1}\right)^{h(\mu)}\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty}\left[e^{-\frac{h(\mu)}{2\sigma_1^{\,2}}(x-\mu_1)^2+\frac{h(\mu)}{2\sigma_0^{\,2}}(x-\mu_0)^2-\frac{1}{2\sigma^2}(x-\mu)^2}\right]dx=1 \qquad (3.2.1)$$

Simplifying the above we get

$$h(\mu)\left[\sigma^2(\mu_0-\mu_1)^2\right]+\frac{1}{h(\mu)}\left[\begin{array}{l}2\sigma_0^{\,2}\sigma^2\log\sigma_1+2\sigma_0^{\,2}\sigma^2\log\sigma_0-\sigma_0^{\,2}\sigma^2\log\sigma_0^{\,2}\sigma_1^{\,2}-2\sigma_1^{\,2}\sigma^2\log\sigma_1\\-2\sigma_1^{\,2}\sigma^2\log\sigma_0+\sigma_1^{\,2}\sigma^2\log\sigma_0^{\,2}\sigma_1^{\,2}-2\sigma_0^{\,2}\sigma_1^{\,2}\log\sigma_1+2\sigma_0^{\,2}\sigma_1^{\,2}\log\sigma_0\end{array}\right]$$

$$+\frac{1}{h^2(\mu)}\left[2\sigma_0^{\,2}\sigma_1^{\,2}\log\sigma_1+2\sigma_0^{\,2}\sigma_1^{\,2}\log\sigma_0-\sigma_0^{\,2}\sigma_1^{\,2}\log\sigma_0^{\,2}\sigma_1^{\,2}\right]+\frac{\sigma_0^{\,2}\sigma^2}{h(\mu)}\left[\log h(\mu)\sigma_1^{\,2}\sigma^2-\log h(\mu)\sigma_0^{\,2}\sigma^2\right]$$

$$+\frac{\sigma_1^{\,2}\sigma^2}{h(\mu)}\left[\log h(\mu)\sigma_0^{\,2}\sigma^2-\log h(\mu)\sigma_1^{\,2}\sigma^2\right]+\frac{\sigma_0^{\,2}\sigma_1^{\,2}}{h^2(\mu)}\left[\log h(\mu)\sigma_1^{\,2}\sigma^2-\log h(\mu)\sigma_0^{\,2}\sigma^2\right]$$

$$= 2\sigma_0^2\sigma^2 \log\sigma_1 - 2\sigma_0^2\sigma^2 \log\sigma_0 - 2\sigma_1^2\sigma^2 \log\sigma_1 + 2\sigma_1^2\sigma^2 \log\sigma_0 - \sigma_1^2(\mu_0 - \mu)^2 + \sigma_0^2(\mu_1 - \mu)^2$$

To develop SPRT based on the statistic U we have used normal approximation to the distribution of U. To examine the performance of the SPRT for small values of n we have considered the graph of the OC function for small values of n equal to n=3,4,5,6 and 7.The computations and the graphs are presented below

3.3. Mean and Variance under H_0 and H_1

The values of Variance of U under H_0 and H_1 are as follows:

	Under H_0		Under H_1
	Var(U)		Var(U)
$k_0 = 0.5$	$n = 3$ 0.09515	$k_0 = 0.5$	$n = 3$ 0.0952
	$n = 4$ 0.51392		$n = 4$ 0.08284
	$n = 5$ 0.1929		$n = 5$ 0.06222
	$n = 6$ 0.10472		$n = 6$ 0.05317
	$n = 7$ 0.08693		$n = 7$ 0.05460
	Var(U)		Var(U)
$k_0 = 0.6$	$n = 3$ 0.145	$k_0 = 0.6$	$n = 3$ 0.145017
	$n = 4$ 0.44948		$n = 4$ 0.13059
	$n = 5$ 0.1892		$n = 5$ 0.09513
	$n = 6$ 0.1218		$n = 6$ 0.07343
	$n = 7$ 0.11775		$n = 7$ 0.071653

		Var(U)			Var(U)
$k_0 = 0.7$	$n = 3$	0.1905	$k_0 = 0.7$	$n = 3$	0.19046
	$n = 4$	0.4012		$n = 4$	0.179832
	$n = 5$	0.1908		$n = 5$	0.12729
	$n = 6$	0.1386		$n = 6$	0.012207
	$n = 7$	0.1424		$n = 7$	0.0857

	μ_0		μ_1
$k_0 = 0.5$	0.8935	$k_0 = 0.5$	0.10649
$k_0 = 0.6$	0.824	$k_0 = 0.6$	0.1759
$k_0 = 0.7$	0.744	$k_0 = 0.7$	0.2559

3.4. OC Function along with Graph

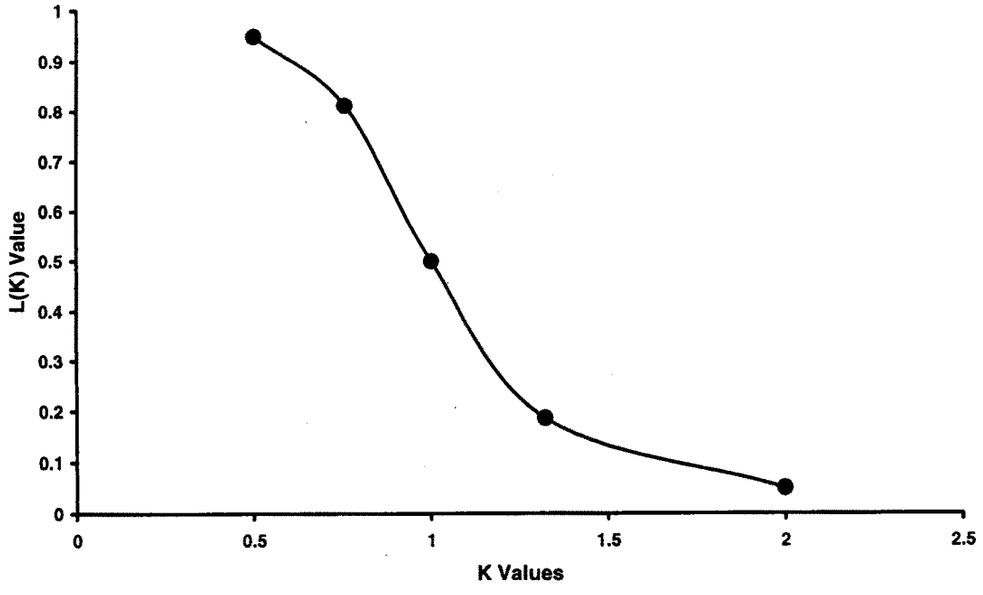
The values of OC function for $k_0 = 0.5$ and $n = 3$ are as follows

Table 3.4.1

	μ	OC function	k
$h(\mu) = -1$	0.106711	0.049998	1.998600
$h(\mu) = -0.5$	0.303380	0.186603	1.32181667
$h(\mu) = 0$	0.500048	0.49999	0.999935
$h(\mu) = 0.5$	0.6967	0.81339	0.75643965
$h(\mu) = 1$	0.89348	0.94999	0.5000427

and the corresponding graph is as follows

Figure 3.4.1.1



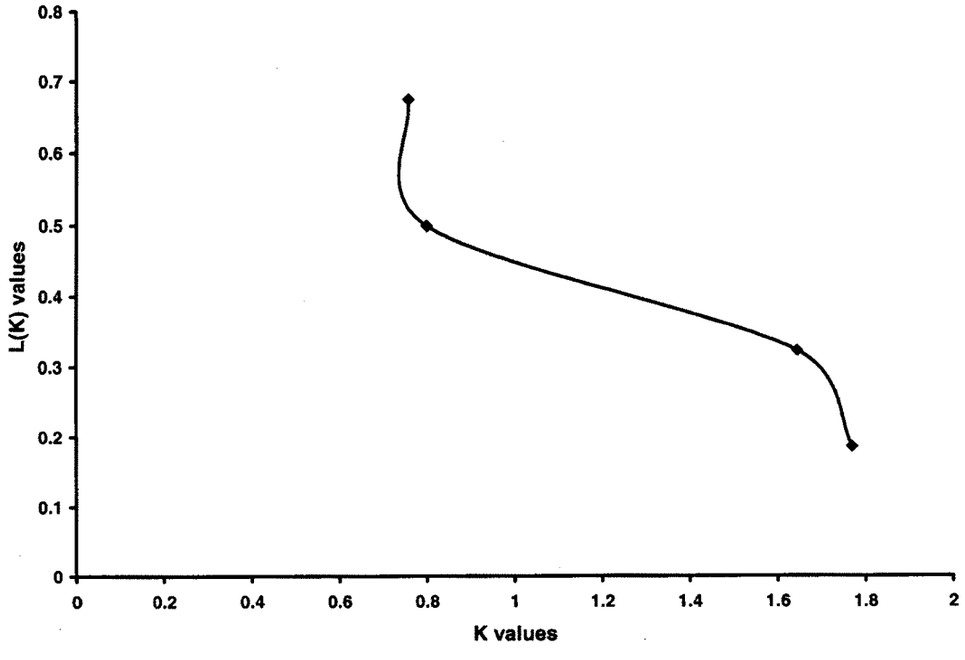
The values of OC function for $k_0 = 0.5$ and $n = 4$ are as follows

Table 3.4.2

	μ	OC function	k
$h(\mu) = -0.5$	0.1085199	0.186603	1.76899
$h(\mu) = -0.25$	0.1822533	0.323852	1.6441686
$h(\mu) = 0$	0.69637278	0.49999	0.7991897
$h(\mu) = 0.25$	1.305038	0.6761430	0.756827

and the corresponding graph is as follows

Figure 3.4.2.1



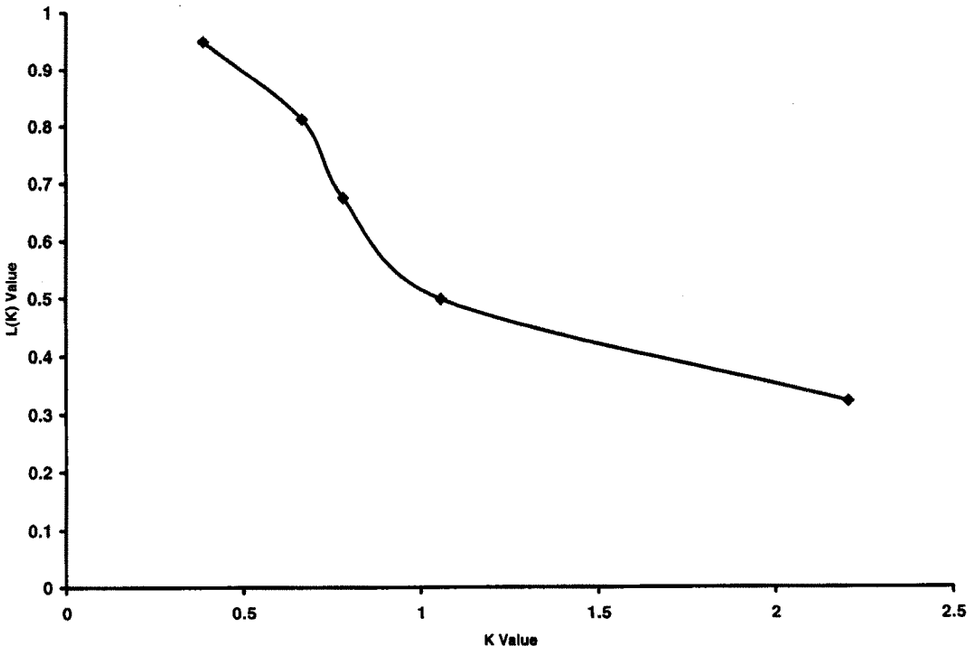
The values of OC function for $k_0 = 0.5$ and $n = 5$ are as follows

Table 3.4.3

	μ	OC function	k
$h(\mu) = -0.25$	0.079619	0.323852	2.20411
$h(\mu) = 0$	0.45986	0.49999	1.05585
$h(\mu) = 0.25$	0.71829	0.676143	0.780822
$(\mu) = 0.5$	0.77256	0.81339	0.665275
$h(\mu) = 1$	0.95224	0.94999	0.386323

and the corresponding graph is as follows

Figure 3.4.3.1



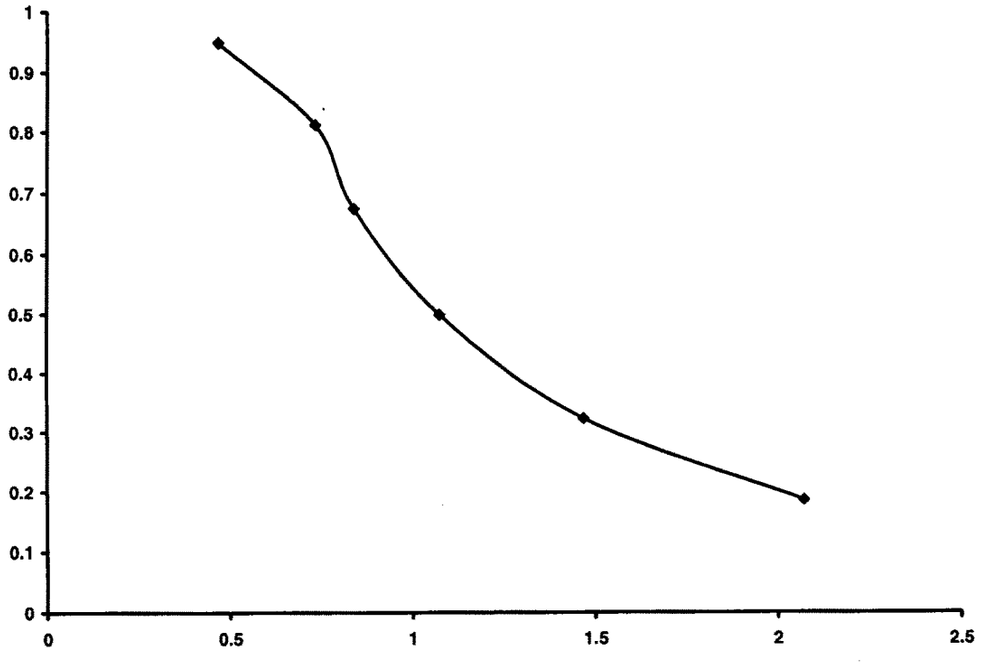
The values of OC function for $k_0 = 0.5$ and $n = 6$ are as follows

Table 3.4.4

	μ	OC function	k
$h(\mu) = -0.5$	0.0962818	0.1866025	2.06968
$h(\mu) = -0.25$	0.24103	0.323852	1.466503
$h(\mu) = 0$	0.44796	0.49999	1.073098
$h(\mu) = 0.25$	0.627647	0.676143	0.838672
$(\mu) = 0.5$	0.71657	0.81339	0.73283
$h(\mu) = 1$	0.912678	0.94999	0.46766

and the corresponding graph is as follows

Figure 3.4.4.1



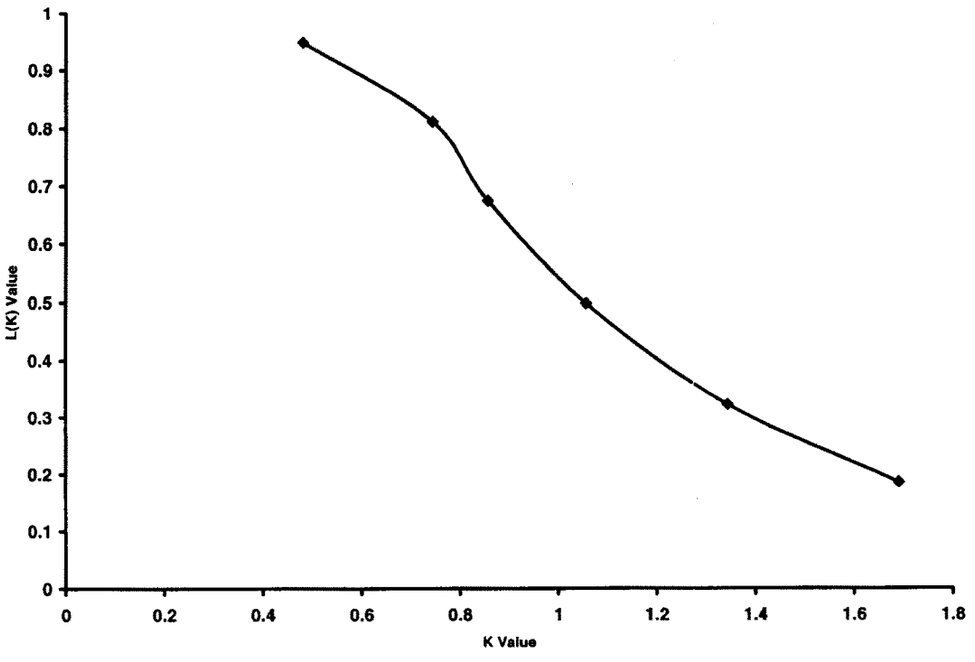
The values of OC function for $k_0 = 0.5$ and $n = 7$ are as follows

Table 3.4.5

	μ	OC function	k
$h(\mu) = -0.5$	0.16966	0.1866025	1.69031
$h(\mu) = -0.25$	0.29346	0.32385	1.34271
$h(\mu) = 0$	0.45967	0.49999	1.0561
$h(\mu) = 0.25$	0.6130402	0.676143	0.856307
$(\mu) = 0.5$	0.706831	0.81339	0.7444
$h(\mu) = 1$	0.90463	0.94999	0.48162

and the corresponding graph is as follows

Figure 3.4.5.1



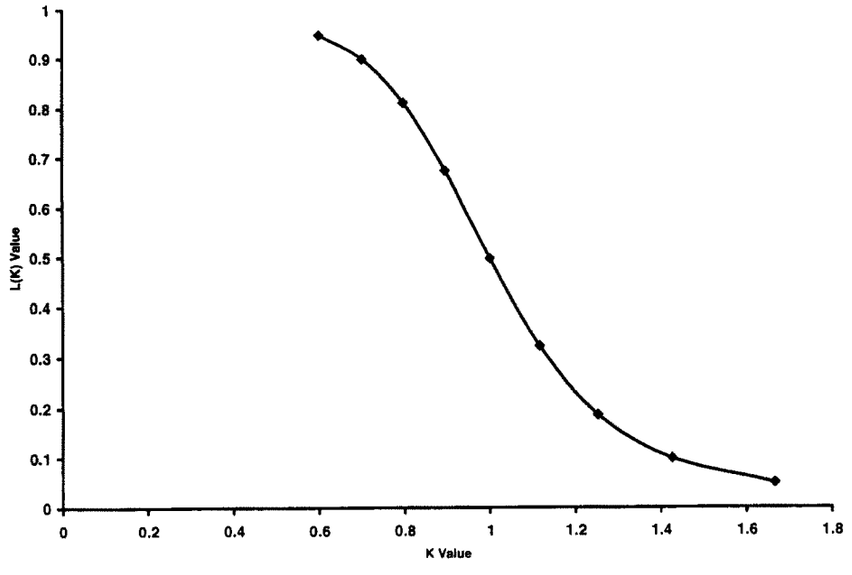
The values of OC function for $k_0 = 0.6$ and $n = 3$ are as follows

Table 3.4.6

	μ	OC function	k
$h(\mu) = -1$	0.1759322	0.04999	1.666
$h(\mu) = -0.75$	0.25693	0.099002	1.426
$h(\mu) = -0.5$	0.33793	0.1866025	1.253
$h(\mu) = -0.25$	0.41894	0.323852	1.1167
$h(\mu) = 0$	0.4999	0.49999	1.00
$h(\mu) = 0.25$	0.58099	0.676143	0.8955
$h(\mu) = 0.5$	0.66198	0.8133938	0.7976
$h(\mu) = 0.75$	0.74299	0.9009947	0.7012
$h(\mu) = 1$	0.82400	0.94999	0.600

and the corresponding graph is as follows

Figure 3.4.6.1



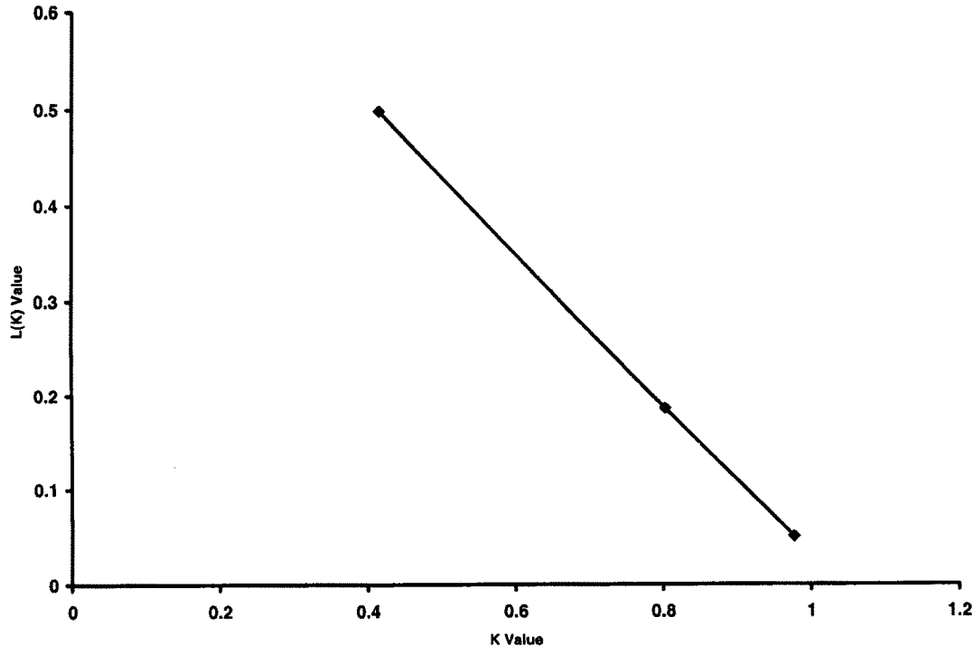
The values of OC function for $k_0 = 0.6$ and $n = 4$ are as follows

Table 3.4.7

	μ	OC function	k
$h(\mu) = -1$	0.5173	0.049998	0.97688
$h(\mu) = -0.5$	0.65777	0.1866225	0.80265
$h(\mu) = 0$	0.9395	0.4999	0.41559

and the corresponding graph is as follows

Figure 3.4.7.1



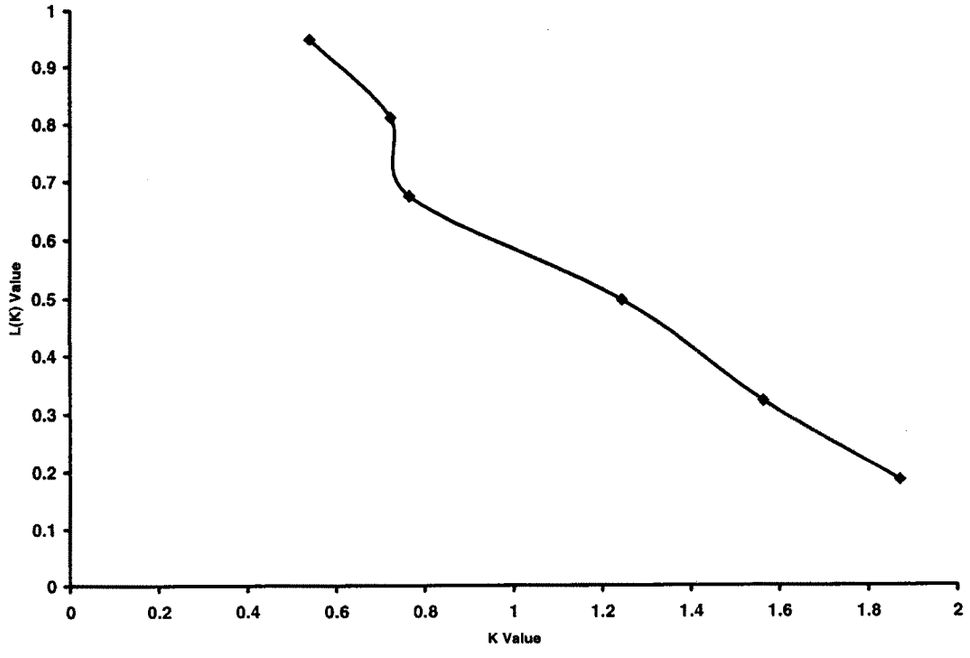
The values of OC function for $k_0 = 0.6$ and $n = 5$ are as follows

Table 3.4.8

	μ	OC function	k
$h(\mu) = -0.5$	0.128914	0.1866025	1.87069
$h(\mu) = -0.25$	0.206867	0.323852	1.56323
$h(\mu) = 0$	0.343068	0.49999	1.24432
$h(\mu) = 0.25$	0.68962	0.676143	0.76483
$h(\mu) = 0.5$	0.725307	0.81339	0.72241
$h(\mu) = 1$	0.8668839	0.94999	0.54073

and the corresponding graph is as follows

Figure 3.4.8.1



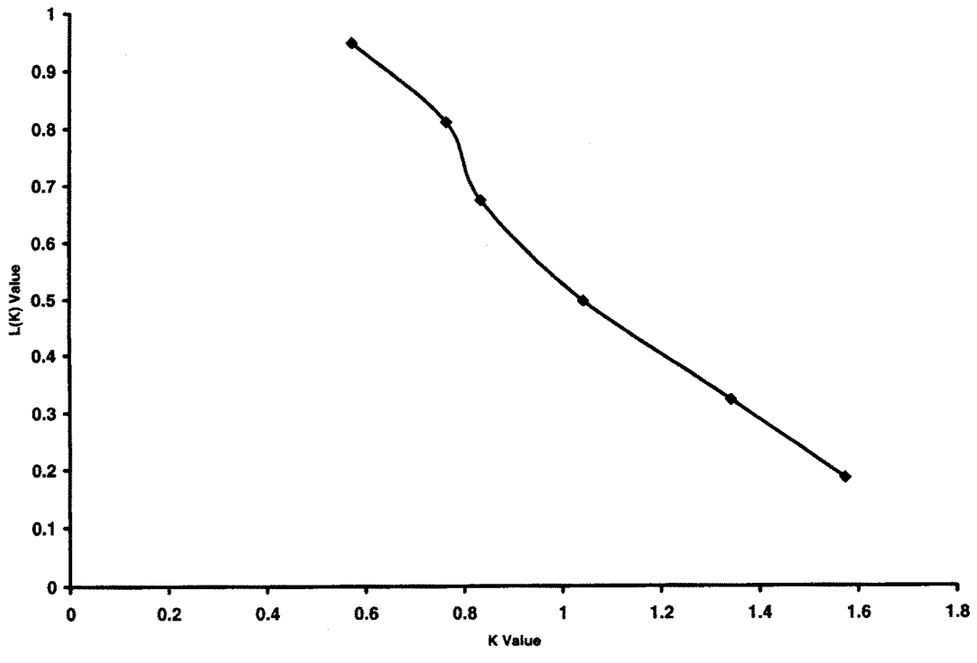
The values of OC function for $k_0 = 0.6$ and $n = 6$ are as follows

Table 3.4.9

	μ	OC function	k
$h(\mu) = -0.5$	0.203544	0.1866025	1.57353
$h(\mu) = -0.25$	0.293942	0.323852	1.34168
$h(\mu) = 0$	0.469685	0.49999	1.04186
$h(\mu) = 0.25$	0.631311	0.676143	0.834268
$h(\mu) = 0.5$	0.69011	0.81339	0.76425
$h(\mu) = 1$	0.84452	0.94999	0.57240

and the corresponding graph is as follows

Figure 3.4.9.1



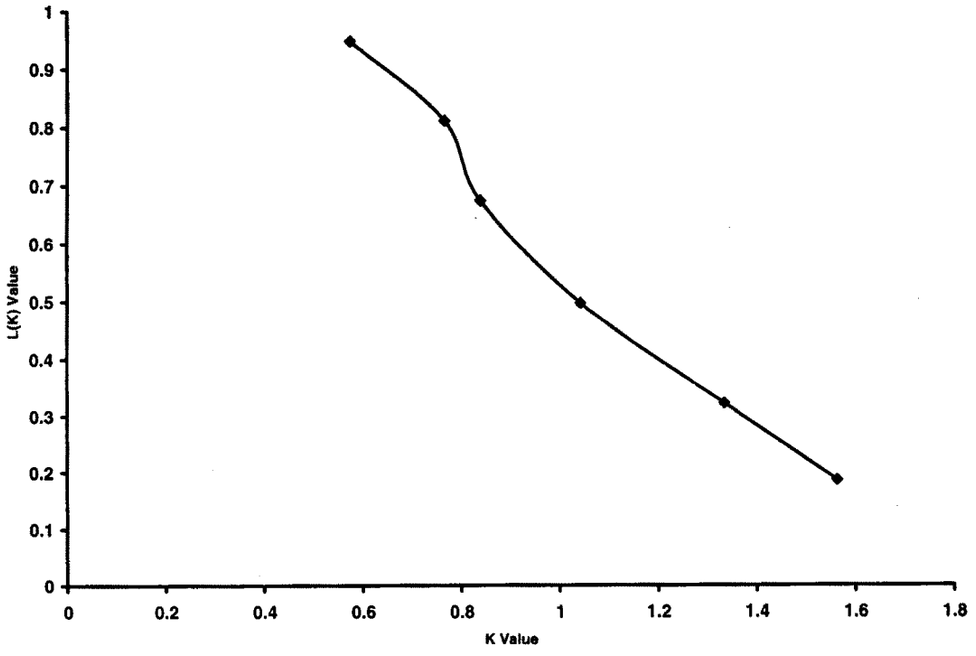
The values of OC function for $k_0 = 0.6$ and $n = 7$ are as follows

Table 3.4.10

	μ	OC function	k
$h(\mu) = -0.5$	0.207018	0.1866025	1.5627
$h(\mu) = -0.25$	0.29766	0.323852	1.3337
$h(\mu) = 0$	0.4697	0.49999	1.04184
$h(\mu) = 0.25$	0.62809	0.676143	0.838139
$h(\mu) = 0.5$	0.68842	0.81339	0.76626
$h(\mu) = 1$	0.84348	0.94999	0.5738

and the corresponding graph is as follows

Figure 3.4.10.1



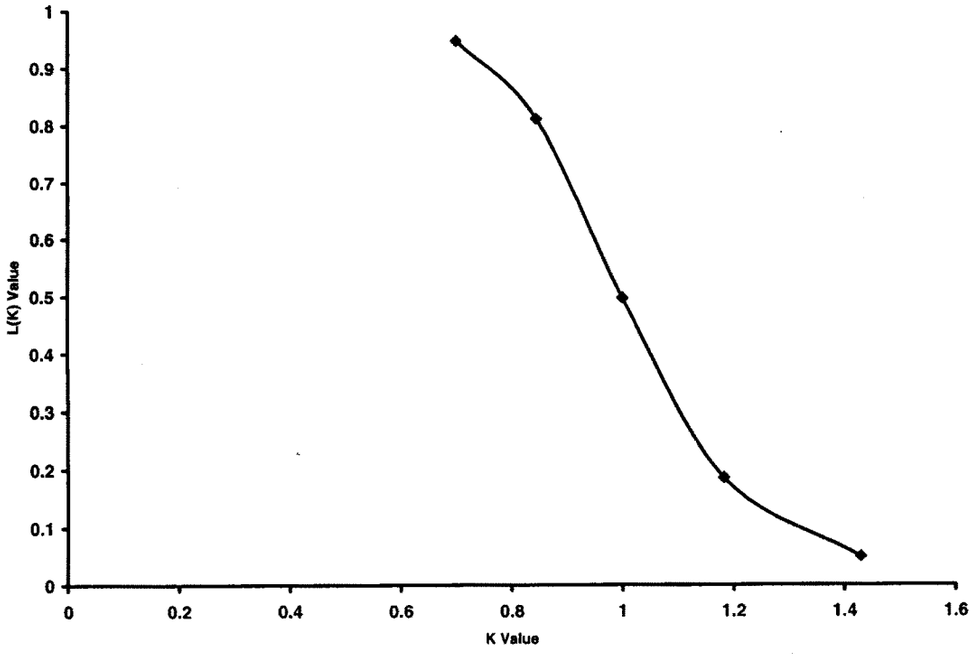
The values of OC function for $k_0 = 0.7$ and $n = 3$ are as follows

Table 3.4.11

	μ	OC function	k
$h(\mu) = -0.5$	0.3778	0.0499	1.4290
$h(\mu) = -1$	0.2558	0.186603	1.183
$h(\mu) = 0$	0.4999	0.49999	1.000
$h(\mu) = 0.5$	0.62199	0.81339	0.845
$h(\mu) = 1$	0.7440053	0.94999	0.700

and the corresponding graph is as follows

Figure 3.4.11.1



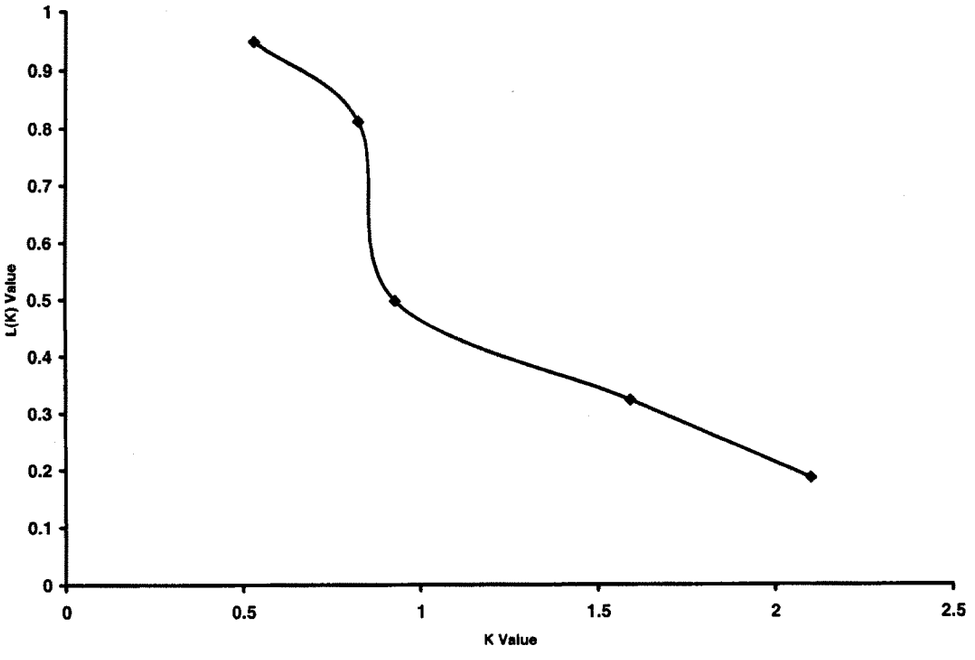
The values of OC function for $k_0 = 0.7$ and $n = 4$ are as follows

Table 3.4.12

	μ	OC function	k
$h(\mu) = -0.5$	0.09222	0.1866025	2.0998
$h(\mu) = -0.25$	0.197942	0.323852	1.59131
$h(\mu) = 0$	0.5549	0.49999	0.92825
$h(\mu) = 0.5$	0.639328	0.81339	0.82465
$h(\mu) = 1$	0.87389	0.94999	0.53038

and the corresponding graph is as follows

Figure 3.4.12.1



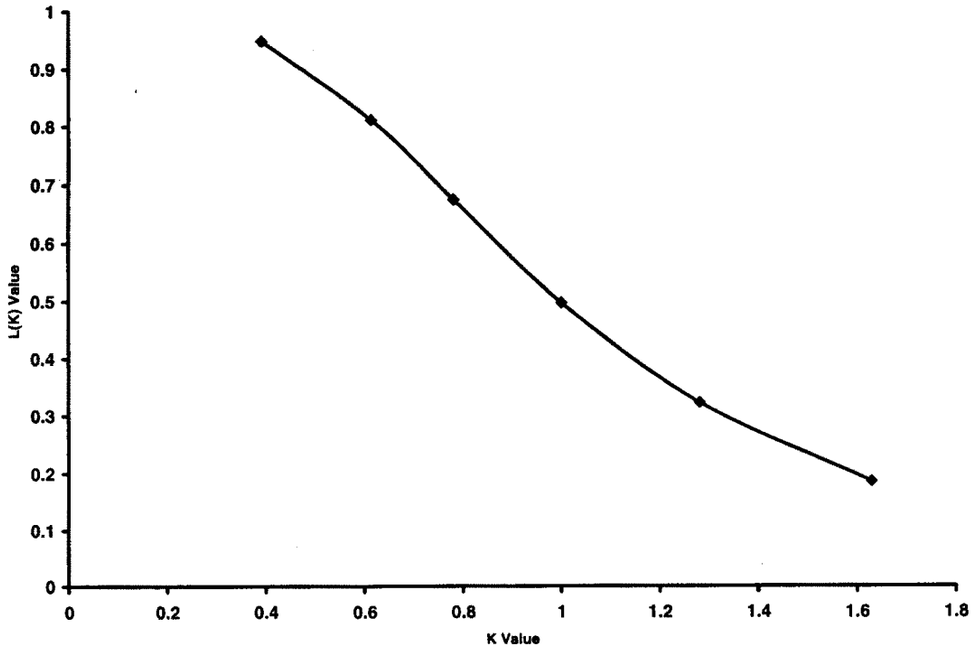
The values of OC function for $k_0 = 0.7$ and $n = 5$ are as follows

Table 3.4.13

	μ	OC function	k
$h(\mu) = -0.5$	0.250543	0.1866025	1.6290
$h(\mu) = -0.25$	0.290509	0.323852	1.2807
$h(\mu) = 0$	0.4754	0.49999	1.00
$h(\mu) = 0.25$	0.66784	0.676143	0.7808
$h(\mu) = 0.5$	0.66829	0.81339	0.613869
$h(\mu) = 1$	0.7668	0.94999	0.3920

and the corresponding graph is as follows

Figure 3.4.13.1



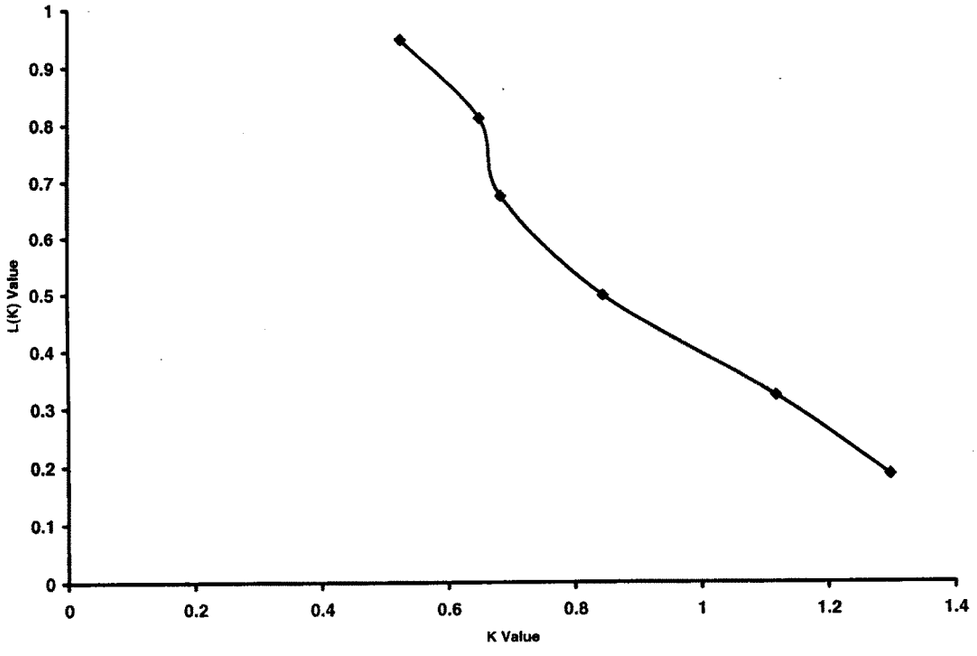
The values of OC function for $k_0 = 0.7$ and $n = 6$ are as follows

Table 3.4.14

	μ	OC function	k
$h(\mu) = -0.5$	0.3156	0.1866025	1.2969
$h(\mu) = -0.25$	0.4188	0.323852	1.1169
$h(\mu) = 0$	0.62285	0.49999	0.8444
$h(\mu) = 0.25$	0.7577	0.676143	0.68343
$h(\mu) = 0.5$	0.78503	0.81339	0.64984
$h(\mu) = 1$	0.87722	0.94999	0.52538

and the corresponding graph is as follows

Figure 3.4.14.1



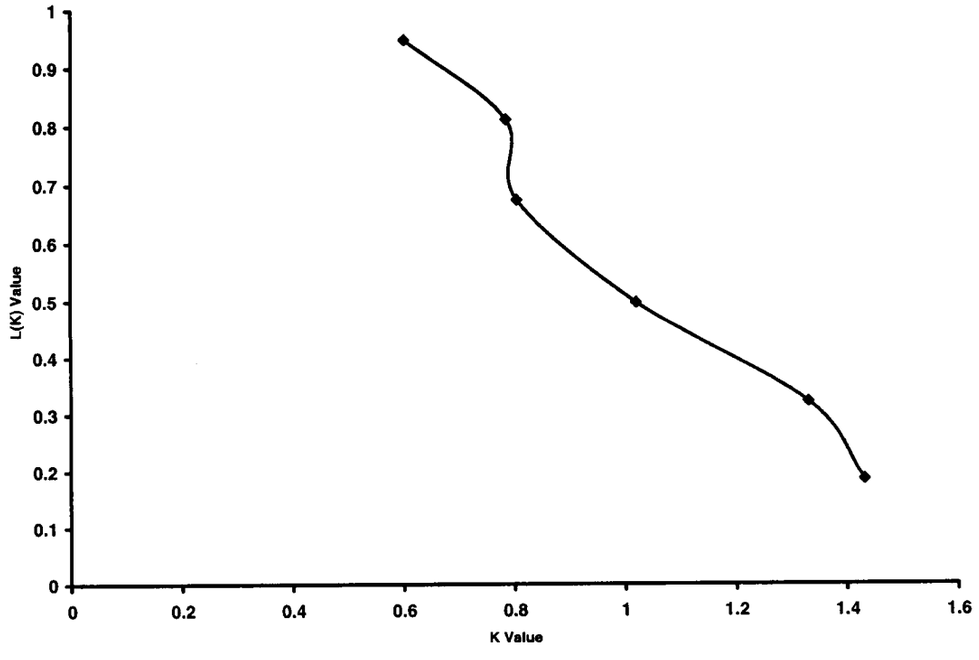
The values of OC function for $k_0 = 0.7$ and $n = 7$ are as follows

Table 3.4.15

	μ	OC function	k
$h(\mu) = -0.5$	0.25509	0.1866025	1.43079
$h(\mu) = -0.25$	0.29942	0.323852	1.3300
$h(\mu) = 0$	0.485509	0.49999	1.0197
$h(\mu) = 0.25$	0.65601	0.676143	0.80474
$h(\mu) = 0.5$	0.67255	0.81339	0.78508
$h(\mu) = 1$	0.775757	0.94999	0.60133

and the corresponding graph is as follows

Figure 3.4.15.1



3.5. Scope for further Research

The SPRT has also been developed for different distributions with shift alternatives i.e. for the Uniform and the exponential distributions. The Mean, the Variance and the OC function have been obtained but still some remaining work to be done in this direction.

CHAPTER-IV

TESTING EXPONENTIALITY AGAINST INCREASING FAILURE RATE AVERAGE ALTERNATIVES

4.1. Introduction

The class of increasing failure rate average (IFRA) distributions plays a central role in the statistical theory of reliability. IFRA distributions naturally arise when coherent systems are formed from components with independent increasing failure rate distributions. The class of increasing failure rate average (IFRA) distributions contains the class of increasing failure rate distributions as subclass. IFRA class of distributions is the smallest class of probability distributions which contains the exponential distribution & is closed under formation of coherent systems. Also these distributions arise as life distributions from various useful shock models.

Let F & G be two probability distributions such that F(0) = G(0) = 0. F is an increasing failure rate average distribution if & only if, for $x > 0$, $0 < b < 1$

$$-\log \bar{F}(bx) < -b \log \bar{F}(x),$$

or equivalently

$$\bar{F}(bx) \geq \{\bar{F}(x)\}^b \qquad (4.1.1)$$

where $\bar{F}(x) = 1 - F(x)$.

The equality in (4.1.1) above holds if & only if F is an exponential distribution.

Tests for exponentiality designed to detect the alternative hypotheses relevant in reliability theory include those of Proschan & Pyke (1967), Bickel & Doksum (1969), Ahmed (1975), Hollander & Proschan (1972, 1975) and Koul (1977, 1978) .

However, the tests that have been developed specifically for testing for increasing failure rate average alternative only is due to Deshpande (1983). Bhadri (2005) proposed a test statistic whose kernel depends on sub sample minima. The test Statistic is given by

$$V(b,k)=\binom{n}{k+1}^{-1}\sum h_b^*(X_{i_1},...X_{i_{k+1}})$$

Where $h_b^*(X_{i_1},...X_{i_{k+1}})=(k+1)^{-1}\sum_{i=1}^{k+1}h_b^i(X_1,...,X_{k+1})$

and

$$h_b^{k+1}(X_1,...,X_{k+1})=\begin{cases}1, & if \quad Min(X_1,...,X_k)>bX_{k+1}\\ 0, & otherwise,\end{cases}$$

as Kernel where b is a fixed number such that 0<b<1, and the summation is taken over all combinations of integers $(i_1,i_2,...,i_k)$ chosen out of integers (1,2,...,n).

Deshpande's test is a special case of Bhadri(2005) with k=1.

The following Lemma gives a characterization of IFRA distribution which is useful in the development of a class of test statistics.

Lemma 4.1.1

F is said to be IFRA distribution if & only if

$\bar{F}^k(bx) \geq \bar{F}^{bk}(x)$, for $x > 0$, $0 < b < 1$, $k \geq 1$

Proof

Let F be a IFRA distribution, then by (4.1.1)

$\bar{F}^k(bx) \geq \bar{F}^{bk}(x)$, for $x > 0$, $0 < b < 1$.

Hence, for $k \geq 1$,

$\bar{F}^k(bx) \geq \bar{F}^{kb}(x)$.

The converse holds if we take $k = 1$.

In Section 2 a new class of statistics is proposed which are U – Statistics whose kernel depends on sub sample median. In section 3, we deal with the mean and asymptotic variance of the statistic U under the null hypothesis. In section 4, we deal with the Pitman ARE comparisons.

4.2 Proposed class of test Statistics

Let $X_1, X_2, X_3, \ldots\ldots, X_n$ be a random sample from a continuous probability distribution with distribution function F such that $F(0) = 0$.

We wish to test the null hypothesis

$H_0 : F(x) = 1 - e^{-\theta x}$ ($x>0, \theta>0$)

with θ unspecified, against the alternative H_1 that F belongs to the increasing failure rate average class but is not exponential.

The above H_0 and H_1 may also be stated as

$H_0 : \bar{F}(bx) = \{\bar{F}(x)\}^b \qquad \{x>0,\ 0 \le b \le 1\}$

$H_1 : \bar{F}(bx) \ge \{\bar{F}(x)\}^b \qquad \{x>0,\ 0 < b < 1\},$

with strict inequality for some x.

Define a parameter

$$M(F) = \int_0^\infty \bar{F}(bx)\, dF(x).$$

If F belongs to H_0, then $M(F) = (b+1)^{-1}$ whereas for all F belonging to H_1

$$M(F) = \int_0^\infty \bar{F}(bx)\, d\,F(x) > \int_0^\infty \{\bar{F}(x)\}^b d\,F(x) = (b+1)^{-1}$$

Hence $M(F) - (b+1)^{-1}$ may be taken as a measure of deviation of F from the null hypothesis of exponentiality.

Define a kernel

$$\varphi_b^{k+1}(x_1, \ldots, x_{k+1}) = \begin{cases} 1, & if \quad Med(x_1, \ldots x_k) > bx_{k+1} \\ 0, & otherwise \end{cases},$$

where b is a fixed number such that $0 < b < 1$.

Define U(b, k) as U-statistic based on the kernel $\varphi_b^{k+1}(x_1, \ldots, x_{k+1})$, that is,

$$U(b, k) = \binom{n}{k+1}^{-1} \sum \varphi_b^*(x_{i_1}, \mathrm{K}, x_{i_{k+1}})$$

where summation is taken over all combinations of integers $(i_1, i_2, \mathrm{K}, i_k)$

chosen out of integers (1, 2, ... , n) and

$$\varphi_b^*(x_1, \mathrm{K}, x_{k+1}) = (k+1)^{-1} \sum_{i=1}^{k+1} \varphi_b^i(x_1, \mathrm{K}, x_{k+1}).$$

It is seen that

$$E[U(b,k)] = E\left[\varphi_b^{k+1}(x_1,\mathrm{K}\,,x_{k+1})\right]$$

$$= P[med(X_1,\mathrm{K}\,,X_k) > bX_{k+1}]$$

$$= \gamma(F), \text{say,}$$

and intuitively large values of the statistic U(b,k) lead to rejection of H_0 against H_1.

4.3 MEAN AND ASYMPTOTIC VARIANCE OF THE STATISTIC U UNDER THE NULL HYPOTHESIS

4.3.1 MEAN

$$E_{H_0}(U) = P[(med(X_1,\mathrm{K}\,,X_k) > bX_{k+1})]$$

$$= \int_0^\infty P[med(X_1,\mathrm{K}\,,X_k) > bx]e^{-x}.dx$$

$$= \int_0^\infty \sum_{i=0}^{a} \binom{k}{i} \left(1-e^{-bx}\right)^i \left(e^{-bx}\right)^{k-i}.e^{-x}.dx$$

where

$$a = \frac{k-1}{2}.$$

Thus, interchanging summation and integration operation, we have

$$E_{H_0}(U) = \sum_{i=0}^{a} \binom{k}{i} \int_0^\infty \left(1-e^{-bx}\right)^i \left(e^{-bx}\right)^{k-i}.e^{-x}.dx\,.$$

Letting $e^{-x} = y$ we get

$$E_{H_0}(U)=\sum_{i=0}^{a}\binom{k}{i}\int_0^1(1-y^b)^i(y^b)^{k-i}.dy.$$

$$= \sum_{i=0}^{a}\binom{k}{i}\int_0^1 z^{k-i}.(1-z)^i.\frac{1}{b}.z^{-\left(\frac{b-1}{b}\right)}.dz \qquad \text{letting } z= y^b$$

$$= \frac{1}{b}.\sum_{i=0}^{a}\binom{k}{i}\int_0^1 z^{k-i-1+\frac{1}{b}}.(1-z)^i.dz$$

$$= \frac{1}{b}\sum_{i=0}^{a}\binom{k}{i}B\left(k-i+\frac{1}{b},i+1\right)=\gamma_0 \text{(say)},$$

where

$$B(a,b) = \int_a^b x^{a-1}.(1-x)^{b-1}.dx.$$

4.3.2 VARIANCE

Now we state one - sample U – statistics theorem due to Hoeffding (1948).

Let $X_1, \ldots X_n$ denote a random sample from some population.

Let γ be an estimable parameter of degree r with symmetric kernel $h(x_1, \ldots x_r)$. If $E[h^2(X_1, \ldots X_r)] < \infty$ and

if $U(X_1, \ldots X_n) = \frac{1}{\binom{n}{r}}\sum_{\beta\in B} h(X_{\beta_1},\ldots,X_{\beta_r})$,

where B consists of subsets of r integers chosen without replacement from {1,...,n}, then $\sqrt{n}[U(X_1,\ldots,X_n)-\gamma]$ has a limiting normal distribution with mean 0 and variance $r^2\xi_1$, provided

$\xi_1 = E[h(X_1,X_2,\ldots X_r)h(X_1,X_{r+1},\ldots X_{2r-1})]-\gamma^2$ is positive.

It follows from the above theorem that

$$\sqrt{n}[U - E(U)] \sim AN(0,(k+1)^2\xi_1),$$

where

$$\xi_1 = Cov[\varphi_b^*(X_1,\Lambda\ ,X_{k+1}),\varphi_b^*(X_1,X_{k+2}\Lambda\ ,X_{2k+1})].$$

$$(k+1)^2\xi_1 = Cov[\sum_{i=1}^{k+1}\varphi_b^i(X_1,\Lambda\ ,X_{k+1}),\sum_{j=1}^{k+1}\varphi_b^j(X_1,X_{k+2}\Lambda\ ,X_{2k+1})], \qquad (4.3.2.1)$$

where

$$\varphi_b^i(X_1,\Lambda\ ,X_{k+1}) = \begin{cases} 1, & if \quad Med(X_{1,\Lambda,}X_{k+1}) > bX_i \\ 0 & otherwise \end{cases}$$

$$\varphi_b^j(X_1,\Lambda\ ,X_{2k+1}) = \begin{cases} 1 & if \quad Med(X_{1,}X_{k+2},...,X_{2k+1}) > bX_j \\ 0 & otherwise \end{cases}$$

Now

$$Cov[\varphi_b^*(X_1,\Lambda\ ,X_{k+1}),\varphi_b^*(X_1,X_{k+2}\Lambda\ ,X_{2k+1})]$$

$$= E[\varphi_b^*(X_1,\Lambda\ ,X_{k+1})\varphi_b^*(X_1,X_{k+2}\Lambda\ ,X_{2k+1})] - \gamma_0^2$$

$$= \int_0^\infty E^2[\varphi_b^*(x,X_2,...,X_{k+1})]e^{-x}dx - \gamma_0^2 . \qquad (4.3.2.2)$$

Now consider

$$E[\varphi_b^*(x,X_2,...,X_{k+1})].$$

$$(k+1)E[\varphi_b^*(x,X_2,...,X_{k+1})] = P[Med(X_2,...,X_{k+1}) > bx] + \sum_{i=2}^{k+1}E[h^i(x,X_2,...,X_{k+1})]$$

(4.3.2.3)

Now,

$$P[Med(X_2,...,X_{k+1}) > bx] = \int_{bx}^\infty \frac{k!}{(a!)^2}[1-e^{-y}]^a[e^{-y}]^a e^{-y}dy . \qquad (4.3.2.3a)$$

Letting u= e^{-y} ,we have

$$P[Med(X_2,\ldots,X_{k+1}) > bx] = \int_0^{e^{-bx}} \frac{k!}{(a!)^2}[1-u]^a[u]^a\,du\,.$$

Now

$$E[h^i(x,X_2,\ldots,X_{k+1})]$$

$$= P[Med(x,X_2,\ldots,X_{i-1},X_{i+1},\ldots,X_{k+1}) > bX_i]$$

$$= \binom{k}{\frac{k-1}{2}} P[Max(X_2,\ldots,X_{\frac{k+1}{2}}) < x < Min(X_{\frac{k+3}{2}},\ldots,X_k), x > bX_i]$$

$$+ k\binom{k-1}{\frac{k-3}{2}} P[Max(x,X_2,\ldots,X_{\frac{k-1}{2}}) < X_{\frac{k+1}{2}} < Min(X_{\frac{k+3}{2}},\ldots,X_k), X_a > bX_i]$$

$$+ k\binom{k-1}{\frac{k-3}{2}} P[Max(X_2,\ldots,X_{\frac{k+1}{2}}) < X_{\frac{k+3}{2}} < Min(x,X_{\frac{k+5}{2}},\ldots,X_k), X_{a+1} > bX_i]$$

$$= \binom{k}{\frac{k-1}{2}} A + k\binom{k-1}{\frac{k-3}{2}} B + k\binom{k-1}{\frac{k-3}{2}} C. \qquad (4.3.2.4)$$

where

$$A = P[Max(X_2,..,X_{i-1},X_{i+1},..,X_{\frac{k+1}{2}}) < x < Min(X_{\frac{k+3}{2}},\ldots,X_k), x > bX_i]$$

$$= P[bX_i < Max(X_2,\ldots,X_{i-1},X_{i+1},\ldots,X_{\frac{k+1}{2}}) < x < Min(X_{\frac{k+3}{2}},\ldots,X_k)]$$

$$+ P[Max(X_2,\ldots,X_{i-1},X_{i+1},\ldots,X_{\frac{k+1}{2}}) < bX_i < x < Min(X_{\frac{k+3}{2}},\ldots,X_k)] \qquad (4.3.2.5)$$

Consider the first term on the right hand side of (4.3.2.5)

$$P[bX_i < Max(X_2,\ldots,X_{i-1},X_{i+1},\ldots,X_{\frac{k+1}{2}}) < x < Min(X_{\frac{k+3}{2}},\ldots,X_k)]$$

$$= P[bX_i < Max(X_2,\ldots,X_{i-1},X_{i+1},\ldots,X_{\frac{k+1}{2}}) < x]P[x < Min(X_{\frac{k+3}{2}},\ldots,X_k]$$

$$= P[X_i < \frac{1}{b} Max(X_2,...,X_{i-1},X_{i+1},...,X_{\frac{k+1}{2}}) < \frac{x}{b}][\overline{F}(x)]^{k-\frac{k+3}{2}+1}$$

$$= [\int_0^{\frac{x}{b}} P[by < Max(X_2,...,X_{i-1},X_{i+1},...,X_{\frac{k+1}{2}}) < x]e^{-y}dy][\overline{F}(x)]^{\frac{k-1}{2}}$$

$$= \left\{\int_0^{\frac{x}{b}} [(1-e^{-x})^{\frac{k+1}{2}-2-1+1} - (1-e^{-yb})^{\frac{k+1}{2}-2-1+1}]e^{-y}dy\right\}[e^{-x}]^{\frac{k-1}{2}}$$

$$= \left\{\int_0^{\frac{x}{b}} [(1-e^{-x})^{\frac{k-3}{2}} e^{-y}dy - \int_0^{\frac{x}{b}} (1-e^{-yb})^{\frac{k-3}{2}} e^{-y}dy\right\}[e^{-x}]^{\frac{k-1}{2}}.$$

Consider

$$\int_0^{\frac{x}{b}} (1-e^{-yb})^{\frac{k-3}{2}} e^{-y}dy$$

$$= \int_{e^{-\frac{x}{b}}}^{1} (1-z^b)^{\frac{k-3}{2}} dz \text{ letting } e^{-y} = z$$

$$= \int_{e^{-\frac{x}{b}}}^{1} \sum_{i=0}^{\frac{k-3}{2}} \binom{k-3}{2}{i} (-1)^i z^{bi} dz$$

$$= \sum_{i=0}^{\frac{k-3}{2}} \binom{k-3}{2}{i} (-1)^i \left[\frac{1-(e^{-\frac{x}{b}})^{ib+1}}{ib+1}\right]$$

Thus we get

$$P[bX_i < Max(X_2,...,X_{i-1},X_{i+1},...,X_{\frac{k+1}{2}}) < x < Min(X_{\frac{k+3}{2}},...,X_k)]$$

$$=[(1-e^{-x})^{\frac{k-3}{2}}(1-(e^{-\frac{x}{b}}))][e^{-x}]^{\frac{k-1}{2}}-\left[\sum_{i=0}^{\frac{k-3}{2}}\binom{k-3}{2}{i}(-1)^i\left[\frac{1-(e^{-\frac{x}{b}})^{ib+1}}{ib+1}\right]\right][e^{-x}]^{\frac{k-1}{2}}.$$

Now consider the second term on the right hand side of (4.3.2.5).

$$P[Max(X_2,...,X_{i-1},X_{i+1},...,X_{\frac{k+1}{2}})<bX_i<x<Min(X_{\frac{k+3}{2}},...,X_k)]$$

$$=P[Max(X_2,...,X_{i-1},X_{i+1},...,X_{\frac{k+1}{2}})<bX_i<x]P[x<Min(X_{\frac{k+3}{2}},...,X_k)]$$

$$=P[\frac{1}{b}Max(X_2,...,X_{i-1},X_{i+1},...,X_{\frac{k+1}{2}})<X_i<\frac{x}{b}]\overline{F}^{\frac{k-1}{2}}(x)$$

$$=\left\{\int_0^{\frac{x}{b}}P[Max(X_2,...,X_{i-1},X_{i+1},...,X_{\frac{k+1}{2}})<by]e^{-y}dy\right\}\overline{F}^{\frac{k-1}{2}}(x)$$

$$=\left\{\int_0^{\frac{x}{b}}(1-e^{-by})^{\frac{k-3}{2}}e^{-y}dy\right\}[e^{-x}]^{\frac{k-1}{2}}$$

$$=\left[\sum_{i=0}^{\frac{k-3}{2}}\binom{k-3}{2}{i}(-1)^i\left[\frac{1-(e^{-\frac{x}{b}})^{ib+1}}{ib+1}\right]\right](e^{-x})^{\frac{k-1}{2}}.$$

Thus by adding the two terms on the right hand side of (4.3.2.5), we have

$$\mathbf{A}=[(1-e^{-x})^{\frac{k-3}{2}}(1-(e^{-\frac{x}{b}}))][e^{-x}]^{\frac{k-1}{2}}-$$

$$\left[\sum_{i=0}^{\frac{k-3}{2}}\binom{k-3}{2}{i}(-1)^i\left[\frac{1-(e^{-\frac{x}{b}})^{ib+1}}{ib+1}\right]\right][e^{-x}]^{\frac{k-1}{2}}+\left[\sum_{i=0}^{\frac{k-3}{2}}\binom{k-3}{2}{i}(-1)^i\left[\frac{1-(e^{-\frac{x}{b}})^{ib+1}}{ib+1}\right]\right](e^{-x})^{\frac{k-1}{2}}$$

$$=[(1-e^{-x})^{\frac{k-3}{2}}(1-(e^{-\frac{x}{b}}))][e^{-x}]^{\frac{k-1}{2}}.$$

Now Consider

$$\mathbf{B} = P[Max(x, X_2,..., X_{i-1}, X_{i+1},...X_{\frac{k-1}{2}}) < X_{\frac{k+1}{2}} < Min(X_{\frac{k+3}{2}},..., X_k), X_{\frac{k+1}{2}} > bX_i]$$

$$= P[bX_i < Max(x, X_2,..., X_{i-1}, X_{i+1},..., X_{\frac{k-1}{2}}) < X_{\frac{k+1}{2}} < Min(X_{\frac{k+3}{2}},..., X_k)]$$

$$+ P[Max(x, X_2,..., X_{i-1}, X_{i+1},..., X_{\frac{k-1}{2}}) < bX_i < X_{\frac{k+1}{2}} < Min(X_{\frac{k+3}{2}},..., X_k)]$$

$$= B_1 + B_2 \quad \text{(say)} \qquad (4.3.2.6)$$

Now consider the first term on the right hand side of (4.3.2.6)

$$B_1 = P[bX_i < Max(x, X_2,..., X_{i-1}, X_{i+1},..., X_{\frac{k-1}{2}}) < X_{\frac{k+1}{2}} < Min(X_{\frac{k+3}{2}},..., X_k)]$$

$$= P[bX_i < x < Max(X_2,..., X_{i-1}, X_{i+1},..., X_{\frac{k-1}{2}}) < X_{\frac{k+1}{2}} < Min(X_{\frac{k+3}{2}},..., X_k)]$$

$$+ P[bX_i < Max(X_2,..., X_{i-1}, X_{i+1},..., X_{\frac{k-1}{2}}) < x < X_{\frac{k+1}{2}} < Min(X_{\frac{k+3}{2}},..., X_k)]$$

$$+ P[x < bX_i < Max(X_2,..., X_{i-1}, X_{i+1},..., X_{\frac{k-1}{2}}) < X_{\frac{k+1}{2}} < Min(X_{\frac{k+3}{2}},..., X_k)]$$

$$+ P[Max(X_2,..., X_{i-1}, X_{i+1},..., X_{\frac{k-1}{2}}) < bX_i < x < X_{\frac{k+1}{2}} < Min(X_{\frac{k+3}{2}},..., X_k)]$$

$$= B_{11} + B_{12} + B_{13} + B_{14} \quad \text{say.} \qquad (4.3.2.7)$$

Now consider the first term on the right hand side of (4.3.2.7)

$$B_{11} = P[bX_i < x < Max(X_2,..., X_{i-1}, X_{i+1},..., X_{\frac{k-1}{2}}) < X_{\frac{k+1}{2}} < Min(X_{\frac{k+3}{2}},..., X_k)]$$

$$= P[bX_i < x]P[x < Max(X_2,..., X_{i-1}, X_{i+1},..., X_{\frac{k-1}{2}}) < X_{\frac{k+1}{2}} < Min(X_{\frac{k+3}{2}},..., X_k)]$$

$$= F\left(\frac{x}{b}\right)\left[\int_{x}^{\infty}\int_{x}^{x_3}\left[F^{\frac{k-1}{2}-2-1+1}(x_2) - F^{\frac{k-1}{2}-2-1+1}(x)\right]dF(x_2)\left(k-\frac{k+3}{2}+1\right)F^{k-\frac{k+3}{2}+1-1}(x_3)dF(x_3)\right]$$

$$= \left[1-e^{-\frac{x}{b}}\right]\left[\int_{x}^{\infty}\int_{x}^{x_3}\left[\left(1-e^{-x_2}\right)^{\frac{k-5}{2}} - \left(1-e^{-x}\right)^{\frac{k-5}{2}}\right]e^{-x_2}dx_2\left(\frac{k-1}{2}\right)\left(e^{-x_3}\right)^{\frac{k-3}{2}}e^{-x_3}dx_3\right]$$

$$= \left[1-e^{-\frac{x}{b}}\right]\left[\int_{x}^{\infty}\int_{x}^{x_3}\left[\left(1-e^{-x_2}\right)^{\frac{k-5}{2}}\right]e^{-x_2}dx_2\left(\frac{k-1}{2}\right)\left(e^{-x_3}\right)^{\frac{k-3}{2}}e^{-x_3}dx_3\right]$$

$$-\left[1-e^{-\frac{x}{b}}\right]\left[\int_{x}^{\infty}\int_{x}^{x_3}\left[\left(1-e^{-x}\right)^{\frac{k-5}{2}}\right]e^{-x_2}dx_2\left(\frac{k-1}{2}\right)\left(e^{-x_3}\right)^{\frac{k-3}{2}}e^{-x_3}dx_3\right].$$

Next consider second term on the right hand side of (4.3.2.7)

$$B_{12} = P[bX_i < Max(X_2,\ldots,X_{i-1},X_{i+1},\ldots,X_{\frac{k-1}{2}}) < x < X_{\frac{k+1}{2}} < Min(X_{\frac{k+3}{2}},\ldots,X_k)]$$

$$= P[bX_i < Max(X_2,\ldots,X_{i-1},X_{i+1},\ldots,X_{\frac{k-1}{2}}) < x]P[x < X_{\frac{k+1}{2}} < Min(X_{\frac{k+3}{2}},\ldots,X_k)]$$

$$= \left[\int_{0}^{\frac{x}{b}} P\left[by \le Max\left(X_2,\ldots,X_{i-1},X_{i+1},\ldots,X_{\frac{k+1}{2}}\right) < x\right]e^{-y}dy\right]\left[\int_{x}^{\infty}\left(e^{-x_2}\right)^{\frac{k-1}{2}}e^{-x_2}dx_2\right]$$

$$= \left[\int_{0}^{\frac{x}{b}}\left\{\left[1-e^{-x}\right]^{\frac{k-5}{2}} - \left[1-e^{-yb}\right]^{\frac{k-5}{2}}\right\}e^{-y}dy\right]\left[\int_{x}^{\infty}\left(e^{-x_2}\right)^{\frac{k-1}{2}}e^{-x_2}dx_2\right].$$

Consider

$$\int_{x}^{\infty}\left(e^{-x_2}\right)^{\frac{k-1}{2}}e^{-x_2}dx_2$$

$$= \frac{\left(e^{-x}\right)^{\frac{k+1}{2}}}{\frac{k+1}{2}} \qquad \text{letting } e^{-x_2} = z.$$

Thus we get

$$B_{12}=\left[\int_0^{\frac{x}{b}}\left\{\left[1-e^{-x}\right]^{\frac{k-5}{2}}\right\}e^{-y}dy\left(\frac{\left(e^{-x}\right)^{\frac{k+1}{2}}}{\frac{k+1}{2}}\right)\right]-\left[\int_0^{\frac{x}{b}}\left\{\left[1-e^{-yb}\right]^{\frac{k-5}{2}}\right\}e^{-y}dy\left(\frac{\left(e^{-x}\right)^{\frac{k+1}{2}}}{\frac{k+1}{2}}\right)\right]$$

Now consider

$$B_{13}=P[x<bX_i<Max(X_2,\ldots,X_{i-1},X_{i+1},\ldots,X_{\frac{k-1}{2}})<X_{\frac{k+1}{2}}<Min(X_{\frac{k+3}{2}},\ldots,X_k)]$$

$$=\int_{\frac{x}{b}}^{\infty}\int_{by}^{\infty}\int_{x_1}^{\infty}\int_{x_2}^{\infty}\left(\frac{k-1}{2}\right)\left(e^{-x_3}\right)^{\frac{k-3}{2}}e^{-x_3}dx_3e^{-x_2}dx_2\left(\frac{k-5}{2}\right)\left(1-e^{-x_1}\right)^{\frac{k-7}{2}}e^{-x_1}dx_1e^{-y}dy$$

$$=\left(\frac{k-1}{2}\right)\left(\frac{k-5}{2}\right)\frac{1}{\left(\frac{k-1}{2}\right)\left(\frac{k+1}{2}\right)}\int_{\frac{x}{b}}^{\infty}\int_{by}^{\infty}\left(1-e^{-x_1}\right)^{\frac{k-7}{2}}\left(e^{-x_1}\right)\left(e^{-x_1}\right)^{\frac{k+1}{2}}dx_1e^{-y}dy$$

Further consider

$$B_{14}=P[Max(X_2,\ldots,X_{i-1},X_{i+1},\ldots,X_{\frac{k-1}{2}})<bX_i<x<X_{\frac{k+1}{2}}<Min(X_{\frac{k+3}{2}},\ldots,X_k)]$$

$$=P[Max(X_2,\ldots,X_{i-1},X_{i+1},\ldots,X_{\frac{k-1}{2}})<bX_i<x]P[x<X_{\frac{k+1}{2}}<Min(X_{\frac{k+3}{2}},\ldots,X_k)]$$

$$=\left[\int_0^{\frac{x}{b}}P\left[Max\left(X_2,\ldots,X_{i-1},X_{i+1},\ldots,X_{\frac{k-1}{2}}\right)<by\right]e^{-y}dy\right]\left[\int_x^{\infty}\overline{F}^{k-\frac{k+3}{2}+1}(x_2)dF(x_2)\right]$$

$$=\int_0^{\frac{x}{b}}\left\{\left[1-e^{-by}\right]^{\frac{k-5}{2}}\right\}e^{-y}dy\left(\frac{\left(e^{-x}\right)^{\frac{k+1}{2}}}{\frac{k+1}{2}}\right).$$

Now consider second term on the r.h.s. of (4.3.2.6)

$$B_2=P[Max(x,X_2,\ldots,X_{i-1},X_{i+1},\ldots,X_{\frac{k-1}{2}})<bX_i<X_{\frac{k+1}{2}}<Min(X_{\frac{k+3}{2}},\ldots,X_k)$$

$$=P[x<Max(X_2,\ldots,X_{i-1},X_{i+1},\ldots,X_{\frac{k-1}{2}})<bX_i<X_{\frac{k+1}{2}}<Min(X_{\frac{k+3}{2}},\ldots,X_k)]$$

$$+\ P[Max(X_2,...,X_{i-1},X_{i+1},...,X_{\frac{k-1}{2}}) < x < bX_i < X_{\frac{k+1}{2}} < Min(X_{\frac{k+3}{2}},...,X_k)]$$

$$=B_{21}+B_{22} \quad \text{say} \qquad (4.3.2.6b)$$

Now consider

$$B_{21} = P[x < Max(X_2,...,X_{i-1},X_{i+1},...,X_{\frac{k-1}{2}}) < bX_i < X_{\frac{k+1}{2}} < Min(X_{\frac{k+3}{2}},...,X_k)]$$

$$=\int_{x}^{\infty}\int_{\frac{x_1}{b}}^{\infty}\int_{by}^{\infty}\int_{x_2}^{\infty}\left(\frac{k-1}{2}\right)\left(e^{-x_3}\right)^{\frac{k-3}{2}}e^{-x_3}dx_3e^{-x_2}dx_2e^{-y}dy\left(\frac{k-5}{2}\right)\left(1-e^{-x_1}\right)^{\frac{k-7}{2}}e^{-x_1}dx_1$$

$$=\left(\frac{k-1}{2}\right)\left(\frac{k-5}{2}\right)\left(\frac{1}{\left(\frac{k-1}{2}\right)}\right)\left(\frac{1}{\left(\frac{k+1}{2}\right)}\right)\left(\frac{1}{\left(\frac{k+3}{2}\right)}\right)\int_{x}^{\infty}\left(e^{-x_1}\right)^{\frac{k+3}{2}}\left(1-e^{-x_1}\right)^{\frac{k-7}{2}}e^{-x_1}dx_1 \cdot$$

Further

$$B_{22} = P[Max(X_2,...,X_{i-1},X_{i+1},...,X_{\frac{k-1}{2}}) < x < bX_i < X_{\frac{k+1}{2}} < Min(X_{\frac{k+3}{2}},...,X_k)]$$

$$=\left(1-e^{-x}\right)^{\frac{k-5}{2}}\left[\int_{\frac{x}{b}}^{\infty}\int_{by}^{\infty}\int_{x_2}^{\infty}\left(\frac{k-1}{2}\right)\left(e^{-x_3}\right)^{\frac{k-3}{2}}e^{-x_3}dx_3e^{-x_2}dx_2e^{-y}dy\right]$$

$$=\left(1-e^{-x}\right)^{\frac{k-5}{2}}\left(\left(\frac{1}{\left(\frac{k+1}{2}\right)}\right)\left(\frac{1}{\left(\frac{k+3}{2}\right)}\right)\left(e^{-x}\right)^{\frac{k-3}{2}}\right).$$

Now we get B by adding B1 and B2.

$$B=\left[1-e^{-\frac{x}{b}}\right]\left[\int_{x}^{\infty}\int_{x}^{x_3}\left[\left(1-e^{-x_2}\right)^{\frac{k-5}{2}}\right]e^{-x_2}dx_2\left(\frac{k-1}{2}\right)\left(e^{-x_3}\right)^{\frac{k-3}{2}}e^{-x_3}dx_3\right]$$

$$-\left[1-e^{-\frac{x}{b}}\right]\left[\int_{x}^{\infty}\int_{x}^{x_3}\left[\left(1-e^{-x}\right)^{\frac{k-5}{2}}\right]e^{-x_2}dx_2\left(\frac{k-1}{2}\right)\left(e^{-x_3}\right)^{\frac{k-3}{2}}e^{-x_3}dx_3\right]$$

$$+\left[\int_0^{\frac{x}{b}}\left\{\left[1-e^{-x}\right]^{\frac{k-5}{2}}\right\}e^{-y}dy\left(\frac{\left(e^{-x}\right)^{\frac{k+1}{2}}}{\frac{k+1}{2}}\right)\right]$$

$$+\left(\frac{k-1}{2}\right)\left(\frac{k-5}{2}\right)\frac{1}{\left(\frac{k-1}{2}\right)\left(\frac{k+1}{2}\right)}\int_{\frac{x}{b}}^{\infty}\int_{by}^{\infty}\left(1-e^{-x_1}\right)^{\frac{k-7}{2}}\left(e^{-x_1}\right)\left(e^{-x_1}\right)^{\frac{k+1}{2}}dx_1e^{-y}dy$$

$$+\left(\frac{k-1}{2}\right)\left(\frac{k-5}{2}\right)\left(\frac{1}{\left(\frac{k-1}{2}\right)}\right)\left(\frac{1}{\left(\frac{k+1}{2}\right)}\right)\left(\frac{1}{\left(\frac{k+3}{2}\right)}\right)\int_x^{\infty}\left(e^{-x_1}\right)^{\frac{k+3}{2}}\left(1-e^{-x_1}\right)^{\frac{k-7}{2}}e^{-x_1}dx_1$$

$$+\left(1-e^{-x}\right)^{\frac{k-5}{2}}\left(\left(\frac{1}{\left(\frac{k+1}{2}\right)}\right)\left(\frac{1}{\left(\frac{k+3}{2}\right)}\right)\left(e^{-x}\right)^{\frac{k-3}{2}}\right).$$

Now consider C

$$C = P[Max(X_2,\ldots,X_{i-1},X_{i+1},\ldots,X_{\frac{k+1}{2}}) < X_{\frac{k+3}{2}} < Min(x,X_{\frac{k+5}{2}},\ldots,X_k),X_{\frac{k+3}{2}} > bX_i]$$

$$= P[bX_i < Max(X_2,\ldots,X_{i-1},X_{i+1},\ldots,X_{\frac{k+1}{2}}) < X_{\frac{k+3}{2}} < x < Min(X_{\frac{k+5}{2}},\ldots,X_k)]$$

$$+ P[bX_i < Max(X_2,\ldots,X_{i-1},X_{i+1},\ldots,X_{\frac{k+1}{2}}) < X_{\frac{k+3}{2}} < Min(X_{\frac{k+5}{2}},\ldots,X_k) < x]$$

$$+ P[Max(X_2,\ldots,X_{i-1},X_{i+1},\ldots,X_{\frac{k+1}{2}}) < bX_i < X_{\frac{k+3}{2}} < x < Min(X_{\frac{k+5}{2}},\ldots,X_k)]$$

$$+ P[Max(X_2,\ldots,X_{i-1},X_{i+1},\ldots,X_{\frac{k+1}{2}}) < bX_i < X_{\frac{k+3}{2}} < Min(X_{\frac{k+5}{2}},\ldots,X_k) < x]$$

$$=C_1+C_2+C_3+C_4 \quad \text{say} \qquad (4.3.2.7)$$

Now consider

$$C_1 = P[bX_i < Max(X_2,\ldots,X_{i-1},X_{i+1},\ldots,X_{\frac{k+1}{2}}) < X_{\frac{k+3}{2}} < x < Min(X_{\frac{k+5}{2}},\ldots,X_k)]$$

$$= P[bX_i < Max(X_2,...,X_{i-1},X_{i+1},...,X_{\frac{k+1}{2}}) < X_{\frac{k+3}{2}} < x]P[x < Min(X_{\frac{k+5}{2}},...,X_k)]$$

$$= \left[\int_0^{\frac{x}{b}}\int_{by}^{x}\int_{x_1}^{x} e^{-x_2}dx_2\left(\frac{k-3}{2}\right)\left(1-e^{-x_1}\right)^{\frac{k-5}{2}}e^{-x_1}dx_1e^{-y}dy\right]\left(e^{-x}\right)^{\frac{k-3}{2}}$$

$$= \left\{\left(\frac{k-3}{2}\right)\int_0^{\frac{x}{b}}\int_{by}^{x} e^{-x_1}\left(1-e^{-x_1}\right)^{\frac{k-5}{2}}e^{-x_1}dx_1e^{-y}dy - \left(\frac{k-3}{2}\right)\int_0^{\frac{x}{b}}\int_{by}^{x} e^{-x}\left(1-e^{-x_1}\right)^{\frac{k-5}{2}}e^{-x_1}dx_1e^{-y}dy\right\}\left(e^{-x}\right)^{\frac{k-3}{2}}$$

Now consider C2

$$C_2 = P[bX_i < Max(X_2,...,X_{i-1},X_{i+1},...,X_{\frac{k+1}{2}}) < X_{\frac{k+3}{2}} < Min(X_{\frac{k+5}{2}},...,X_k) < x]$$

$$= \int_0^{\frac{x}{b}}\int_{by}^{x}\int_{x_1}^{x}\int_{x_2}^{x}\left(\frac{k-3}{2}\right)\left(e^{-x_3}\right)^{\frac{k-5}{2}}e^{-x_3}dx_3e^{-x_2}dx_2\left(\frac{k-3}{2}\right)\left(1-e^{-x_1}\right)^{\frac{k-5}{2}}e^{-x_1}dx_1e^{-y}dy$$

$$= \int_0^{\frac{x}{b}}\int_{by}^{x}\frac{\left(e^{-x_1}\right)^{\frac{k-1}{2}}}{\left(\frac{k-1}{2}\right)}\left(\frac{k-3}{2}\right)\left(1-e^{-x_1}\right)^{\frac{k-5}{2}}e^{-x_1}dx_1e^{-y}dy$$

$$-\int_0^{\frac{x}{b}}\int_{by}^{x}\frac{\left(e^{-x}\right)^{\frac{k-1}{2}}}{\left(\frac{k-1}{2}\right)}\left(\frac{k-3}{2}\right)\left(1-e^{-x_1}\right)^{\frac{k-5}{2}}e^{-x_1}dx_1e^{-y}dy$$

$$-\int_0^{\frac{x}{b}}\int_{by}^{x}\left(e^{-x}\right)^{\frac{k-3}{2}}e^{-x_1}\left(\frac{k-3}{2}\right)\left(1-e^{-x_1}\right)^{\frac{k-5}{2}}e^{-x_1}dx_1e^{-y}dy$$

$$+\int_0^{\frac{x}{b}}\int_{by}^{x}\left(e^{-x}\right)^{\frac{k-3}{2}}e^{-x}\left(\frac{k-3}{2}\right)\left(1-e^{-x_1}\right)^{\frac{k-5}{2}}e^{-x_1}dx_1e^{-y}dy$$

Now consider C3

$$C_3 = P[Max(X_2,...,X_{i-1},X_{i+1},...,X_{\frac{k+1}{2}}) < bX_i < X_{\frac{k+3}{2}} < x < Min(X_{\frac{k+5}{2}},...,X_k)]$$

$$= \left[\int_0^x \int_{\frac{x_1}{b}}^{\frac{x}{b}} \int_{by}^{x} e^{-x_2} dx_2 e^{-y} dy \left(\frac{k-3}{2} \right) \left(1 - e^{-x_1}\right)^{\frac{k-5}{2}} e^{-x_1} dx_1 \right] \left(e^{-x}\right)^{\frac{k-3}{2}}$$

$$= \left[\int_0^x \int_{\frac{x_1}{b}}^{\frac{x}{b}} e^{-by} e^{-y} dy \left(\frac{k-3}{2} \right) \left(1 - e^{x_1}\right)^{\frac{k-5}{2}} e^{-x_1} dx_1 \right] \left(e^{-x}\right)^{\frac{k-3}{2}}$$

$$- \left[\int_0^x \int_{\frac{x_1}{b}}^{\frac{x}{b}} e^{-x} e^{-y} dy \left(\frac{k-3}{2} \right) \left(1 - e^{x_1}\right)^{\frac{k-5}{2}} e^{-x_1} dx_1 \right] \left(e^{-x}\right)^{\frac{k-3}{2}}$$

Now consider C4

$$C_4 = P[Max(X_2,\ldots,X_{i-1},X_{i+1},\ldots,X_{\frac{k+1}{2}}) < bX_i < X_{\frac{k+3}{2}} < Min(X_{\frac{k+5}{2}},\ldots,X_k) < x]$$

$$= \int_0^x \int_{\frac{x_1}{b}}^{\frac{x}{b}} \int_{by}^{x} \int_{x_2}^{x} \left(\frac{k-3}{2} \right) \left(e^{-x_3}\right)^{\frac{k-5}{2}} e^{-x_3} dx_3 e^{-x_2} dx_2 e^{-y} dy \left(\frac{k-3}{2} \right) \left(1 - e^{-x_1}\right)^{\frac{k-5}{2}} e^{-x_1} dx_1$$

$$= \int_0^x \int_{\frac{x_1}{b}}^{\frac{x}{b}} \frac{\left(e^{-by}\right)^{\frac{k-1}{2}}}{\left(\frac{k-1}{2}\right)} \left(\frac{k-3}{2} \right) \left(1 - e^{-x_1}\right)^{\frac{k-5}{2}} e^{-x_1} dx_1 e^{-y} dy$$

$$- \int_0^x \int_{\frac{x_1}{b}}^{\frac{x}{b}} \frac{\left(e^{-x}\right)^{\frac{k-1}{2}}}{\left(\frac{k-1}{2}\right)} \left(\frac{k-3}{2} \right) \left(1 - e^{-x_1}\right)^{\frac{k-5}{2}} e^{-x_1} dx_1 e^{-y} dy$$

$$- \int_0^x \int_{\frac{x_1}{b}}^{\frac{x}{b}} \left(e^{-x}\right)^{\frac{k-3}{2}} e^{-by} \left(\frac{k-3}{2} \right) \left(1 - e^{-x_1}\right)^{\frac{k-5}{2}} e^{-x_1} dx_1 e^{-y} dy$$

$$+ \int_0^x \int_{\frac{x_1}{b}}^{\frac{x}{b}} \left(e^{-x}\right)^{\frac{k-3}{2}} e^{-x} \left(\frac{k-3}{2} \right) \left(1 - e^{-x_1}\right)^{\frac{k-5}{2}} e^{-x_1} dx_1 e^{-y} dy$$

Now we get C by adding C1, C2, C3, and C4 as

$$C=\int_0^{\frac{x}{b}}\int_{by}^{x}\frac{\left(e^{-x_1}\right)^{\frac{k-1}{2}}}{\left(\frac{k-1}{2}\right)}\left(\frac{k-3}{2}\right)\left(1-e^{-x_1}\right)^{\frac{k-5}{2}}e^{-x_1}dx_1e^{-y}dy$$

$$-\int_0^{\frac{x}{b}}\int_{by}^{x}\frac{\left(e^{-x}\right)^{\frac{k-1}{2}}}{\left(\frac{k-1}{2}\right)}\left(\frac{k-3}{2}\right)\left(1-e^{-x_1}\right)^{\frac{k-5}{2}}e^{-x_1}dx_1e^{-y}dy$$

$$+\int_0^{x}\int_{\frac{x_1}{b}}^{\frac{x}{b}}\frac{\left(e^{-by}\right)^{\frac{k-1}{2}}}{\left(\frac{k-1}{2}\right)}\left(\frac{k-3}{2}\right)\left(1-e^{-x_1}\right)^{\frac{k-5}{2}}e^{-x_1}dx_1e^{-y}dy$$

$$-\int_0^{x}\int_{\frac{x_1}{b}}^{\frac{x}{b}}\frac{\left(e^{-x}\right)^{\frac{k-1}{2}}}{\left(\frac{k-1}{2}\right)}\left(\frac{k-3}{2}\right)\left(1-e^{-x_1}\right)^{\frac{k-5}{2}}e^{-x_1}dx_1e^{-y}dy$$

Now by adding A, B and C we get

$$\left(\frac{k}{\frac{k-1}{2}}\right)A+k\left(\frac{\frac{k-1}{k-3}}{2}\right)B+k\left(\frac{\frac{k-1}{k-3}}{2}\right)C$$

$$=\left(\frac{k}{\frac{k-1}{2}}\right)\left[(1-e^{-x})^{\frac{k-3}{2}}\left(1-(e^{-\frac{x}{b}})\right)\right]\left[e^{-x}\right]^{\frac{k-1}{2}}$$

$$+k\left(\frac{\frac{k-1}{k-3}}{2}\right)\left[1-e^{-\frac{x}{b}}\right]\left[\int_x^{\infty}\int_x^{x_3}\left[\left(1-e^{-x_2}\right)^{\frac{k-5}{2}}\right]e^{-x_2}dx_2\left(\frac{k-1}{2}\right)\left(e^{-x_3}\right)^{\frac{k-3}{2}}e^{-x_3}dx_3\right]$$

$$-\left[1-e^{-\frac{x}{b}}\right]\left[\int_x^{\infty}\int_x^{x_3}\left[\left(1-e^{-x}\right)^{\frac{k-5}{2}}\right]e^{-x_2}dx_2\left(\frac{k-1}{2}\right)\left(e^{-x_3}\right)^{\frac{k-3}{2}}e^{-x_3}dx_3\right]$$

$$+\left[\int_0^{\frac{x}{b}}\left\{\left[1-e^{-x}\right]^{\frac{k-5}{2}}\right\}e^{-y}dy\left(\frac{\left(e^{-x}\right)^{\frac{k+1}{2}}}{\frac{k+1}{2}}\right)\right]$$

$$+\left(\frac{k-1}{2}\right)\left(\frac{k-5}{2}\right)\frac{1}{\left(\frac{k-1}{2}\right)\left(\frac{k+1}{2}\right)}\int_{\frac{x}{b}}^{\infty}\int_{by}^{\infty}\left(1-e^{-x_1}\right)^{\frac{k-7}{2}}\left(e^{-x_1}\right)\left(e^{-x_1}\right)^{\frac{k+1}{2}}dx_1e^{-y}dy$$

$$+\left(\frac{k-1}{2}\right)\left(\frac{k-5}{2}\right)\left(\frac{1}{\left(\frac{k-1}{2}\right)}\right)\left(\frac{1}{\left(\frac{k+1}{2}\right)}\right)\left(\frac{1}{\left(\frac{k+3}{2}\right)}\right)\int_{x}^{\infty}\left(e^{-x_1}\right)^{\frac{k+3}{2}}\left(1-e^{-x_1}\right)^{\frac{k-7}{2}}e^{-x_1}dx_1$$

$$+\left(1-e^{-x}\right)^{\frac{k-5}{2}}\left(\left(\frac{1}{\left(\frac{k+1}{2}\right)}\right)\left(\frac{1}{\left(\frac{k+3}{2}\right)}\right)\left(e^{-x}\right)^{\frac{k-3}{2}}\right)$$

$$+k\left(\frac{k-1}{k-3}\right)\int_{0}^{\frac{x}{b}}\int_{by}^{x}\frac{\left(e^{-x_1}\right)^{\frac{k-1}{2}}}{\left(\frac{k-1}{2}\right)}\left(\frac{k-3}{2}\right)\left(1-e^{-x_1}\right)^{\frac{k-5}{2}}e^{-x_1}dx_1e^{-y}dy$$

$$-\int_{0}^{\frac{x}{b}}\int_{by}^{x}\frac{\left(e^{-x}\right)^{\frac{k-1}{2}}}{\left(\frac{k-1}{2}\right)}\left(\frac{k-3}{2}\right)\left(1-e^{-x_1}\right)^{\frac{k-5}{2}}e^{-x_1}dx_1e^{-y}dy$$

$$+\int_{0}^{x}\int_{\frac{x_1}{b}}^{\frac{x}{b}}\frac{\left(e^{-by}\right)^{\frac{k-1}{2}}}{\left(\frac{k-1}{2}\right)}\left(\frac{k-3}{2}\right)\left(1-e^{-x_1}\right)^{\frac{k-5}{2}}e^{-x_1}dx_1e^{-y}dy$$

$$-\int_{0}^{x}\int_{\frac{x_1}{b}}^{\frac{x}{b}}\frac{\left(e^{-x}\right)^{\frac{k-1}{2}}}{\left(\frac{k-1}{2}\right)}\left(\frac{k-3}{2}\right)\left(1-e^{-x_1}\right)^{\frac{k-5}{2}}e^{-x_1}dx_1e^{-y}dy$$

=Z (Say).

Thus from 4.3.2.3a and 4.3.2.4 we get right hand side of (4.3.2.3) equal to

$$\left(\int_{0}^{e^{-bx}}\frac{k!}{(a!)^2}[1-u]^a[u]^a\,du\right)+\text{ kZ(Say),}$$

Then finally by substituting in 4.3.2.2 we get

$$Cov[h_b^*(X_1,\Lambda\ ,X_{k+1}),h_b^*(X_1,X_{k+2}\Lambda\ ,X_{2k+1})]$$

$$= \int_0^{\infty} \left[\left(\int_0^{e^{-bx}} \frac{k!}{(a!)^2} [1-u]^a [u]^a \, du \right) + kZ(\text{Say}) \right]^2 e^{-x} dx - \gamma_0^2$$

where γ_0 is the null mean.

Now we consider the special cases. When k=3, we have

$$E_{H_0}(U) = P[Med(X_1, X_2, X_3) > bX_4]$$

$$= \int_0^{\infty} P[Med(X_1, X_2, X_3) > bx] e^{-x} dx$$

We have

$$P[Med(X_1, X_2, X_3) > bx] = e^{-3bx} + 3(1 - e^{-bx}) e^{-2bx}$$

Thus

$$E_{H_0}(U) = \int_0^{\infty} \left[e^{-3bx} + 3(1 - e^{-bx}) e^{-2bx} \right] e^{-x} dx .$$

Letting e^{-x} =u we get

$$E_{H_0}(U) = \frac{(5b+1)}{(2b+1)(3b+1)}$$

$$= \mu_0(3) \text{ (say).}$$

Now,

$$(k+1)^2 \xi_1 = Cov\left[h_b^*(X_1, X_2, X_3, X_4), h_b^*(X_1, X_5, X_6, X_7) \right]$$

where

$$h_b^*(X_1, X_2, X_3, X_4) = \frac{1}{4} \sum_{i=1}^{4} h_b^i(X_1, X_2, X_3, X_4),$$

$$= \frac{1}{4} \left[h_b^1(X_1, X_2, X_3, X_4) + h_b^2(X_1, X_2, X_3, X_4) + h_b^3(X_1, X_2, X_3, X_4) + h_b^4(X_1, X_2, X_3, X_4) \right].$$

where

$$h_b^1(X_1,X_2,X_3,X_4)=\begin{cases}1, & if \quad Med(X_2,X_3,X_4)>bX_1\\ 0, & otherwise\end{cases}$$

$$h_b^2(X_1,X_2,X_3,X_4)=\begin{cases}1, & if \quad Med(X_1,X_3X_4)>bX_2\\ 0, & otherwise\end{cases}$$

$$h_b^3(X_1,X_2,X_3,X_4)=\begin{cases}1, & if \quad Med(X_1,X_2,X_4)>bX_3\\ 0, & otherwise\end{cases}$$

$$h_b^4(X_1,X_2,X_3,X_4)=\begin{cases}1, & if \quad Med(X_1,X_2,X_3)>bX_4\\ 0, & otherwise\end{cases}$$

and

$$h_b^*(X_1,X_5,X_6,X_7)=\frac{1}{4}\sum_{j=1}^{4}h_b^j(X_1,X_5,X_6,X_7)$$

$$=\frac{1}{4}\left[h_b^1(X_1,X_5,X_6,X_7)+h_b^5(X_1,X_5,X_6,X_7)+h_b^6(X_1,X_5,X_6,X_7)+h_b^7(X_1,X_5,X_6,X_7)\right]$$

where

$$h_b^1(X_1,X_5,X_6,X_7)=\begin{cases}1, & if \quad Med(X_5,X_6,X_7)>bX_1\\ 0, & otherwise\end{cases}$$

$$h_b^5(X_1,X_5,X_6,X_7)=\begin{cases}1, & if \quad Med(X_1,X_6,X_7)>bX_5\\ 0, & otherwise\end{cases}$$

$$h_b^6(X_1,X_5,X_6,X_7)=\begin{cases}1, & if \quad Med(X_1,X_5,X_7)>bX_6\\ 0, & otherwise\end{cases}$$

$$h_b^7(X_1,X_5,X_6,X_7)=\begin{cases}1, & if \quad Med(X_1,X_5,X_6)>bX_7\\ 0, & otherwise\end{cases}$$

Now,

$$Cov\left[h_b^*(X_1,X_2,X_3,X_4),h_b^*(X_1,X_5,X_6,X_7)\right]$$

$$=E\left[h_b^*(X_1,X_2,X_3,X_4)h_b^*(X_1,X_5,X_6,X_7)\right]-\mu_0^2(3).$$

Further

$$16\,E\left[h_b^*(X_1,X_2,X_3,X_4)h_b^*(X_1,X_5,X_6,X_7)\right]=a_1+a_2+\ldots+a_{16} \text{ (say). (4.3.2.8)}$$

where

$$a_1=E\left[h_b^1(X_1,X_2,X_3,X_4)h_b^1(X_1,X_5,X_6,X_7)\right]$$

$$=P[Med(X_2,X_3,X_4)>bX_1, Med(X_5,X_6,X_7)>bX_1$$

$$=\int P^2[Med(X_2,X_3,X_4)>bx]e^{-x}dx$$

Now,

$$P[Med(X_2,X_3,X_4)>bx]$$

$$=\sum_{i=0}^{1}\binom{3}{i}F^i(bx)\overline{F}^{3-i}(bx)$$

$$=3\overline{F}^2(bx)-2\overline{F}^3(bx)$$

Thus

$$a_1=\int_0^\infty\left[3\overline{F}^2(bx)-2\overline{F}^3(bx)\right]^2e^{-x}dx$$

$$=\frac{9}{(4b+1)}+\frac{4}{(6b+1)}-\frac{12}{(5b+1)}$$

$$a_2=E\left[h_b^1(X_1,X_2,X_3,X_4)h_b^5(X_1,X_5,X_6,X_7)\right]$$

$$=P[Med(X_2,X_3,X_4)>bX_1, Med(X_1,X_6,X_7)>bX_5]$$

$$=\int_0^\infty P[Med(X_2,X_3,X_4)>bx, Med(x,X_6,X_7)>bX_5]e^{-x}dx$$

$$=\int_0^\infty P[Med(X_2,X_3,X_4)>bx]P[Med(x,X_6,X_7)>bX_5]e^{-x}dx$$

Now,

$$P[Med(X_2,X_3,X_4)>bx]=\left[3\overline{F}^2(bx)-2\overline{F}^3(bx)\right] \qquad (4.3.2.8a)$$

and

$$P[Med(x, X_6, X_7) > bX_5]$$

$$= 2P[X_6 < x < X_7, x > bX_5] + 2P[x < X_6 < X_7, X_6 > bX_5] + 2P[X_6 < X_7 < x, X_7 > bX_5]$$

$$= 2S_1 + 2S_2 + 2S_3 \qquad (4.3.2.8b)$$

Now consider the first term on the r.h.s. of (4.3.2.8b)

$$S_1 = P[X_6 < x < X_7, x > bX_5]$$

$$= P[X_6 < x < X_7]P[x > bX_5]$$

$$= e^{-x}\left(1 - e^{-x}\right)\left(1 - e^{-\frac{x}{b}}\right)$$

Now consider the second term on the r.h.s of (4.3.2.8b)

$$S_2 = P[x < X_6 < X_7, X_6 > bX_5]$$

$$= \int_x^{\infty} P[x < z < X_7, z > bX_5] e^{-z} dz$$

$$= \int_x^{\infty} e^{-z}\left(1 - e^{-\frac{z}{b}}\right) e^{-z} dz$$

$$= \int_0^{e^{-x}} y\left(1 - y^{\frac{1}{b}}\right) dy \quad \text{letting } e^{-z} = y$$

$$= \frac{e^{-2x}}{2} - \frac{\left(e^{-x}\right)^{\frac{2b+1}{b}}}{\left(\frac{2b+1}{b}\right)}.$$

Next consider the third term on the r.h.s.of (4.3.2.8b)

$$S_3 = P[X_6 < X_7 < x, X_7 > bX_5]$$

$$= \int_0^x P[X_6 < y < x, y > bX_5]e^{-y}dy$$

$$= \int_0^x \left(1-e^{-y}\right)\left(1-e^{-\frac{y}{b}}\right)e^{-y}dy$$

$$= \int_{e^{-x}}^1 (1-z)\left(1-z^{\frac{1}{b}}\right)dz \qquad \text{letting } e^{-y}=z$$

$$= 1-e^{-x} - \frac{1}{\frac{b+1}{b}} + \frac{\left(e^{-x}\right)^{\frac{b+1}{b}}}{\frac{b+1}{b}} - \frac{1}{2} + \frac{e^{-2x}}{2} + \frac{1}{\frac{2b+1}{b}} - \frac{\left(e^{-x}\right)^{\frac{2b+2}{b}}}{\frac{2b+1}{b}}$$

Adding S_1,S_2 and S_3 we have (4.3.2.8b)

$$P[Med(x, X_6, X_7) > bX_5] =$$

$$\left[-2\left(e^{-x}\right)^{\frac{b+1}{b}} + 2\left(e^{-x}\right)^{\frac{2b+1}{b}} - \frac{2b\left(e^{-x}\right)^{\frac{2b+1}{b}}}{(2b+1)} - \frac{2b}{(b+1)} + \frac{2b\left(e^{-x}\right)^{\frac{b+1}{b}}}{(b+1)} + 1 + \frac{2b}{(2b+1)} - \frac{2b\left(e^{-x}\right)^{\frac{2b+1}{b}}}{(2b+1)}\right]$$

Thus from (4.3.2.8a) and (4.3.2.8b) we have

$$a_2 = \int_0^\infty \left(3e^{-2bx} - 2e^{-3bx}\right)\left(\left[-2\left(e^{-x}\right)^{\frac{b+1}{b}} + 2\left(e^{-x}\right)^{\frac{2b+1}{b}} - \frac{2b\left(e^{-x}\right)^{\frac{2b+1}{b}}}{(2b+1)} - \frac{2b}{(b+1)} + \frac{2b\left(e^{-x}\right)^{\frac{b+1}{b}}}{(b+1)} + 1 + \frac{2b}{(2b+1)} - \frac{2b\left(e^{-x}\right)^{\frac{2b+1}{b}}}{(2b+1)}\right]\right)e^{-x}dx$$

$$= \frac{-6b}{(2b^2+2b+1)} + \frac{6b}{(2b^2+3b+1)} - \frac{6b^2}{(2b+1)(2b^2+3b+1)} - \frac{6b}{(b+1)(2b+1)} + \frac{6b^2}{(b+1)(2b^2+2b+1)}$$

$$+ \frac{3}{(2b+1)} + \frac{6b}{(2b+1)^2} - \frac{6b^2}{(2b+1)(2b^2+3b+1)} + \frac{4b}{(3b^2+2b+1)} - \frac{4b}{(3b^2+3b+1)}$$

$$+ \frac{4b^2}{(2b+1)(3b^2+3b+1)} + \frac{4b}{(3b+1)(b+1)} - \frac{4b^2}{(b+1)(3b^2+2b+1)} - \frac{2}{(3b+1)} - \frac{4b}{(3b+1)(2b+1)} + \frac{4b^2}{(2b+1)(3b^2+3b+1)}$$

$$a_3 = E\left[h_b^1(X_1, X_2, X_3, X_4)h_b^6(X_1, X_5, X_6, X_7)\right]$$

=same as a_2

$$a_4 = E\left[h_b^1(X_1, X_2, X_3, X_4)h_b^7(X_1, X_5, X_6, X_7)\right]$$

=same as a_2

$$a_5 = E\left[h_b^2(X_1, X_2, X_3, X_4)h_b^1(X_1, X_5, X_6, X_7)\right]$$

=same as a_2

$$a_6 = E\left[h_b^2(X_1, X_2, X_3, X_4)h_b^5(X_1, X_5, X_6, X_7)\right]$$

$$= P[Med(X_1, X_3, X_4) > bX_2, Med(X_1, X_6, X_7) > bX_5]$$

$$= \int_0^\infty P[Med(x, X_3, X_4) > bX_2, Med(x, X_6, X_7) > bX_5]e^{-x}dx$$

Now,

$$P[Med(x, X_3, X_4) > bX_2]$$

$$= -2\left(e^{-x}\right)^{\frac{b+1}{b}} + 2\left(e^{-x}\right)^{\frac{2b+1}{b}} - \frac{2b\left(e^{-x}\right)^{\frac{2b+1}{b}}}{(2b+1)} - \frac{2b}{(b+1)} + \frac{2b\left(e^{-x}\right)^{\frac{b+1}{b}}}{(b+1)} + 1 + \frac{2b}{(2b+1)} - \frac{2b\left(e^{-x}\right)^{\frac{2b+1}{b}}}{(2b+1)}$$

Thus

$$a_6 = \int_0^\infty \left[-2\left(e^{-x}\right)^{\frac{b+1}{b}} + 2\left(e^{-x}\right)^{\frac{2b+1}{b}} - \frac{2b\left(e^{-x}\right)^{\frac{2b+1}{b}}}{(2b+1)} - \frac{2b}{(b+1)} + \frac{2b\left(e^{-x}\right)^{\frac{b+1}{b}}}{(b+1)} + 1 + \frac{2b}{(2b+1)} - \frac{2b\left(e^{-x}\right)^{\frac{2b+1}{b}}}{(2b+1)}\right]^2 e^{-x}dx$$

$$= \frac{4b}{(b+1)^2(3b+2)} + \frac{4b}{(2b+1)^2(5b+2)} + 1 + \frac{4b^2}{(b+1)^2} + \frac{4b^2}{(2b+1)^2} - \frac{4b}{(2b+1)} - \frac{4b}{(b+1)} + \frac{4b}{(3b+1)}$$

$$+ \frac{16b^3}{(b+1)(2b+1)(3b+1)} + \frac{8b^2}{(b+1)(2b+1)} - \frac{16b^3}{(2b+1)^2(3b+1)} - \frac{8b^3}{(b+1)^2(2b+1)} - \frac{8b^2}{(b+1)(3b+1)}$$

$$a_7 = E\left[h_b^2(X_1, X_2, X_3, X_4)h_b^6(X_1, X_5, X_6, X_7)\right]$$

=same as a_6

$$a_8 = E\left[h_b^2(X_1, X_2, X_3, X_4)h_b^7(X_1, X_5, X_6, X_7)\right]$$

=same as a_6

$$a_9 = E\left[h_b^3(X_1, X_2, X_3, X_4)h_b^1(X_1, X_5, X_6, X_7)\right]$$

=same as a_2

$$a_{10} = E\left[h_b^3(X_1, X_2, X_3, X_4)h_b^5(X_1, X_5, X_6, X_7)\right]$$

=same as a_6

$$a_{11} = E\left[h_b^3(X_1, X_2, X_3, X_4)h_b^6(X_1, X_5, X_6, X_7)\right]$$

=same as a_6

$$a_{12} = E\left[h_b^3(X_1, X_2, X_3, X_4)h_b^7(X_1, X_5, X_6, X_7)\right]$$

=same as a_6

$$a_{13} = E\left[h_b^4(X_1, X_2, X_3, X_4)h_b^1(X_1, X_5, X_6, X_7)\right]$$

=same as a_2

$$a_{14} = E\left[h_b^4(X_1, X_2, X_3, X_4)h_b^5(X_1, X_5, X_6, X_7)\right]$$

=same as a_6

$$a_{15} = E\left[h_b^4(X_1, X_2, X_3, X_4)h_b^6(X_1, X_5, X_6, X_7)\right]$$

=same as a_6

$$a_{16} = E\left[h_b^4(X_1, X_2, X_3, X_4)h_b^7(X_1, X_5, X_6, X_7)\right]$$

=same as a_6

Now adding the 16 terms and subtracting $16\mu_0^2$ we have

$16\xi_1$ =(4.3.2.8)-$16\mu_0^2$(3)

$$= \left[4 \frac{\left(\begin{array}{l} 16+496b+988758b^5+2594180b^6+7574172b^8+53137b^3+6732b^2 \\ +8714723b^9+274281b^4+5084192b^7+5712714b^{11} \\ +7892997b^{10}+1273860b^{13}+3229542b^{12}+259200b^{14} \end{array} \right)}{\left[\begin{array}{l} (4b+1)(6b+1)(5b+1)(3b^2+3b+1)(3b^2+2b+1)(b+1)(2b+1)^2 \\ (2b^2+2b+1)(5b+2)(3b+2) \end{array} \right]} \right] - 16\left(\frac{(5b+1)}{(2b+1)(3b+1)} \right)^2$$

$$= 12b^3 \frac{\begin{pmatrix} 15402121b^8 + 9298525b^7 + 11242746b^{11} + 19367133b^9 \\ +17811846b^{10} + 31 + 4339925b^6 + 428502b^4 + 905b \\ +11435b^2 + 85227b^3 + 1566024b^5 + 4339980b^{12} + 777600b^{13} \end{pmatrix}}{\begin{bmatrix} (4b+1)(6b+1)(5b+1)(3b^2+3b+1)(3b^2+2b+1) \\ (b+1)(2b+1)^2(2b^2+2b+1)(5b+2)(3b+2)(3b+1)^2 \end{bmatrix}}$$

$=B^*$ (Say)

We calculate ξ_1 for different values of b.

Thus we get

b	$B^*=16\xi_1$
0.1	0.0448
0.2	0.199
0.3	0.421
0.4	0.678
0.5	0.9532
0.6	1.233
0.7	1.511
0.8	1.781
0.9	2.039

Now for the case k=5, we have

$$E_{H_0}(U) = P[Med(X_1, X_2, X_3, X_4, X_5) > bX_6]$$

$$= \int_0^\infty P[Med(X_1, X_2, X_3, X_4, X_5) > bx] e^{-x} dx .$$

We have

$$P[Med(X_1, X_2, X_3, X_4, X_5) > bx]$$

$$= \sum_{i=0}^{a-1} \binom{k}{i} F^i(bx)\left[\overline{F}(bx)\right]^{k-i} \text{ where k=5,a=} \frac{k+1}{2}$$

$$= \sum_{i=0}^{2} \binom{5}{i} F^i(bx)\left[\overline{F}(bx)\right]^{5-i}$$

$$= \left[\overline{F}(bx)\right]^5 + 5F(bx)\left[\overline{F}(bx)\right]^4 + 10F^2(bx)\left[\overline{F}(bx)\right]^3$$

$$= e^{-5bx} + 5\left(1-e^{-bx}\right)e^{-4bx} + 10\left(1-e^{-bx}\right)^2 e^{-3bx}$$

$$= 6e^{-5bx} - 15e^{-4bx} + 10e^{-3bx}.$$

Thus

$$P[Med(X_1, X_2, X_3, X_4, X_5) > bx] = 6e^{-5bx} - 15e^{-4bx} + 10e^{-3bx} \quad .$$

Thus

$$E_{H_0}(U) = \int_0^\infty \left[6e^{-5bx} - 15e^{-4bx} + 10e^{-3bx}\right] e^{-x} dx .$$

Letting e^{-x} =u we get

$$E_{H_0}(U) = \frac{6}{(5b+1)} - \frac{15}{(4b+1)} + \frac{10}{(3b+1)}$$

$$= \mu_0(5) \text{ say.}$$

Now

$$(k+1)^2 \xi_1 = Cov\left[h_b^*(X_1, X_2, ..., X_6), h_b^*(X_1, X_7, ..., X_{11})\right]$$

where

$$h_b^*(X_1, X_2, ..., X_6)$$

$$= \frac{1}{6} \sum_{i=1}^{6} h_b^i(X_1, X_2, ..., X_6)$$

$$= \frac{1}{6}\left[h_b^1(X_1, X_2, ..., X_6) + h_b^2(X_1, X_2, ..., X_6) + ... + h_b^6(X_1, X_2, ..., X_6)\right]$$

and

$$h_b^1(X_1, X_2,..., X_6)=\begin{cases}1, & if \quad Med(X_2,..., X_6) > bX_i \\ 0 & otherwise\end{cases}$$

$$h_b^2(X_1, X_2,..., X_6)=\begin{cases}1, & if \quad Med(X_1, X_3,...X_6) > bX_2 \\ 0 & otherwise\end{cases}$$

and so on upto

$$h_b^6(X_1, X_2,..., X_6)=\begin{cases}1, & if \quad Med(X_1, X_2,...X_5) > bX_6 \\ 0 & otherwise\end{cases}$$

and

$$h_b^*(X_1, X_7,..., X_{11})$$

$$=\frac{1}{6}\sum_{j=1}^{6} h_b^j(X_1, X_7,..., X_{11})$$

$$=\frac{1}{6}\left[h_b^1(X_1, X_7,..., X_{11})+h_b^7(X_1, X_7,..., X_{11})+...+h_b^{11}(X_1, X_7,..., X_{11})\right]$$

where

$$h_b^1(X_1, X_7,..., X_{11})=\begin{cases}1, & if \quad Med(X_7,..., X_{11}) > bX_1 \\ 0 & otherwise\end{cases}$$

$$h_b^7(X_1, X_7,..., X_{11})=\begin{cases}1, & if \quad Med(X_1, X_8,..., X_{11}) > bX_7 \\ 0, & otherwise\end{cases}$$

and so on upto

$$h_b^{11}(X_1, X_7,..., X_{11})=\begin{cases}1, & if \quad Med(X_1, X_7,..., X_{10}) > bX_{11} \\ 0, & otherwise\end{cases}$$

Now

$$Cov\left[h_b^*(X_1, X_2,..., X_6), h_b^*(X_1, X_7,..., X_{11})\right]$$

$$= E\left[h_b^*(X_1, X_2,..., X_6) h_b^*(X_1, X_7,..., X_{11})\right] - \mu_0^2 .$$

Now,

$$36\, E\left[h_b^*(X_1, X_2,..., X_6) h_b^*(X_1, X_7,..., X_{11})\right] = b_1 + b_2 + ... + b_{36} \qquad (4.3.2.9)$$

where

$$b_1 = E\left[h_b^1(X_1, X_2, ..., X_6)h_b^1(X_1, X_7, ..., X_{11})\right]$$

$$= P[Med(X_2, X_3, X_4, X_5, X_6) > bX_1, Med(X_7, X_8, X_9, X_{10}, X_{11}) > bX_1]$$

$$= \int_0^\infty P^2(Med(X_2, X_3, X_4, X_5, X_6) > bx)e^{-x}dx.$$

we have

$$P[Med(X_2, X_3, X_4, X_5, X_6) > bx] = 6e^{-5bx} - 15e^{-4bx} + 10e^{-3bx}.$$

Therefore

$$b_1 = \int_0^\infty \left[6e^{-5bx} - 15e^{-4bx} + 10e^{-3bx}\right]^2 e^{-x}dx$$

$$= \frac{(1 + 40b + 635b^2 + 4880b^3 + 16164b^4)}{(6b+1)(7b+1)(8b+1)(9b+1)(10b+1)},$$

$$b_2 = E\left[h_b^1(X_1, X_2, ..., X_6)h_b^7(X_1, X_7, ..., X_{11})\right]$$

$$= P[Med(X_2, ..., X_6) > bX_1, Med(X_1, X_8, ..., X_{11}) > bX_7]$$

$$= \int_0^\infty P(Med(X_2, ..., X_6) > bx)P(Med(x, X_8, ..., X_{11}) > bX_7)e^{-x}dx$$

Now,

$$P[Med(X_2, X_3, X_4, X_5, X_6) > bx] = 6e^{-5bx} - 15e^{-4bx} + 10e^{-3bx}$$

and

$$P[Med(x, X_8, ..., X_{11}) > bX_7]$$

$$= 6P[Max(X_8, X_9) < x < Min(X_{10}, X_{11}), x > bX_7]$$
$$+ 12P[Max(x, X_8) < X_9 < Min(X_{10}, X_{11}), X_9 > bX_7]$$
$$+ 12P[Max(X_8, X_9) < X_{10} < Min(x, X_{11}), X_{10} > bX_7]$$

$$= 6r_1 + 12r_2 + 12r_3$$

Now consider

$$r_1 = P[Max(X_8, X_9) < x < Min(X_{10}, X_{11}), x > bX_7]$$

$$= \begin{matrix} P[bX_7 < Max(X_8, X_9) < x < Min(X_{10}, X_{11})] + \\ P[Max(X_8, X_9) < bX_7 < x < Min(X_{10}, X_{11})] \end{matrix}$$

$$= r_{11} + r_{12} \quad \text{say}$$

Now consider

$$r_{11} = P[bX_7 < Max(X_8, X_9) < x < Min(X_{10}, X_{11})]$$

$$= P[bX_7 < Max(X_8, X_9) < x]P[x < Min(X_{10}, X_{11})]$$

$$= \left[\int_0^{\frac{x}{b}} P[by < Max(X_8, X_9) < x]e^{-y}dy\right]\left[[\overline{F}(x)]^2\right]$$

$$= \left[\int_0^{\frac{x}{b}} \left[\left(1-e^{-x}\right)^2 - \left(1-e^{-by}\right)^2\right]e^{-y}dy\right]\left[e^{-x}\right]^2$$

$$= \left(1-e^{-x}\right)^2\left(1-e^{-\frac{x}{b}}\right)e^{-2x} - \left[\left(1-e^{-\frac{x}{b}}\right) - \frac{2\left(1-e^{-\frac{x}{b}(b+1)}\right)}{(b+1)} + \frac{\left(1-e^{-\frac{x}{b}(2b+1)}\right)}{(2b+1)}\right]e^{-2x}$$

and

$$r_{12} = P[Max(X_8, X_9) < bX_7 < x < Min(X_{10}, X_{11})]$$

$$= P[Max(X_8, X_9) < bX_7 < x]P[x < Min(X_{10}, X_{11})]$$

$$= \left[\int_0^{\frac{x}{b}} P[Max(X_8, X_9) < by]e^{-y}dy\right]\left[[\overline{F}(x)]^2\right]$$

$$= \left[\int_0^{\frac{x}{b}} \left[\left(1-e^{-by}\right)^2\right] e^{-y}\,dy\right]\left[e^{-x}\right]^2$$

$$= \left[\left(1-e^{-\frac{x}{b}}\right) - \frac{2\left(1-e^{-\frac{x}{b}(b+1)}\right)}{(b+1)} + \frac{\left(1-e^{-\frac{x}{b}(2b+1)}\right)}{(2b+1)}\right] e^{-2x}$$

Thus adding r_{11} and r_{12} we have

$$r_1 = \left(1-e^{-x}\right)^2\left(1-e^{-\frac{x}{b}}\right)e^{-2x}$$

Now consider

$$r_2 = P[Max(x, X_8) < X_9 < Min(X_{10}, X_{11}), X_9 > bX_7]$$

$$= \begin{array}{l} P[bX_7 < Max(x, X_8) < X_9 < Min(X_{10}, X_{11})] + \\ P[Max(x, X_8) < bX_7 < X_9 < Min(X_{10}, X_{11})] \end{array}$$

$$= r_{21} + r_{22}$$

Now consider

$$r_{21} = P[bX_7 < Max(x, X_8) < X_9 < Min(X_{10}, X_{11})]$$

$$= \begin{array}{l} P[bX_7 < x < X_8 < X_9 < Min(X_{10}, X_{11})] + \\ P[bX_7 < X_8 < x < X_9 < Min(X_{10}, X_{11})] + \\ P[x < bX_7 < X_8 < X_9 < Min(X_{10}, X_{11})] + \\ P[X_8 < bX_7 < x < X_9 < Min(X_{10}, X_{11})] \end{array}$$

$$= r_{211} + r_{212} + r_{213} + r_{214}$$

Now consider

$$r_{211} = P[bX_7 < x < X_8 < X_9 < Min(X_{10}, X_{11})]$$

$$= P[bX_7 < x]P[x < X_8 < X_9 < Min(X_{10}, X_{11})]$$

$$= F\left(\frac{x}{b}\right)\left[\int_{x}^{\infty}\int_{x_8}^{\infty} P[x_9 < Min(X_{10}, X_{11})]dF(x_9)dF(x_8)\right]$$

$$= \left[1 - e^{-\frac{x}{b}}\right]\left[\int_{x}^{\infty}\int_{x_8}^{\infty} \left(\overline{F}(x_9)\right)^2 dF(x_9)dF(x_8)\right]$$

$$= \left[1 - e^{-\frac{x}{b}}\right]\frac{\left(e^{-x}\right)^4}{12}$$

$$= \frac{e^{-4x}}{12} - \frac{e^{-4x-\frac{x}{b}}}{12}$$

Now consider

$$r_{211} = P[bX_7 < X_8 < x < X_9 < Min(X_{10}, X_{11})]$$

$$= P[bX_7 < X_8 < x]P[x < X_9 < Min(X_{10}, X_{11})]$$

$$= \left[\int_{0}^{x} F\left(\frac{x_8}{b}\right)dF(x_8)\right]\left[\int_{x}^{\infty} \left(\overline{F}(x_9)\right)^2 dF(x_9)\right]$$

$$= \left[\left(1 - e^{-x}\right) - \frac{b\left(1 - e^{-x\left(\frac{b+1}{b}\right)}\right)}{(b+1)}\right]\frac{e^{-3x}}{3}$$

$$= \frac{e^{-3x}}{3} - \frac{e^{-4x}}{3} - \frac{be^{-3x}}{3(b+1)} + \frac{be^{-4x-\frac{x}{b}}}{3(b+1)}$$

Next consider

$$r_{213} = P[x < bX_7 < X_8 < X_9 < Min(X_{10}, X_{11})]$$

$$= \int_{\frac{x}{b}}^{\infty}\int_{bx_7}^{\infty}\int_{x_8}^{\infty}\int_{x_9}^{\infty} 2e^{-z}e^{-z}dze^{-x_9}dx_9e^{-x_8}dx_8e^{-x_7}dx_7$$

$$= \frac{e^{-4x-\frac{x}{b}}}{12(4b+1)}$$

and

$$r_{214} = P[X_8 < bX_7 < x < X_9 < Min(X_{10}, X_{11})]$$

$$= P[X_8 < bX_7 < x]P[x < X_9 < Min(X_{10}, X_{11})]$$

$$= \left[\int_0^{\frac{x}{b}} P[X_8 < bx_7]e^{-x_7}dx_7\right]\left[\frac{e^{-3x}}{3}\right]$$

$$= \left[\int_0^{\frac{x}{b}} \left(1-e^{-bx_7}\right)e^{-x_7}dx_7\right]\left[\frac{e^{-3x}}{3}\right]$$

$$= \left[\left(1-e^{-\frac{x}{b}}\right) - \frac{\left(1-e^{-\frac{x}{b}(b+1)}\right)}{(b+1)}\right]\frac{e^{-3x}}{3} \qquad \text{letting } e^{-x_7} = y$$

$$= \frac{e^{-3x}}{3} - \frac{e^{-3x-\frac{x}{b}}}{3} - \frac{e^{-3x}}{3(b+1)} + \frac{e^{-4x-\frac{x}{b}}}{3(b+1)}$$

Now adding $r_{211}, r_{212}, r_{213}$ and r_{214} we get r_{21}

$$r_{21} = \frac{e^{-3x}}{3} - \frac{e^{-4x}}{4} - \frac{e^{-3x-\frac{x}{b}}}{3} + \frac{(3b+1)e^{-4x-\frac{x}{b}}}{3(4b+1)}$$

Now consider

$$r_{22} = P[Max(x, X_8) < bX_7 < X_9 < Min(X_{10}, X_{11})]$$

$$= \begin{matrix} P[x < X_8 < bX_7 < X_9 < Min(X_{10}, X_{11})] \\ + P[X_8 < x < bX_7 < X_9 < Min(X_{10}, X_{11})] \end{matrix}$$

$$= r_{221} + r_{222}$$

Now consider

$$r_{221} = P[x < X_8 < bX_7 < X_9 < Min(X_{10}, X_{11})]$$

$$= \int_{x}^{\infty}\int_{\frac{x_8}{b}}^{\infty}\int_{bx_7}^{\infty}\int_{x_9}^{\infty} 2e^{-z}e^{-z}dze^{-x_9}dx_9e^{-x_7}dx_7e^{-x_8}dx_8$$

$$= \frac{e^{-x\left(\frac{4b+1}{b}\right)}}{3(3b+1)\left(\frac{4b+1}{b}\right)}$$

and

$$r_{222} = P[X_8 < x < bX_7 < X_9 < Min(X_{10}, X_{11})]$$

$$= P[X_8 < x]P[x < bX_7 < X_9 < Min(X_{10}, X_{11})]$$

$$= \left(1 - e^{-x}\right)\left[\int_{\frac{x}{b}}^{\infty}\int_{bx_7}^{\infty}\int_{x_9}^{\infty} 2e^{-z}e^{-z}dze^{-x_9}dx_9e^{-x_7}dx_7\right]$$

$$= \left(1 - e^{-x}\right)\left(\frac{e^{-\frac{x}{b}(3b+1)}}{3(3b+1)}\right)$$

Thus by adding r_{221} and r_{222} we have

$$r_{22} = \frac{e^{-3x-\frac{x}{b}}}{3(3b+1)} - \frac{e^{-4x-\frac{x}{b}}}{3(4b+1)}$$

Now by adding r_{21} and r_{22} we have

$$r_2 = \frac{e^{-3x}}{3} - \frac{e^{-4x}}{4} - \frac{be^{-3x-\frac{x}{b}}}{(3b+1)} + \frac{be^{-4x-\frac{x}{b}}}{(4b+1)}$$

Now consider r_3

$$r_3 = P[Max(X_8, X_9) < X_{10} < Min(x, X_{11}), X_{10} > bX_7]$$

$$= \begin{matrix} P[bX_7 < Max(X_8, X_9) < X_{10} < Min(x, X_{11})] + \\ P[Max(X_8, X_9) < bX_7 < X_{10} < Min(x, X_{11})] \end{matrix}$$

$$= \begin{matrix} P[bX_7 < Max(X_8, X_9) < X_{10} < x < X_{11}] + \\ P[bX_7 < Max(X_8, X_9) < X_{10} < X_{11} < x] + \\ P[Max(X_8, X_9) < bX_7 < X_{10} < x < X_{11}] + \\ P[Max(X_8, X_9) < bX_7 < X_{10} < X_{11} < x] \end{matrix}$$

$$= r_{31} + r_{32} + r_{33} + r_{34}$$

Now consider

$$r_{31} = P[bX_7 < Max(X_8, X_9) < X_{10} < x < X_{11}]$$

$$= P[bX_7 < Max(X_8, X_9) < X_{10} < x] P[x < X_{11}]$$

$$= \left[\int_0^{\frac{x}{b}} \int_{bx_7}^{x} \int_{z}^{x} e^{-x_{10}} dx_{10} 2\left(1 - e^{-z}\right) e^{-z} dz e^{-x_7} dx_7 \right] \left[e^{-x} \right]$$

$$= \begin{matrix} \frac{(5b+1)e^{-x}}{3(2b+1)(3b+1)} - \frac{(3b+1)e^{-2x}}{(b+1)(2b+1)} + e^{-3x} - \frac{e^{-4x}}{3} \\ - \frac{2b^2 e^{-3x-\frac{x}{b}}}{(b+1)(2b+1)} + \frac{2b^2 e^{-4x-\frac{x}{b}}}{(2b+1)(3b+1)} \end{matrix}$$

Now consider

$$r_{32} = P[bX_7 < Max(X_8, X_9) < X_{10} < X_{11} < x]$$

$$= \int_0^{\frac{x}{b}} \int_{bx_7}^{x} \int_{z}^{x} \int_{x_{10}}^{x} e^{-x_{11}} dx_{11} e^{-x_{10}} dx_{10} \left(2 - 2e^{-z}\right) e^{-z} dz e^{-x_7} dx_7$$

$$= \begin{matrix} \frac{(7b+1)}{12(3b+1)(4b+1)} - \frac{(5b+1)e^{-x}}{3(2b+1)(3b+1)} + \frac{(3b+1)e^{-2x}}{2(b+1)(2b+1)} - \frac{e^{-3x}}{3} + \frac{e^{-4x}}{12} \\ + \frac{6b^3 e^{-3x-\frac{x}{b}}}{3(b+1)(2b+1)(3b+1)} - \frac{2b^3 e^{-4x-\frac{x}{b}}}{(2b+1)(3b+1)(4b+1)} \end{matrix}$$

Now consider

$$r_{33} = P[Max(X_8, X_9) < bX_7 < X_{10} < x < X_{11}]$$

$$= P[Max(X_8, X_9) < bX_7 < X_{10} < x]P[x < X_{11}]$$

$$= \left[\int_0^x \int_{\frac{z}{b}}^{\frac{x}{b}} \int_{bx_7}^{x} e^{-x_{10}} dx_{10} e^{-x_7} dx_7 2\left(1-e^{-z}\right)e^{-z} dz\right]\left[e^{-x}\right]$$

$$= \frac{2b^2e^{-x}}{(b+1)(2b+1)(3b+1)} - \frac{2b^2e^{-2x}}{(b+1)(2b+1)} + \frac{be^{-2x-\frac{x}{b}}}{(b+1)} - \frac{2be^{-3x-\frac{x}{b}}}{(b+1)(2b+1)} + \\ + \frac{be^{-4x-\frac{x}{b}}}{(2b+1)(3b+1)}$$

Now consider

$$r_{34} = P[Max(X_8, X_9) < bX_7 < X_{10} < X_{11} < x]$$

$$= \int_0^x \int_{\frac{z}{b}}^{\frac{x}{b}} \int_{bx_7}^{x} \int_{x_{10}}^{x} e^{-x_{11}} dx_{11} e^{-x_{10}} dx_{10} e^{-x_7} dx_7 2\left(1-e^{-z}\right)e^{-z} dz$$

$$= \frac{b^2}{(2b+1)(3b+1)(4b+1)} - \frac{2b^2e^{-x}}{(b+1)(2b+1)(3b+1)} + \frac{b^2e^{-2x}}{(b+1)(2b+1)} \\ - \frac{2b^2e^{-2x-\frac{x}{b}}}{2(b+1)(2b+1)} + \frac{2b^2e^{-3x-\frac{x}{b}}}{(b+1)(2b+1)(3b+1)} - \frac{b^2e^{-4x-\frac{x}{b}}}{(2b+1)(3b+1)(4b+1)}$$

Now adding r_{31}, r_{32}, r_{33} and r_{34} we have

$$r_3 = \frac{\left(26b^2+9b+1\right)}{12(2b+1)(3b+1)(4b+1)} - \frac{e^{-2x}}{2} + \frac{2e^{-3x}}{3} - \frac{e^{-4x}}{4} + \frac{be^{-2x-\frac{x}{b}}}{(2b+1)} \\ - \frac{2be^{-3x-\frac{x}{b}}}{(3b+1)} + \frac{be^{-4x-\frac{x}{b}}}{(4b+1)}$$

Thus now by adding r_1, r_2 and r_3 we have

$$P[Med(x, X_8, ..., X_{11}) > bX_7]$$

$$= 6r_1 + 12r_2 + 12r_3$$

$$= \frac{\left(26b^2+9b+1\right)}{(2b+1)(3b+1)(4b+1)} - \frac{6e^{-2x-\frac{x}{b}}}{(2b+1)} + \frac{12e^{-3x-\frac{x}{b}}}{(3b+1)} - \frac{6e^{-4x-\frac{x}{b}}}{(4b+1)}$$

Thus finally we get

$$b_2=\int_0^{\infty}\left[6e^{-5bx}-15e^{-4bx}+10e^{-3bx}\right]\left[\frac{(26b^2+9b+1)}{(2b+1)(3b+1)(4b+1)}-\frac{6e^{-2x-\frac{x}{b}}}{(2b+1)}+\frac{12e^{-3x-\frac{x}{b}}}{(3b+1)}-\frac{6e^{-4x-\frac{x}{b}}}{(4b+1)}\right]e^{-x}dx$$

$$=\frac{\left[\begin{array}{l}5922000b^{18}+48648300b^{17}+185458260b^{16}+449383187b^{15}+768347941b^{14}\\+971883891b^{13}+936201885b^{12}+701114952b^{11}+414219816b^{10}\\+194837638b^9+73244138b^8+21970558b^7+5220242b^6+968844b^5\\137316b^4+14343b^3+1041b^2+47b+1\end{array}\right]}{\begin{array}{l}(5b^2+3b+1)(5b^2+5b+1)(4b^2+4b+1)(3b^2+3b+1)(3b^2+5b+1)\\(5b+1)(3b+1)(5b^2+4b+1)(4b^2+3b+1)(4b+1)^2(3b^2+4b+1)\end{array}},$$

$$b_3=E\left[h_b^1(X_1,X_2,...,X_6)h_b^8(X_1,X_7,...,X_{11})\right]$$

=same as b_2

$$b_4=E\left[h_b^1(X_1,X_2,...,X_6)h_b^9(X_1,X_7,...,X_{11})\right]$$

=same as b_2

$$b_5=E\left[h_b^1(X_1,X_2,...,X_6)h_b^{10}(X_1,X_7,...,X_{11})\right]$$

=same as b_2

$$b_6=E\left[h_b^1(X_1,X_2,...,X_6)h_b^{11}(X_1,X_7,...,X_{11})\right]$$

=same as b_2

$$b_7=E\left[h_b^2(X_1,X_2,...,X_6)h_b^1(X_1,X_7,...,X_{11})\right]$$

=same as b_2

$$b_8=E\left[h_b^2(X_1,X_2,...,X_6)h_b^7(X_1,X_7,...,X_{11})\right]$$

$$=P\left[Med(X_1,X_3,...,X_6)>bX_2,Med(X_1,X_8,...,X_{11})>bX_7\right]$$

$$=\int_0^{\infty}\left[P\left[Med(x,X_3,..,X_6)>bX_2\right]\right]\left[P\left[Med(x,X_8,..,X_{11})>bX_7\right]\right]e^{-x}dx$$

Now we have

$$P[Med(x,X_3,..,X_6)>bX_2]=\frac{(26b^2+9b+1)}{(2b+1)(3b+1)(4b+1)}-\frac{6e^{-2x-\frac{x}{b}}}{(2b+1)}+\frac{12e^{-3x-\frac{x}{b}}}{(3b+1)}-\frac{6e^{-4x-\frac{x}{b}}}{(4b+1)}$$

Also

$$P[Med(x,X_8,..,X_{11})>bX_7]=\frac{(26b^2+9b+1)}{(2b+1)(3b+1)(4b+1)}-\frac{6e^{-2x-\frac{x}{b}}}{(2b+1)}+\frac{12e^{-3x-\frac{x}{b}}}{(3b+1)}-\frac{6e^{-4x-\frac{x}{b}}}{(4b+1)}$$

Thus

$$b_8=\int_0^\infty\left[\frac{(26b^2+9b+1)}{(2b+1)(3b+1)(4b+1)}-\frac{6e^{-2x-\frac{x}{b}}}{(2b+1)}+\frac{12e^{-3x-\frac{x}{b}}}{(3b+1)}-\frac{6e^{-4x-\frac{x}{b}}}{(4b+1)}\right]^2e^{-x}dx$$

$$=\begin{array}{l}\frac{(26b^2+9b+1)^2}{(2b+1)^2(3b+1)^2(4b+1)^2}+\frac{36b}{(2b+1)^2(5b+2)}+\frac{144b}{(3b+1)^2(7b+2)}+\frac{36b}{(4b+1)^2(9b+2)}\\ -\frac{12b(26b^2+9b+1)}{(2b+1)^2(3b+1)^2(4b+1)}+\frac{24b(26b^2+9b+1)}{(2b+1)(3b+1)^2(4b+1)^2}-\frac{12b(26b^2+9b+1)}{(2b+1)(3b+1)(4b+1)^2}\\ -\frac{72b}{(2b+1)(3b+1)^2}+\frac{72b}{(2b+1)(4b+1)(7b+2)}-\frac{72b}{(3b+1)(4b+1)^2}\end{array},$$

$$b_9=E\left[h_b^2(X_1,X_2,...,X_6)h_b^8(X_1,X_7,...,X_{11})\right]$$

=same as b_8,

$$b_{10}=E\left[h_b^2(X_1,X_2,...,X_6)h_b^9(X_1,X_7,...,X_{11})\right]$$

=same as b_8,

$$b_{11}=E\left[h_b^2(X_1,X_2,...,X_6)h_b^{10}(X_1,X_7,...,X_{11})\right]$$

=same as b_8,

$$b_{12}=E\left[h_b^2(X_1,X_2,...,X_6)h_b^{11}(X_1,X_7,...,X_{11})\right]$$

=same as b_8,

$$b_{13}=E\left[h_b^3(X_1,X_2,...,X_6)h_b^1(X_1,X_7,...,X_{11})\right]$$

=same as b_2,

$$b_{14}=E\left[h_b^3(X_1,X_2,...,X_6)h_b^7(X_1,X_7,...,X_{11})\right]$$

=same as b_8,

$$b_{15} = E\left[h_b^3(X_1, X_2, ..., X_6) h_b^8(X_1, X_7, ..., X_{11})\right]$$

=same as b_8,

$$b_{16} = E\left[h_b^3(X_1, X_2, ..., X_6) h_b^9(X_1, X_7, ..., X_{11})\right]$$

=same as b_8,

$$b_{17} = E\left[h_b^3(X_1, X_2, ..., X_6) h_b^{10}(X_1, X_7, ..., X_{11})\right]$$

=same as b_8,

$$b_{18} = E\left[h_b^3(X_1, X_2, ..., X_6) h_b^{11}(X_1, X_7, ..., X_{11})\right]$$

=same as b_8,

$$b_{19} = E\left[h_b^4(X_1, X_2, ..., X_6) h_b^1(X_1, X_7, ..., X_{11})\right]$$

=same as b_2,

$$b_{20} = E\left[h_b^4(X_1, X_2, ..., X_6) h_b^7(X_1, X_7, ..., X_{11})\right]$$

=same as b_8,

$$b_{21} = E\left[h_b^4(X_1, X_2, ..., X_6) h_b^8(X_1, X_7, ..., X_{11})\right]$$

=same as b_8,

$$b_{22} = E\left[h_b^4(X_1, X_2, ..., X_6) h_b^9(X_1, X_7, ..., X_{11})\right]$$

=same as b_8,

$$b_{23} = E\left[h_b^4(X_1, X_2, ..., X_6) h_b^{10}(X_1, X_7, ..., X_{11})\right]$$

=same as b_8,

$$b_{24} = E\left[h_b^4(X_1, X_2, ..., X_6) h_b^{11}(X_1, X_7, ..., X_{11})\right]$$

=same as b_8,

$$b_{25} = E\left[h_b^5(X_1, X_2, \ldots, X_6) h_b^1(X_1, X_7, \ldots, X_{11})\right]$$

=same as b_2,

$$b_{26} = E\left[h_b^5(X_1, X_2, \ldots, X_6) h_b^7(X_1, X_7, \ldots, X_{11})\right]$$

=same as b_8,

$$b_{27} = E\left[h_b^5(X_1, X_2, \ldots, X_6) h_b^8(X_1, X_7, \ldots, X_{11})\right]$$

=same as b_8,

$$b_{28} = E\left[h_b^5(X_1, X_2, \ldots, X_6) h_b^9(X_1, X_7, \ldots, X_{11})\right]$$

=same as b_8,

$$b_{29} = E\left[h_b^5(X_1, X_2, \ldots, X_6) h_b^{10}(X_1, X_7, \ldots, X_{11})\right]$$

=same as b_8,

$$b_{30} = E\left[h_b^5(X_1, X_2, \ldots, X_6) h_b^{11}(X_1, X_7, \ldots, X_{11})\right]$$

=same as b_8,

$$b_{31} = E\left[h_b^6(X_1, X_2, \ldots, X_6) h_b^1(X_1, X_7, \ldots, X_{11})\right]$$

=same as b_2,

$$b_{32} = E\left[h_b^6(X_1, X_2, \ldots, X_6) h_b^7(X_1, X_7, \ldots, X_{11})\right]$$

=same as b_8,

$$b_{33} = E\left[h_b^6(X_1, X_2, \ldots, X_6) h_b^8(X_1, X_7, \ldots, X_{11})\right]$$

=same as b_8,

$$b_{34} = E\left[h_b^6(X_1, X_2, \ldots, X_6) h_b^9(X_1, X_7, \ldots, X_{11})\right]$$

=same as b_8,

$$b_{35} = E\left[h_b^6(X_1, X_2, \ldots, X_6) h_b^{10}(X_1, X_7, \ldots, X_{11})\right]$$

=same as b_8,

$$b_{36} = E\left[h_b^6(X_1, X_2, \ldots, X_6)h_b^{11}(X_1, X_7, \ldots, X_{11})\right]$$

=same as b_8,

Now adding the 36 terms and subtracting $36\mu_0^2$ we have

$36\xi_1$ =(4.3.2.9)-$36\mu_0^2$

$$= 36\frac{\left(\begin{array}{l} 8+780b+18329976000000b^{27}+1129547175486062 5b^{16}+2206644280655084b^{14} \\ +5417635657356273b^{15}+1050525b^3+41156661 16b^6+216867 33b^4+3375098 34b^5 \\ +36042b^2+199195137855482 02b^{17}+2157102191790b^9+120314680259 10b^{10} \\ +225661187238576b^{12}+764836746248446b^{13}+565994193356 12b^{11}+295269272917862 28b^{18} \\ +40352118981b^7+323924253017b^8+364598651123627 69b^{19}+370358554932090 41b^{20} \\ +304140962778009 48b^{21}+196992331740006 20b^{22}+970149827371584 0b^{23}+342582998950200 0b^{24} \\ +779956907400000b^{25}+900197874000000b^{26} \end{array}\right)}{\left[\begin{array}{l} (5b+2)(7b+2)(9b+2)(3b+1)^2(5b+1)(5b^2+3b+1)(5b^2+4b+1)(5b^2+5b+1)(4b^2+3b+1) \\ (4b^2+4b+1)(4b+1)^2(3b^2+3b+1)(b+1)(3b^2+5b+1)(1+10b)(1+9b)(1+8b)(1+7b)(1+6b) \end{array}\right]} - 36\left(\left[\frac{6}{(5b+1)}\right]-\left[\frac{15}{(4b+1)}\right]+\left[\frac{10}{(3b+1)}\right]\right)^2$$

$$= 1800b^5\frac{\left(\begin{array}{l} 51441284808 00b^{19}+8201920303920b^{20}+17908663072b^7+4947082826400b^{21} \\ -13858730807108b^{17}+33+88953434076b^8+2833547320b^6+98792b^2+14635005480 00b^{22} \\ -5077133904274b^{18}-13188060427571b^{16}+2603856374600b^{11}+1082145752590b^{10} \\ +2162162b^3+2710b+4683307255905b^{12}+5666835402514b^{13}+32155837b^4 \\ +2741281958572b^{14}-5052888006434b^{15}+347252388b^5+349377165696b^9+183299760000b^{23} \end{array}\right)}{\left[\begin{array}{l} (5b+2)(7b+2)(9b+2)(3b+1)^2(5b+1)^2(5b^2+3b+1)(5b^2+4b+1)(5b^2+5b+1)(4b^2+3b+1) \\ (4b^2+4b+1)(4b+1)^2(3b^2+3b+1)(b+1)(3b^2+5b+1)(1+10b)(1+9b)(1+8b)(1+7b)(1+6b) \end{array}\right]}$$

=B^{**}(Say).

We calculate ξ_1 for different values of b.

Thus we get

b	$B^{**}=36\xi_1$
0.1	**0.00817**
0.2	**0.03865**
0.3	**0.05606**
0.4	**0.05293**
0.5	**0.03966**
0.6	**0.02503**
0.7	**0.01321**
0.8	**0.00535**
0.9	**0.00120**

Similarly, for the case k=7 we have

$$E_{H_0}(U) = P[Med(X_1, X_2, ..., X_7) > bX_8]$$

$$= \int_0^\infty P[Med(X_1, X_2, ..., X_7) > bx] e^{-x} dx \ .$$

We have

$$P[Med(X_1, X_2, ..., X_7) > bx]$$

$$= \sum_{i=0}^{a-1} \binom{k}{i} F^i(bx) [\bar{F}(bx)]^{k-i}$$

$$= \sum_{i=0}^{3} \binom{7}{i} F^i(bx) [\bar{F}(bx)]^{7-i}$$

Thus

$$P[Med(X_1, X_2, ..., X_7) > bx] = 35e^{-4bx} - 84e^{-5bx} + 70e^{-6bx} - 20e^{-7bx}$$

Thus

$$E_{H_0}(U) = \int_0^\infty [35e^{-4bx} - 84e^{-5bx} + 70e^{-6bx} - 20e^{-7bx}] e^{-x} dx$$

Letting e^{-x} =u we get

$$E_{H_0}(U) = \frac{35}{(4b+1)} - \frac{84}{(5b+1)} + \frac{70}{(6b+1)} - \frac{20}{(7b+1)}$$

$$= \mu_0(7) \quad \text{say.}$$

Now,

$$(k+1)^2 \xi_1 = Cov[h_b^*(X_1, X_2, ..., X_8), h_b^*(X_1, X_9, ..., X_{15})]$$

where

$$h_b^*(X_1, X_2, ..., X_8)$$

$$= \frac{1}{8} \sum_{i=1}^{8} h_b^i(X_1, X_2, ..., X_8)$$

$$= \frac{1}{8}\left[h_b^1(X_1, X_2, ..., X_8) + h_b^2(X_1, X_2, ..., X_8) + ... + h_b^8(X_1, X_2, ..., X_8)\right]$$

and

$$h_b^1(X_1, X_2, ..., X_8) = \begin{cases} 1, & if \quad Med(X_2, ...X_8) > bX_1 \\ 0, & otherwise \end{cases}$$

$$h_b^2(X_1, X_2, ..., X_8) = \begin{cases} 1, & if \quad Med(X_1, X_3, .., X_8) > bX_2 \\ 0, & otherwise \end{cases}$$

and so on upto

$$h_b^8(X_1, X_2, ..., X_8) = \begin{cases} 1, & if \quad Med(X_1, X_2, ..., X_7) > bX_8 \\ 0, & otherwise \end{cases}$$

Further

$$h_b^*(X_1, X_9, ..., X_{15})$$

$$= \frac{1}{8}\sum_{j=1}^{8} h_b^j(X_1, X_9, ..., X_{15})$$

$$= \frac{1}{8}\left[h_b^1(X_1, X_9, ..., X_{15}) + h_b^9(X_1, X_9, ..., X_{15}) + ... + h_b^{15}(X_1, X_9, ..., X_{15})\right]$$

where

$$h_b^1(X_1, X_9, ..., X_{15}) = \begin{cases} 1, & if \quad Med(X_9, ..., X_{15}) > bX_1 \\ 0, & otherwise \end{cases}$$

$$h_b^9(X_1, X_9, ..., X_{15}) = \begin{cases} 1, & if \quad Med(X_1, X_{10}, ..., X_{15}) > bX_9 \\ 0, & otherwise \end{cases}$$

and so on upto

$$h_b^{15}(X_1, X_9, ..., X_{15}) = \begin{cases} 1, & if \quad Med(X_1, X_9, ..., X_{14}) > bX_{15} \\ 0, & otherwise \end{cases}$$

Now

$$Cov\left[h_b^*(X_1, X_2, ..., X_8), h_b^*(X_1, X_9, ..., X_{15})\right]$$

$$= E\left[h_b^*(X_1, X_2, ..., X_8) h_b^*(X_1, X_9, ..., X_{15})\right] - \mu_0^2 (7).$$

Now

$$64\, E\left[h_b^*(X_1, X_2, ..., X_8) h_b^*(X_1, X_9, ..., X_{15})\right] = c_1 + c_2 + ... + c_{64} \qquad (4.3.2.10)$$

where

$$c_1 = E\left[h_b^1(X_1, ..., X_8) h_b^1(X_1, X_9, ..., X_{15})\right]$$

$$= P\left[Med(X_2, ..., X_8) > bX \;, Med(X_9, ..., X_{15}) > bX_1\right]$$

$$= \int_0^\infty \left\{P[Med(X_2, ..., X_8) > bx] P[Med(X_9, ..., X_{15}) > bx]\right\} e^{-x} dx$$

$$= \int_0^\infty P^2[Med(X_2, ..., X_8) > bx] e^{-x} dx$$

Now we have

$$= P[Med\,(X_2, ..., X_8) > bx] = 35e^{-4bx} - 84e^{-5bx} + 70e^{-6bx} - 20e^{-7bx}$$

Thus

$$c_1 = \int_0^\infty \left[35e^{-4bx} - 84e^{-5bx} + 70e^{-6bx} - 20e^{-7bx}\right]^2 e^{-x} dx$$

$$= \frac{(35)^2}{(8b+1)} + \frac{(84)^2}{(10b+1)} + \frac{(70)^2}{(12b+1)} + \frac{(20)^2}{(14b+1)} - \frac{2(35)(84)}{(9b+1)} + \frac{2(35)(70)}{(10b+1)} - \frac{2(35)(20)}{(11b+1)} - \frac{2(70)(84)}{(11b+1)} + \frac{2(20)(84)}{(12b+1)} - \frac{2(70)(20)}{(13b+1)}$$

Now consider

$$c_2 = E\left[h_b^1(X_1, ..., X_8) h_b^9(X_1, X_9, ..., X_{15})\right]$$

$$= P[Med(X_2, ..., X_8) > bX_1, Med(X_1, X_{10}, ..., X_{15}) > bX_9]$$

$$= \int_0^\infty \left\{P[Med(X_2, ..., X_8) > bx] P[Med(x, X_{10}, ..., X_{15}) > bX_9]\right\} e^{-x} dx$$

Now we have

$$= P[Med\,(X_2,\ldots,X_8) > bx] = 35e^{-4bx} - 84e^{-5bx} + 70e^{-6bx} - 20e^{-7bx}$$

and

$$P[Med(x, X_{10},\ldots,X_{15}) > bX_9] =$$

$$\binom{7-1}{4-1} P[Max(X_{10}, X_{11}, X_{12}) < x < Min(X_{13}, X_{14}, X_{15}), x > bX_9] +$$
$$(7-1)\binom{7-2}{4-2} P[Max(x, X_{10}, X_{11}) < X_{12} < Min(X_{13}, X_{14}, X_{15}), X_{12} > bX_9] +$$
$$(7-1)\binom{7-2}{4-1} P[Max(X_{10}, X_{11}, X_{12}) < X_{13} < Min(x, X_{14}, X_{15}), X_{13} > bX_9]$$

$$= \binom{7-1}{4-1} P_1 + (7-1)\binom{7-2}{4-2} P_2 + (7-1)\binom{7-2}{4-1} P_3$$

Now consider

$$P_1 = P[Max(X_{10}, X_{11}, X_{12}) < x < Min(X_{13}, X_{14}, X_{15}), x > bX_9]$$

$$= \begin{matrix} P[bX_9 < Max(X_{10}, X_{11}, X_{12}) < x < Min(X_{13}, X_{14}, X_{15})] + \\ P[Max(X_{10}, X_{11}, X_{12}) < bX_9 < x < Min(X_{13}, X_{14}, X_{15})] \end{matrix}$$

$$= P_{11} + P_{12}$$

Now consider

$$P_{11} = P[bX_9 < Max(X_{10}, X_{11}, X_{12}) < x < Min(X_{13}, X_{14}, X_{15})]$$

$$= P[bX_9 < Max(X_{10}, X_{11}, X_{12}) < x < Min(X_{13}, X_{14}, X_{15})]$$

$$= \left[\int_0^{\frac{x}{b}} P[by < Max(X_{10}, X_{11}, X_{12}) < x] e^{-y} dy\right] [\bar{F}(x)]^3$$

$$= \left[\int_0^{\frac{x}{b}} \left[(1-e^{-x})^3 - (1-e^{-by})^3\right] e^{-y} dy\right] [e^{-3x}]$$

$$= \int_0^{\frac{x}{b}} e^{-3x}(1-e^{-x})^3 e^{-y} dy - \int_0^{\frac{x}{b}} e^{-3x}(1-e^{-by})^3 e^{-y} dy$$

Now consider

$$P_{12} = P[Max(X_{10}, X_{11}, X_{12}) < bX_9 < x < Min(X_{13}, X_{14}, X_{15})]$$

$$= P[Max(X_{10}, X_{11}, X_{12}) < bX_9 < x]P[x < Min(X_{13}, X_{14}, X_{15})]$$

$$= \left[\int_0^{\frac{x}{b}} P[Max(X_{10}, X_{11}, X_{12}) < by] e^{-y} dy\right] \left[e^{-3x}\right]$$

$$= \left[\int_0^{\frac{x}{b}} \left[\left(1 - e^{-by}\right)^3\right] e^{-y} dy\right] \left[e^{-3x}\right]$$

Now adding P_{11} and P_{12} we have

$$P_1 = e^{-3x} - 3e^{-4x} + 3e^{-5x} - e^{-6x} - e^{-3x-\frac{x}{b}} + 3e^{-4x-\frac{x}{b}} - 3e^{-5x-\frac{x}{b}} + e^{-6x-\frac{x}{b}}$$

Now consider

$$P_2 = P[Max(x, X_{10}, X_{11}) < X_{12} < Min(X_{13}, X_{14}, X_{15}), X_{12} > bX_9]$$

$$= \begin{matrix} P[bX_9 < Max(x, X_{10}, X_{11}) < X_{12} < Min(X_{13}, X_{14}, X_{15})] + \\ P[Max(x, X_{10}, X_{11}) < bX_9 < X_{12} < Min(X_{13}, X_{14}, X_{15})] \end{matrix}$$

$$= P_{21} + P_{22}$$

Now consider

$$P_{21} = P[bX_9 < Max(x, X_{10}, X_{11}) < X_{12} < Min(X_{13}, X_{14}, X_{15})]$$

$$= \begin{matrix} P[bX_9 < x < Max(X_{10}, X_{11}) < X_{12} < Min(X_{13}, X_{14}, X_{15})] + \\ P[bX_9 < Max(X_{10}, X_{11}) < x < X_{12} < Min(X_{13}, X_{14}, X_{15})] + \\ P[x < bX_9 < Max(X_{10}, X_{11}) < X_{12} < Min(X_{13}, X_{14}, X_{15})] + \\ P[Max(X_{10}, X_{11}) < bX_9 < x < X_{12} < Min(X_{13}, X_{14}, X_{15})] \end{matrix}$$

$$= P_{211} + P_{212} + P_{213} + P_{214}$$

Now consider

$$P_{211} = P[bX_9 < x < Max(X_{10}, X_{11}) < X_{12} < Min(X_{13}, X_{14}, X_{15})]$$

$$= P[bX_9 < x]P[x < Max(X_{10}, X_{11}) < X_{12} < Min(X_{13}, X_{14}, X_{15})]$$

$$= F\left(\frac{x}{b}\right)\left[\int_x^\infty\int_z^\infty [\overline{F}(x_{12})]^3 dF(x_{12}) 2F(z) dF(z)\right]$$

$$= \left[1 - e^{-\frac{x}{b}}\right]\left[\int_x^\infty \int_0^{\overline{F}(z)} u^3 du 2F(z) dF(z)\right] \qquad \text{letting } \overline{F}(x_{12}) = u$$

$$= \frac{e^{-5x}}{10} - \frac{e^{-6x}}{12} - \frac{e^{-5x-\frac{x}{b}}}{10} + \frac{e^{-6x-\frac{x}{b}}}{12}$$

Now consider

$$P_{212} = P[bX_9 < Max(X_{10}, X_{11}) < x < X_{12} < Min(X_{13}, X_{14}, X_{15})]$$

$$= P[bX_9 < Max(X_{10}, X_{11}) < x]P[x < X_{12} < Min(X_{13}, X_{14}, X_{15})]$$

$$= \left[\int_0^x 2F\left(\frac{z}{b}\right)F(z)dF(z)\right]\left[\int_x^\infty [\overline{F}(x_{12})]^3 dF(x_{12})\right]$$

$$= \frac{e^{-4x}}{4} - \frac{e^{-5x}}{2} + \frac{e^{-6x}}{4} - \frac{be^{-4x}}{2(b+1)} + \frac{be^{-5x-\frac{x}{b}}}{2(b+1)} + \frac{be^{-4x}}{2(2b+1)} - \frac{be^{-6x-\frac{x}{b}}}{2(2b+1)}$$

Now consider

$$P_{213} = P[x < bX_9 < Max(X_{10}, X_{11}) < X_{12} < Min(X_{13}, X_{14}, X_{15})]$$

$$= \int_{\frac{x}{b}}^\infty \int_{bx_9}^\infty \int_z^\infty \int_{x_{12}}^\infty 3e^{-2y} e^{-y} dy e^{-x_{12}} dx_{12} 2(1 - e^{-z}) e^{-z} dz e^{-x_9} dx_9$$

$$= \frac{e^{-5x-\frac{x}{b}}}{10(5b+1)} - \frac{e^{-6x-\frac{x}{b}}}{12(6b+1)}$$

Now consider

$$P_{214} = P[Max(X_{10}, X_{11}) < bX_9 < x < X_{12} < Min(X_{13}, X_{14}, X_{15})]$$

$$= P[Max(X_{10}, X_{11}) < bX_9 < x]P[x < X_{12} < Min(X_{13}, X_{14}, X_{15})]$$

$$= \left[\int_0^{\frac{x}{b}} P[Max(X_{10}, X_{11}) < bX_9] e^{-x_9} dx_9\right]\left[\frac{e^{-4x}}{4}\right]$$

$$= \frac{2b^2 e^{-4x}}{4(b+1)(2b+1)} - \frac{e^{-4x-\frac{x}{b}}}{4} + \frac{2e^{-5x-\frac{x}{b}}}{4(b+1)} - \frac{e^{-6x-\frac{x}{b}}}{4(2b+1)}$$

Now adding $P_{211}, P_{212}, P_{213}$ and P_{214} we have

$$P_{21} = \frac{e^{-4x}}{4} - \frac{2e^{-5x}}{5} + \frac{e^{-6x}}{6} - \frac{e^{-4x-\frac{x}{b}}}{4} + \frac{(4b+1)e^{-5x-\frac{x}{b}}}{2(5b+1)} - \frac{(4b+1)e^{-6x-\frac{x}{b}}}{4(6b+1)}$$

Now consider P_{22}

$$P_{22} = P[Max(x, X_{10}, X_{11}) < bX_9 < X_{12} < Min(X_{13}, X_{14}, X_{15})]$$

$$= \begin{matrix} P[x < Max(X_{10}, X_{11}) < bX_9 < X_{12} < Min(X_{13}, X_{14}, X_{15})] + \\ P[Max(X_{10}, X_{11}) < x < bX_9 < X_{12} < Min(X_{13}, X_{14}, X_{15})] \end{matrix}$$

$$= P_{221} + P_{222}$$

Now consider

$$P_{221} = P[x < Max(X_{10}, X_{11}) < bX_9 < X_{12} < Min(X_{13}, X_{14}, X_{15})]$$

$$= \int_x^{\infty}\int_{\frac{x}{b}}^{\infty}\int_{bx_9}^{\infty}\int_{x_{12}}^{\infty} 3\overline{F}^2(y)dF(y)dF(x_{12})dF(x_9)2F(z)dF(z)$$

$$= \frac{be^{-5x-\frac{x}{b}}}{2(4b+1)(5b+1)} - \frac{be^{-6x-\frac{x}{b}}}{2(4b+1)(6b+1)}$$

Now consider

$$P_{222} = P[Max(X_{10}, X_{11}) < x < bX_9 < X_{12} < Min(X_{13}, X_{14}, X_{15})]$$

$$= P[Max(X_{10}, X_{11}) < x]P[x < bX_9 < X_{12} < Min(X_{13}, X_{14}, X_{15})]$$

$$= (1-e^{-x})^2 \left[\int_{\frac{x}{b}}^{\infty} \int_{bx_9}^{\infty} \int_{x_{12}}^{\infty} 3e^{-2y} e^{-y} dy e^{-x_{12}} dx_{12} e^{-x_9} dx_9 \right]$$

$$= \frac{e^{-4x-\frac{x}{b}}}{4(4b+1)} - \frac{2e^{-5x-\frac{x}{b}}}{4(4b+1)} + \frac{e^{-6x-\frac{x}{b}}}{4(4b+1)}$$

Now adding P_{221} and P_{222} we have

$$P_{22} = \frac{e^{-4x-\frac{x}{b}}}{4(4b+1)} - \frac{e^{-5x-\frac{x}{b}}}{2(5b+1)} + \frac{e^{-6x-\frac{x}{b}}}{4(6b+1)}$$

Thus now adding P_{21} and P_{22} we have

$$P_2 = \frac{e^{-4x}}{4} - \frac{2e^{-5x}}{5} + \frac{e^{-6x}}{6} - \frac{be^{-4x-\frac{x}{b}}}{(4b+1)} + \frac{2be^{-5x-\frac{x}{b}}}{(5b+1)} - \frac{be^{-6x-\frac{x}{b}}}{(6b+1)}$$

Now consider P_3

$$P_3 = P[Max(X_{10}, X_{11}, X_{12}) < X_{13} < Min(x, X_{14}, X_{15}), X_{13} > bX_9]$$

$$= \begin{matrix} P[bX_9 < Max(X_{10}, X_{11}, X_{12}) < X_{13} < Min(x, X_{14}, X_{15})] + \\ P[Max(X_{10}, X_{11}, X_{12}) < bX_9 < X_{13} < Min(x, X_{14}, X_{15})] \end{matrix}$$

$$= \begin{matrix} P[bX_9 < Max(X_{10}, X_{11}, X_{12}) < X_{13} < x < Min(X_{14}, X_{15})] + \\ P[bX_9 < Max(X_{10}, X_{11}, X_{12}) < X_{13} < Min(X_{14}, X_{15}) < x] + \\ P[Max(X_{10}, X_{11}, X_{12}) < bX_9 < X_{13} < x < Min(X_{14}, X_{15})] + \\ P[Max(X_{10}, X_{11}, X_{12}) < bX_9 < X_{13} < Min(X_{14}, X_{15}) < x] \end{matrix}$$

$$= P_{31} + P_{32} + P_{33} + P_{34}$$

Now consider

$$P_{31} = P[bX_9 < Max(X_{10}, X_{11}, X_{12}) < X_{13} < x < Min(X_{14}, X_{15})]$$

$$= P[bX_9 < Max(X_{10}, X_{11}, X_{12}) < X_{13} < x] P[x < Min(X_{14}, X_{15})]$$

$$= \left[\int_0^{\frac{x}{b}} \int_{bx_9}^{x} \int_{z}^{x} e^{-x_{13}} dx_{13} 3(1-e^{-z})^2 e^{-z} dz e^{-x_9} dx_9 \right] (e^{-2x})$$

$$= \frac{(26b^2+9b+1)e^{-2x}}{4(2b+1)(3b+1)(4b+1)} - \frac{(7b^2+6b+1)e^{-3x}}{(b+1)(2b+1)(3b+1)} + \frac{3e^{-4x}}{2} - e^{-5x} + \frac{e^{-6x}}{4} - \frac{6b^2 e^{-4x-\frac{x}{b}}}{2(b+1)(2b+1)} + \frac{6b^2 e^{-5x-\frac{x}{b}}}{(2b+1)(3b+1)} - \frac{12b^2 e^{-6x-\frac{x}{b}}}{4(3b+1)(4b+1)}$$

Similarly Next consider

$$P_{32} = P[bX_9 < Max(X_{10}, X_{11}, X_{12}) < X_{13} < Min(X_{14}, X_{15}) < x]$$

$$= \int_0^{\frac{x}{b}} \int_{bx_9}^{x} \int_{z}^{x} \int_{x_{13}}^{x} 2e^{-y} e^{-y} dy e^{-x_{13}} dx_{13} 3(1-e^{-z})^2 e^{-z} dz e^{-x_9} dx_9$$

then

$$P_{33} = P[Max(X_{10}, X_{11}, X_{12}) < bX_9 < X_{13} < x < Min(X_{14}, X_{15})]$$

$$= P[Max(X_{10}, X_{11}, X_{12}) < bX_9 < X_{13} < x] P[x < Min(X_{14}, X_{15})]$$

$$= \left[\int_0^{x} \int_{\frac{z}{b}}^{\frac{x}{b}} \int_{bx_9}^{x} e^{-x_{13}} dx_{13} e^{-x_9} dx_9 3(1-e^{-z})^2 e^{-z} dz \right] (e^{-2x})$$

and

$$P_{34} = P[Max(X_{10}, X_{11}, X_{12}) < bX_9 < X_{13} < Min(X_{14}, X_{15}) < x]$$

$$= \int_0^{x} \int_{\frac{z}{b}}^{\frac{x}{b}} \int_{bx_9}^{x} \int_{x_{13}}^{x} 2e^{-y} e^{-y} dy e^{-x_{13}} dx_{13} e^{-x_9} dx_9 3(1-e^{-z})^2 e^{-z} dz$$

Now simplifying and adding P_{31}, P_{32}, P_{33} and P_{34} we have

$$P_3 = \frac{1}{60} - \frac{6b^4}{(3b+1)(4b+1)(5b+1)(6b+1)} - \frac{e^{-3x}}{3} + \frac{3e^{-4x}}{4} - \frac{3e^{-5x}}{5} + \frac{e^{-6x}}{6} + \frac{be^{-3x-\frac{x}{b}}}{(3b+1)} - \frac{3be^{-4x-\frac{x}{b}}}{(4b+1)} + \frac{3be^{-5x-\frac{x}{b}}}{(5b+1)} - \frac{be^{-6x-\frac{x}{b}}}{(6b+1)}$$

Thus finally we get

$$P[Med(x, X_{10}, \ldots, X_{15}) > bX_9]$$

$$= 20P_1 + 60P_2 + 60P_3$$

$$= \frac{(342b^3 + 119b^2 + 18b + 1)}{(3b+1)(4b+1)(5b+1)(6b+1)} - \frac{20e^{-3x-\frac{x}{b}}}{(3b+1)} + \frac{60e^{-4x-\frac{x}{b}}}{(4b+1)} - \frac{60e^{-5x-\frac{x}{b}}}{(5b+1)} + \frac{20e^{-6x-\frac{x}{b}}}{(6b+1)}$$

Finally we get

$$c_2 = \int_0^\infty \{P[Med(X_2,\ldots,X_8) > bx]P[Med(x, X_{10},\ldots,X_{15}) > bX_9]\}e^{-x}dx$$

$$= \int_0^\infty \{35e^{-4bx} - 84e^{-5bx} + 70e^{-6bx} - 20e^{-7bx}\} \left\{ \frac{(342b^3 + 119b^2 + 18b + 1)}{(3b+1)(4b+1)(5b+1)(6b+1)} - \frac{20e^{-3x-\frac{x}{b}}}{(3b+1)} + \frac{60e^{-4x-\frac{x}{b}}}{(4b+1)} - \frac{60e^{-5x-\frac{x}{b}}}{(5b+1)} + \frac{20e^{-6x-\frac{x}{b}}}{(6b+1)} \right\} e^{-x}dx$$

$$= \frac{35(342b^3 + 119b^2 + 18b + 1)}{(3b+1)(4b+1)^2(5b+1)(6b+1)} - \frac{(20)(35)b}{(3b+1)(4b^2+4b+1)} + \frac{(60)(35)b}{(4b+1)(4b^2+5b+1)} - \frac{(60)(35)b}{(5b+1)(4b^2+6b+1)}$$
$$+ \frac{(20)(35)b}{(6b+1)(4b^2+7b+1)} - \frac{84(342b^3 + 119b^2 + 18b + 1)}{(3b+1)(4b+1)(5b+1)^2(6b+1)} + \frac{(20)(84)b}{(3b+1)(5b^2+4b+1)} - \frac{(60)(84)b}{(4b+1)(5b^2+5b+1)}$$
$$+ \frac{(60)(84)b}{(5b+1)(5b^2+6b+1)} - \frac{(20)(84)b}{(6b+1)(5b^2+7b+1)} + \frac{70(342b^3 + 119b^2 + 18b + 1)}{(3b+1)(4b+1)(5b+1)(6b+1)^2} - \frac{(20)(70)b}{(3b+1)(6b^2+4b+1)}$$
$$+ \frac{(60)(70)b}{(4b+1)(6b^2+5b+1)} - \frac{(60)(70)b}{(5b+1)(6b^2+6b+1)} + \frac{(20)(70)b}{(6b+1)(6b^2+7b+1)} - \frac{20(342b^3 + 119b^2 + 18b + 1)}{(3b+1)(4b+1)(5b+1)(6b+1)(7b+1)}$$
$$+ \frac{(20)(20)b}{(3b+1)(7b^2+4b+1)} - \frac{(60)(20)b}{(4b+1)(7b^2+5b+1)} + \frac{(60)(20)b}{(5b+1)(7b^2+6b+1)} - \frac{(20)(20)b}{(6b+1)(7b^2+7b+1)}$$

$$c_3 = E\left[h_b^1(X_1,\ldots,X_8)h_b^{10}(X_1, X_9,\ldots,X_{15})\right]$$

=same as c_2,

$$c_4 = E\left[h_b^1(X_1, X_2,\ldots,X_8)h_b^{11}(X_1, X_9,\ldots,X_{15})\right]$$

=same as c_2,

$$c_5 = E\left[h_b^1(X_1, X_2,\ldots,X_8)h_b^{12}(X_1, X_9,\ldots,X_{15})\right]$$

=same as c_2,

$$c_6 = E\left[h_b^1(X_1, X_2,\ldots,X_8)h_b^{13}(X_1, X_9,\ldots,X_{15})\right]$$

=same as c_2,

$$c_7 = E\left[h_b^1(X_1, X_2, ..., X_8) h_b^{14}(X_1, X_9, ..., X_{15})\right]$$

=same as c_2,

$$c_8 = E\left[h_b^1(X_1, X_2, ..., X_8) h_b^{15}(X_1, X_9, ..., X_{15})\right]$$

=same as c_2,

$$c_9 = E\left[h_b^2(X_1, X_2, ..., X_8) h_b^1(X_1, X_9, ..., X_{15})\right]$$

=same as c_2,

$$c_{10} = E\left[h_b^2(X_1, X_2, ..., X_8) h_b^9(X_1, X_9, ..., X_{15})\right]$$

$$= P[Med(X_1, X_3, ..., X_8) > bX_2, Med(X_1, X_{10}, ..., X_{15}) > bX_9]$$

$$= \int_0^\infty P[Med(x, X_3, ..., X_8) > bX_2] P[Med(x, X_{10}, ..., X_{15}) > bX_9] e^{-x} dx$$

Now we have

$$P[Med(x, X_3, ..., X_8) > bX_2]$$

$$= \frac{(342b^3 + 119b^2 + 18b + 1)}{(3b+1)(4b+1)(5b+1)(6b+1)} - \frac{20e^{-3x-\frac{x}{b}}}{(3b+1)} + \frac{60e^{-4x-\frac{x}{b}}}{(4b+1)} -$$
$$\frac{60e^{-5x-\frac{x}{b}}}{(5b+1)} + \frac{20e^{-6x-\frac{x}{b}}}{(6b+1)}$$

and

$$P[Med(x, X_{10}, ..., X_{15}) > bX_9]$$

$$= \frac{(342b^3 + 119b^2 + 18b + 1)}{(3b+1)(4b+1)(5b+1)(6b+1)} - \frac{20e^{-3x-\frac{x}{b}}}{(3b+1)} + \frac{60e^{-4x-\frac{x}{b}}}{(4b+1)} -$$
$$\frac{60e^{-5x-\frac{x}{b}}}{(5b+1)} + \frac{20e^{-6x-\frac{x}{b}}}{(6b+1)}$$

Thus we have

$$c_{10} = \int_0^\infty P[Med(x, X_3, ..., X_8) > bX_2] P[Med(x, X_{10}, ..., X_{15}) > bX_9] e^{-x} dx$$

$$= \int_0^\infty \left[\frac{(342b^3+119b^2+18b+1)}{(3b+1)(4b+1)(5b+1)(6b+1)} - \frac{20e^{-3x-\frac{x}{b}}}{(3b+1)} + \frac{60e^{-4x-\frac{x}{b}}}{(4b+1)} - \frac{60e^{-5x-\frac{x}{b}}}{(5b+1)} + \frac{20e^{-6x-\frac{x}{b}}}{(6b+1)} \right]^2 e^{-x} dx$$

$$= \frac{(342b^3+119b^2+18b+1)^2}{(3b+1)^2(4b+1)^2(5b+1)^2(6b+1)^2} + \frac{(20)^2 b}{(3b+1)^2(7b+2)} + \frac{(60)^2 b}{(4b+1)^2(9b+2)} + \frac{(60)^2 b}{(5b+1)^2(11b+2)}$$
$$+ \frac{(20)^2 b}{(6b+1)^2(13b+2)} - \frac{2(20)(342b^3+119b^2+18b+1)b}{(3b+1)^2(4b+1)^2(5b+1)(6b+1)} + \frac{2(60)(342b^3+119b^2+18b+1)b}{(3b+1)(4b+1)^2(5b+1)^2(6b+1)}$$
$$- \frac{2(60)(342b^3+119b^2+18b+1)b}{(3b+1)(4b+1)(5b+1)^2(6b+1)^2} + \frac{2(20)(342b^3+119b^2+18b+1)b}{(3b+1)(4b+1)(5b+1)(6b+1)^2(7b+1)} - \frac{(20)(60)b}{(3b+1)(4b+1)^2}$$
$$+ \frac{2(20)(60)b}{(3b+1)(5b+1)(9b+2)} - \frac{(20)(20)b}{(3b+1)(5b+1)(6b+1)} - \frac{(60)(60)b}{(4b+1)(5b+1)^2} + \frac{2(20)(60)b}{(4b+1)(6b+1)(11b+2)}$$
$$- \frac{(20)(60)b}{(5b+1)(6b+1)^2},$$

$$c_{11} = E\left[h_b^2(X_1, X_2, \ldots, X_8) h_b^{10}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{12} = E\left[h_b^2(X_1, X_2, \ldots, X_8) h_b^{11}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{13} = E\left[h_b^2(X_1, X_2, \ldots, X_8) h_b^{12}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{14} = E\left[h_b^2(X_1, X_2, \ldots, X_8) h_b^{13}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{15} = E\left[h_b^2(X_1, X_2, \ldots, X_8) h_b^{14}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{16} = E\left[h_b^2(X_1, X_2, \ldots, X_8) h_b^{15}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{17} = E\left[h_b^3(X_1, X_2, \ldots, X_8) h_b^{1}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_2,

$$c_{18} = E\left[h_b^3(X_1, X_2, ..., X_8) h_b^9(X_1, X_9, ..., X_{15})\right]$$

=same as c_{10},

$$c_{19} = E\left[h_b^3(X_1, X_2, ..., X_8) h_b^{10}(X_1, X_9, ..., X_{15})\right]$$

=same as c_{10},

$$c_{20} = E\left[h_b^3(X_1, X_2, ..., X_8) h_b^{11}(X_1, X_9, ..., X_{15})\right]$$

=same as c_{10},

$$c_{21} = E\left[h_b^3(X_1, X_2, ..., X_8) h_b^{12}(X_1, X_9, ..., X_{15})\right]$$

=same as c_{10},

$$c_{22} = E\left[h_b^3(X_1, X_2, ..., X_8) h_b^{13}(X_1, X_9, ..., X_{15})\right]$$

=same as c_{10},

$$c_{23} = E\left[h_b^3(X_1, X_2, ..., X_8) h_b^{14}(X_1, X_9, ..., X_{15})\right]$$

=same as c_{10},

$$c_{24} = E\left[h_b^3(X_1, X_2, ..., X_8) h_b^{15}(X_1, X_9, ..., X_{15})\right]$$

=same as c_{10},

$$c_{25} = E\left[h_b^4(X_1, X_2, ..., X_8) h_b^1(X_1, X_9, ..., X_{15})\right]$$

=same as c_2,

$$c_{26} = E\left[h_b^4(X_1, X_2, ..., X_8) h_b^9(X_1, X_9, ..., X_{15})\right]$$

=same as c_{10},

$$c_{27} = E\left[h_b^4(X_1, X_2, ..., X_8) h_b^{10}(X_1, X_9, ..., X_{15})\right]$$

=same as c_{10},

$$c_{28} = E\left[h_b^4(X_1, X_2, \ldots, X_8) h_b^{11}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{29} = E\left[h_b^4(X_1, X_2, \ldots, X_8) h_b^{12}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{30} = E\left[h_b^4(X_1, X_2, \ldots, X_8) h_b^{13}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{31} = E\left[h_b^4(X_1, X_2, \ldots, X_8) h_b^{14}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{32} = E\left[h_b^4(X_1, X_2, \ldots, X_8) h_b^{15}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{33} = E\left[h_b^5(X_1, X_2, \ldots, X_8) h_b^{1}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{2},

$$c_{34} = E\left[h_b^5(X_1, X_2, \ldots, X_8) h_b^{9}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{35} = E\left[h_b^5(X_1, X_2, \ldots, X_8) h_b^{10}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{36} = E\left[h_b^5(X_1, X_2, \ldots, X_8) h_b^{11}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{37} = E\left[h_b^5(X_1, X_2, \ldots, X_8) h_b^{12}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{38} = E\left[h_b^5(X_1, X_2, \ldots, X_8) h_b^{13}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{39} = E\left[h_b^5(X_1, X_2, \ldots, X_8) h_b^{14}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{40} = E\left[h_b^5(X_1, X_2, \ldots, X_8) h_b^{15}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{41} = E\left[h_b^6(X_1, X_2, \ldots, X_8) h_b^{15}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_2,

$$c_{42} = E\left[h_b^6(X_1, X_2, \ldots, X_8) h_b^9(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{43} = E\left[h_b^6(X_1, X_2, \ldots, X_8) h_b^{10}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{44} = E\left[h_b^6(X_1, X_2, \ldots, X_8) h_b^{11}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{45} = E\left[h_b^6(X_1, X_2, \ldots, X_8) h_b^{12}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{46} = E\left[h_b^6(X_1, X_2, \ldots, X_8) h_b^{13}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{47} = E\left[h_b^6(X_1, X_2, \ldots, X_8) h_b^{14}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{48} = E\left[h_b^6(X_1, X_2, \ldots, X_8) h_b^{15}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{49} = E\left[h_b^7(X_1, X_2, \ldots, X_8) h_b^1(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_2,

$$c_{50} = E\left[h_b^7(X_1, X_2, \ldots, X_8) h_b^9(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{51} = E\left[h_b^7(X_1, X_2, \ldots, X_8) h_b^{10}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{52} = E\left[h_b^7(X_1, X_2, \ldots, X_8) h_b^{11}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{53} = E\left[h_b^7(X_1, X_2, \ldots, X_8) h_b^{12}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{54} = E\left[h_b^7(X_1, X_2, \ldots, X_8) h_b^{13}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{55} = E\left[h_b^7(X_1, X_2, \ldots, X_8) h_b^{14}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{56} = E\left[h_b^7(X_1, X_2, \ldots, X_8) h_b^{15}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{57} = E\left[h_b^8(X_1, X_2, \ldots, X_8) h_b^1(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_2,

$$c_{58} = E\left[h_b^8(X_1, X_2, \ldots, X_8) h_b^9(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{59} = E\left[h_b^8(X_1, X_2, \ldots, X_8) h_b^{10}(X_1, X_9, \ldots, X_{15})\right]$$

=same as c_{10},

$$c_{60} = E\left[h_b^8(X_1, X_2, ..., X_8)h_b^{11}(X_1, X_9, ..., X_{15})\right]$$

=same as c_{10},

$$c_{61} = E\left[h_b^8(X_1, X_2, ..., X_8)h_b^{12}(X_1, X_9, ..., X_{15})\right]$$

=same as c_{10},

$$c_{62} = E\left[h_b^8(X_1, X_2, ..., X_8)h_b^{13}(X_1, X_9, ..., X_{15})\right]$$

=same as c_{10},

$$c_{63} = E\left[h_b^8(X_1, X_2, ..., X_8)h_b^{14}(X_1, X_9, ..., X_{15})\right]$$

=same as c_{10},

$$c_{64} = E\left[h_b^8(X_1, X_2, ..., X_8)h_b^{15}(X_1, X_9, ..., X_{15})\right]$$

=same as c_{10}.

Now adding the 64 terms and subtracting $64\mu_0^2$(7), we have

$64\xi_1$ =(4.3.2.10)-$64\mu_0^2$

$$= 32\frac{\left(\begin{array}{l}32+506925150462907322948497979450b^{29}+6496b+19976125971053979 2b^{10}\\+636496b^{2}+40117136b^{3}+1828813970b^{4}+64281302174b^{5}+1813494419526b^{6}\\+42206513232855b^{7}+826565439328354b^{8}+13824932339272660b^{9}+2354015729635739142768b^{14}\\+30695408550672489612988921489 6b^{28}+16499514086395229148276215124 5b^{27}\\+37980737716443033024987303 4b^{21}+18159806781752166662061b^{15}\\+95011576360319908290442654735 4b^{31}+2516118844367300146b^{11}\\+33790892702280601210779333516b^{25}+13704147950430478192225813 84b^{22}\\+12511852340305376128512 6b^{16}+79002323616154031262285677564b^{26}\\+12939645295354829903137516440b^{24}+27172143444205718752 6b^{13}\\+77171147109469757846495 2b^{17}+74003367973494785274629925711 8b^{30}\\+94638694218975178316859038b^{20}+44438314233012534551551883 74b^{23}\\+21201584910944242413794405b^{19}+42685897436023984970129 20b^{18}\\+61693806866437379692659267058 4b^{35}+86818773739326734180925708811 2b^{34}\\+10378163615605827823563213090 24b^{33}+10661710292961106899585886554 60b^{32}\\+27825028175540691820b^{12}+45274363140424805775360000b^{42}\\+55820218040292625920000 0b^{43}+63199107843476351009760000 0b^{41}\\+45086554954294502331227328 00b^{40}+20864964773580034937991315840b^{39}\\+69743704659211259317754104320b^{38}+17900268038164668192288485116 8b^{37}\\+36687161778555410332464372064b^{36}\end{array}\right)}{\left[\begin{array}{l}(7b+2)(9b+2)(11b+2)(13b+2)(6b+1)^{2}(2b+1)(4b+1)^{2}(4b^{2}+6b+1)(4b^{2}+7b+1)\\(5b^{2}+4b+1)(5b^{2}+5b+1)(5b+1)^{2}(5b^{2}+7b+1)(6b^{2}+4b+1)(6b^{2}+5b+1)(6b^{2}+6b+1)\\(b+1)(7b+1)(7b^{2}+4b+1)(7b^{2}+5b+1)(7b^{2}+6b+1)(7b^{2}+7b+1)(8b+1)(10b+1)\\(12b+1)(14b+1)(9b+1)(11b+1)(13b+1)\end{array}\right]}$$

$$-64\left[\left[\frac{35}{(4b+1)}\right]-\left[\frac{84}{(5b+1)}\right]+\left[\frac{70}{(6b+1)}\right]-\left[\frac{20}{(7b+1)}\right]\right]^{2}$$

$$= 352800b^{7}\frac{\left(\begin{array}{l}93-10424559060241813075648 56b^{28}+31654460559773124702720b^{35}+17041b+3544140827955087360000b^{37}\\+43971162234542717664 2b^{15}+1413754b^{2}+72256824b^{3}+22835700613582533022132 8b^{30}+59131077025319155508252 8b^{21}\\+14652860161 98b^{6}+54874100105713028998233 6b^{32}+49262227408793290752 00b^{36}+35336177950459407 6b^{11}+41340940275233990b^{10}\\+13024900526406866291386 05b^{20}-45576146246242664058647 2b^{29}+39046740750297614797684 2b^{22}+91509968033828054332b^{14}\\+24966047854094b^{7}+26006091999863606665b^{12}+2585854346b^{4}+45764235713183173613134 8b^{23}+31335765031826660617291 6b^{24}\\+12337160425356101150992 0b^{34}+20951014216341169530328b^{18}+16558654150333360911b^{13}+31774065961069160463705 6b^{33}\\+66692955398107716514 04b^{17}+18375517773593431425 26b^{16}+25064647362138902012249 3b^{31}+69495056588b^{5}+41371398884567 05b^{9}\\-71989673633700006610789 2b^{26}-12131805268973222229652 6b^{25}-11360110474753454901135 20b^{27}+35105077808296 9b^{8}\\+56615530851633039070488 8b^{19}\end{array}\right)}{\left[\begin{array}{l}(7b+2)(9b+2)(11b+2)(13b+2)(6b+1)^{2}(2b+1)(4b+1)^{2}(4b^{2}+6b+1)(4b^{2}+7b+1)(5b^{2}+4b+1)(5b^{2}+5b+1)(5b+1)^{2}\\(5b^{2}+7b+1)(6b^{2}+4b+1)(6b^{2}+5b+1)(6b^{2}+6b+1)(b+1)(7b+1)^{2}(7b^{2}+4b+1)(7b^{2}+5b+1)(7b^{2}+6b+1)(7b^{2}+7b+1)\\(8b+1)(10b+1)(12b+1)(14b+1)(9b+1)(11b+1)(13b+1)\end{array}\right]}$$

$=B^{***}(Say)$

We calculate ξ_1 for different values of b.

Thus we get

b	B***=64ξ_1
0.1	0.005308
0.2	0.03771
0.3	0.06209
0.4	0.06186
0.5	0.04755
0.6	0.03037
0.7	0.016108
0.8	0.006535
0.9	0.001463

4.4 Asymptotic Relative Efficiency

In this section we consider the Pitman's Asymptotic Relative Efficiency. Let $\{S_n\}$, $\{T_{n'}\}$ be sequences of statistics for testing H_0 against H_1,where the indexing corresponds to the number of sample observations n and n' respectively used for the tests. Let θ^* be an arbitrary element in the alternative $\Omega - \omega$ and denote by C_n and $D_{n'}$ the critical regions of size α, $0<\alpha<1$ for the S_n and $T_{n'}$ tests, respectively, that is,

$P_\theta(S_n \in C_n) \leq \alpha$ for all $\theta \in \omega$ and all n

and

$P_\theta(T_{n'} \in D_{n'}) \leq \alpha$ for all $\theta \in \omega$ and all n'

Let $\beta, \alpha < \beta < 1$, be arbitrary, but fixed and define N and N' to be the smallest values of n and n′ respectively for which

$P_{\theta^*}(S_N \in C_N) > \beta$ and $P_{\theta^*}(T_{N'} \in D_{N'}) > \beta$

Now the finite sample size relative efficiency of $\{S_n\}$ with respect to (or relative to) $\{T_{n'}\}$ at the alternative θ^* is taken to be

$$e(S,T/\alpha,\beta,\theta^*,underlyingdistributions) = \frac{N'(\alpha,\beta,\theta^*,underlyingdistributions)}{N(\alpha,\beta,\theta^*,underlyingdistributions)}$$

we refer to $e(S,T/\alpha,\beta,\theta^*,underlyingdistributions)$ as simply the efficiency of S_n relative to $T_{n'}$ and if $e(S,T/\alpha,\beta,\theta^*,underlyingdistributions) > 1$, we say that a test based on S_n is more efficient than one based on $T_{n'}$, if it is less than 1, the opposite is true.

Thus here we define the Pitman Efficiency.

Let $\{S_{n_i}\}$ and $\{T_{n_i'}\}$ be two sequences of test statistics for testing $H_0 : \theta = \theta_0$, where θ_0 is some specified parameter value, against a class of alternatives H_1 at size α, and let $\{\theta_i\}$ be a sequence of alternatives to H_0 such that $\lim_{i \to \infty} \theta_i = \theta_0$. In addition, let $\beta_{S_{n_i}}(\theta_i)$ and $\beta_{T_{n_i'}}(\theta_i)$ be the powers of the tests based on S_{n_i} and $T_{n_i'}$, respectively, at the alternative θ_i. Let $\{n_i\}$ and $\{n_i'\}$ be increasing sequences of positive integers such that the two sequences of tests have the same limiting significance level α and

$$\alpha < \lim_{i \to \infty} \beta_{S_{n_i}}(\theta_i) = \lim_{i \to \infty} \beta_{T_{n_i}}(\theta_i) < 1$$

Then the asymptotic relative efficiency (ARE) of $\{S_{n_i}\}$ relative to $\{T_{n_i}\}$

is ARE(S,T)= $\lim_{i \to \infty} \frac{n_i'}{n_i}$

The quantity

$$K_S = \lim_{n \to \infty} \frac{\mu_{S_n}'(\theta_0)}{\sqrt{n\sigma_{S_n}^2(\theta_0)}}$$

is called the efficacy of the test based on S_n and is denoted by eff(S).

where $\mu_{S_n}(\theta), \sigma_{S_n}^2(\theta)$ are sequences of numbers associated with $\{S_n\}$. These sequences are means and variances of $\{S_n\}$ in many applications.

Using similar notation and terminology for $T_{n'}$ that is we have

$$K_T = \lim_{n' \to \infty} \frac{\mu_{T_n}'(\theta_0)}{\sqrt{n'\sigma_{T_n}^2(\theta_0)}}$$

is called the efficacy of the test based on T_n and is denoted by eff(T).

The expression for ARE of S relative to T is given by

$$\text{ARE(S,T)} = \left[\frac{eff(S)}{eff(T)}\right]^2 .$$

$$= \frac{K_S^2}{K_T^2}$$

For asymptotic relative efficiency comparisons, we have considered three parametric families of distributions, namely, Weibull, Linear Failure Rate and Makeham distributions. These depend upon a real parameter θ in such a way that $\theta=\theta_0$ yields a distribution belonging to null hypothesis whereas $\theta>\theta_0$ yields distribution from the alternative.

These are

(i) Weibull Distribution

$$\overline{F}_\theta(x)=\exp\left(-x^\theta\right), \qquad x>0,\ \theta\geq 1, \theta_0=1$$

(ii) Linear Failure Rate Distribution

$$\overline{F}_\theta(x)=\exp\left(-\left(x+\theta\frac{x^2}{2}\right)\right), \qquad x>0,\ \theta\geq 0, \theta_0=0$$

(iii) Makeham Distribution

$$\overline{F}_\theta(x)=\exp\left(-x+\theta\left(x+e^{-x}-1\right)\right), \quad x>0,\ \theta\geq 0, \theta_0=0$$

First we consider Weibull Distribution given by

$F(x)=1-e^{-x^\theta}$, $\overline{F}(x)=e^{-x^\theta}$ and $f(x)=\theta x^{\theta-1}e^{-x^\theta}$ $x>0$.

For the case k=3,

$$E(U)=P[Med(X_1,X_2,X_3)>bX_4]$$

$$=\int_0^\infty P[Med(X_1,X_2,X_3)>bx]dF(x)$$

$$=\int_0^\infty\left\{\int_{bx}^\infty \frac{3!}{1!1!}F^{2-1}(y)\overline{F}^{2-1}(y)f(y)dy\right\}dF(x)$$

$$=\int_0^\infty\left\{\int_{bx}^\infty 6\left[1-e^{-y^\theta}\right]e^{-y^\theta}\theta y^{\theta-1}e^{-y^\theta}dy\right\}dF(x)$$

Letting $y^{\theta} = t$, we have

$$E(U) = \int_0^{\infty} \left\{ \int_{(bx)^{\theta}}^{\infty} 6\left[1 - e^{-t}\right] e^{-2t} dt \right\} dF(x)$$

$$= \int_0^{\infty} \left\{ \int_{(bx)^{\theta}}^{\infty} 6e^{-2t} dt - \int_{(bx)^{\theta}}^{\infty} 6e^{-3t} dt \right\} dF(x)$$

$$= \int_0^{\infty} \left[3e^{-2(bx)^{\theta}} - 2e^{-3(bx)^{\theta}} \right] \theta x^{\theta-1} e^{-x^{\theta}} dx$$

putting $x^{\theta} = y$ we get

$$E(U) = \int_0^{\infty} \left[3e^{-2(b^{\theta})y} - 2e^{-3(b^{\theta})y} \right] e^{-y} dy$$

$$= \int_0^{\infty} \left[3e^{-y(2b^{\theta}+1)} \right] dy - \int_0^{\infty} \left[2e^{-y(3b^{\theta}+1)} \right] dy$$

$$= \frac{3}{(2b^{\theta}+1)} - \frac{2}{(3b^{\theta}+1)}$$

Now differentiating with respect to 6 we get

$$\frac{d}{d\theta} E(U) = \frac{-3}{(2b^{\theta}+1)^2} 2b^{\theta} \log b + \frac{2}{(3b^{\theta}+1)^2} 3b^{\theta} \log b$$

$$\frac{d}{d6} E(U)\Big|_{\theta=1} = \frac{-6b \log b}{(2b+1)^2} + \frac{6b \log b}{(3b+1)^2}$$

$$= 6b \log b \left\{ \frac{1}{(3b+1)^2} - \frac{1}{(2b+1)^2} \right\}$$

$$= 6b \log b \left\{ \frac{-5b^2 - 2b}{((3b+1)^2((2b+1)^2))} \right\}$$

Now we find

$$\text{Eff}=\frac{\frac{d}{d\theta}E(U)at\theta=1}{\sqrt{Var_{H_0}(U)}}$$

Squaring we get

$$(\text{Eff})^2 = \frac{\left(\frac{d}{d\theta}E(U)at\theta=1\right)^2}{B^*}$$

The following table gives the efficacy values for the case k=3

b	B^*	$\left(\frac{d}{d\theta}E(U)at\theta=1\right)^2$	$\frac{\left(\frac{d}{d\theta}E(U)at\theta=1\right)^2}{B^*}$
0.1	0.0448	0.0201	0.449
0.2	0.199	0.053	0.266
0.3	0.421	0.061	0.145
0.4	0.678	0.0503	0.0742
0.5	0.953	0.035	0.0367
0.6	1.233	0.021	0.0170
0.7	1.511	0.0109	0.0072
0.8	1.781	0.0043	0.0024
0.9	2.039	0.00096	0.00047

Now consider the case k=5.

$$E(U)= P[Med(X_1,X_2,...,X_5)> bX_6]$$

$$= \int_0^\infty P[Med(X_1,X_2,...,X_5)> bx]dF(x)$$

$$= \int_0^\infty \left\{ \int_{bx}^\infty \frac{5!}{2!2!} F^2(y)\overline{F}^2(y)f(y)dy \right\} dF(x)$$

$$= \int_0^{\infty} \left\{ \int_{bx}^{\infty} 30\left[1 - e^{-y^{\theta}}\right]^2 \left(e^{-y^{\theta}}\right)^2 \theta y^{\theta-1} e^{-y^{\theta}} dy \right\} dF(x)$$

Letting $y^{\theta} = t$ we get

$$E(U) = \int_0^{\infty} \left\{ \int_{(bx)^{\theta}}^{\infty} 30\left[1 - e^{-t}\right]^2 e^{-2t} e^{-t} dt \right\} dF(x)$$

$$= \int_0^{\infty} \left\{ \int_{(bx)^{\theta}}^{\infty} 30\left[1 - 2e^{-t} + e^{-2t}\right] e^{-3t} dt \right\} dF(x)$$

$$= \int_0^{\infty} \left\{ \int_{(bx)^{\theta}}^{\infty} 30e^{-3t} dt - \int_{(bx)^{\theta}}^{\infty} 60e^{-4t} dt + \int_{(bx)^{\theta}}^{\infty} 30e^{-5t} dt \right\} dF(x)$$

Letting $e^{-t} = y$ we get

$$= \int_0^{\infty} \left[10e^{-3(bx)^{\theta}} - 15e^{-4(bx)^{\theta}} + 6e^{-5(bx)^{\theta}}\right] \theta x^{\theta-1} e^{-x^{\theta}} dx$$

$$= \int_0^{\infty} \left(10e^{-3(bx)^{\theta}} \theta x^{\theta-1} e^{-x^{\theta}} dx\right) - \int_0^{\infty} \left(15e^{-4(bx)^{\theta}} \theta x^{\theta-1} e^{-x^{\theta}} dx\right) + \int_0^{\infty} \left(6e^{-5(bx)^{\theta}} \theta x^{\theta-1} e^{-x^{\theta}} dx\right)$$

Now differentiating the above with respect to θ and putting $\theta = 1$, we have

$$\frac{d}{d\theta} E(U) at\ \theta = 1 = \frac{-60 b^3 \ln(b)\left(47 b^2 + 24 b + 3\right)}{\left(9b^2 + 6b + 1\right)\left(16 b^2 + 8b + 1\right)\left(25 b^2 + 10 b + 1\right)}$$

Now squaring we get

$$\left[\frac{d}{d\theta} E(U) at\ \theta = 1\right]^2 = \frac{3600 b^2 (\ln(b))^2 \left(47 b^2 + 24 b + 3\right)^2}{\left(9b^2 + 6b + 1\right)^2 \left(16 b^2 + 8b + 1\right)^2 \left(25 b^2 + 10 b + 1\right)^2}$$

Thus we have

$$\text{Eff} = \frac{\frac{d}{d\theta} E(U) at\ \theta = 1}{\sqrt{Var_{H_0}(U)}}$$

Here

$$\text{Eff} = \frac{\frac{d}{d\theta} E(U) at \theta = 1}{\sqrt{B^{**}}}$$

Squaring we get

$$(\text{Eff})^2 = \frac{\left(\frac{d}{d\theta} E(U) at \theta = 1\right)^2}{B^{**}}$$

The values of square of the efficacies are tabulated below.

b	B^{**}	$\left(\frac{d}{d\theta} E(U) at \theta = 1\right)^2$	$\frac{\left(\frac{d}{d\theta} E(U) at \theta = 1\right)^2}{B^{**}}$
0.1	0.00817	0.0118	1.444
0.2	0.03865	0.051	1.319
0.3	0.05606	0.066	1.177
0.4	0.5293	0.058	1.096
0.5	0.03966	0.041	1.034
0.6	0.02503	0.025	0.999
0.7	0.01321	0.013	0.984
0.8	0.00535	0.0049	0.916
0.9	0.001200	0.0011	0.917

For the case k=7, we have

$$E(U) = P[Med(X_1, X_2, ..., X_7) > bX_8]$$

$$= \int_0^\infty P[Med(X_1, X_2, ..., X_7) > bx] dF(x)$$

$$= \int_0^\infty \left\{ \int_{bx}^\infty \frac{7!}{3!3!} F^3(y)\overline{F}^3(y) f(y) dy \right\} dF(x)$$

$$= \int_0^\infty \left\{ \int_{bx}^\infty 140\left[1-e^{-y^\theta}\right]^3 \left(e^{-y^\theta}\right)^3 \theta y^{\theta-1} e^{-y^\theta} dy \right\} dF(x)$$

Letting $y^\theta = t$ we have

$$E(U) = \int_0^\infty \left\{ \int_{(bx)^\theta}^\infty 140\left[1-e^{-t}\right]^3 e^{-3t} e^{-t} dt \right\} dF(x)$$

$$= \int_0^\infty \left\{ \int_{(bx)^\theta}^\infty 140\left[1-3e^{-t}+3e^{-2t}-e^{-3t}\right] e^{-3t} e^{-t} dt \right\} dF(x)$$

$$= \int_0^\infty \left\{ \int_{(bx)^\theta}^\infty 140e^{-3t}e^{-t}dt - \int_{(bx)^\theta}^\infty 140e^{-4t}3e^{-t}dt + \int_{(bx)^\theta}^\infty 140e^{-t}3e^{-5t}dt - \int_{(bx)^\theta}^\infty 140e^{-6t}e^{-t}dt \right\} dF(x)$$

Letting e^{-t} =y, we have,

$$E(U) = \int_0^\infty \left(35e^{-4(bx)^\theta}\theta x^{\theta-1}e^{-x^\theta}dx\right) - \int_0^\infty \left(84e^{-5(bx)^\theta}\theta x^{\theta-1}e^{-x^\theta}dx\right) + \int_0^\infty \left(70e^{-6(bx)^\theta}\theta x^{\theta-1}e^{-x^\theta}dx\right) - \int_0^\infty \left(20e^{-7(bx)^\theta}\theta x^{\theta-1}e^{-x^\theta}dx\right)$$

Now differentiating the above with respect to θ and putting $\theta = 1$ and simplifying, we have

$$\frac{d}{d\theta}E(U)at\,\theta = 1 = -1680b^4\ln(b)^2 \frac{\left(2+319b^3+179b^2+33b\right)}{\left[\left(8b+16b^2+1\right)\left(25b^2+10b+1\right)\left(36b^2+12b+1\right)\left(49b^2+14b+1\right)\right]}$$

Now squaring we get

$$\left[\frac{d}{d\theta}E(U)at\,\theta=1\right]^2 = 2822400b^8\ln(b)^2 \frac{\left(2+319b^3+179b^2+33b\right)^2}{\left[\left(8b+16b^2+1\right)^2\left(25b^2+10b+1\right)^2\left(36b^2+12b+1\right)^2\left(49b^2+14b+1\right)^2\right]}$$

Thus

$$\text{Eff} = \frac{\frac{d}{d\theta}E(U)at\,\theta=1}{\sqrt{Var_{H_0}(U)}}$$

Here

$$\text{Eff} = \frac{\frac{d}{d\theta}E(U)at\theta=1}{\sqrt{B^{***}}}$$

Squaring we get

$$(\text{Eff})^2 = \frac{\left(\frac{d}{d\theta}E(U)at\theta=1\right)^2}{B^{***}}$$

Thus the calculated values of square of the efficacies are as follows:

b	B^{***}	$\left(\frac{d}{d\theta}E(U)at\theta=1\right)^2$	$\frac{\left(\frac{d}{d\theta}E(U)at\theta=1\right)^2}{B^{***}}$
0.1	0.005308	0.007716	1.4537
0.2	0.03771	0.04808	1.2749
0.3	0.06209	0.06931	1.1163
0.4	0.06186	0.062168	1.0049
0.5	0.04755	0.04415	0.9285
0.6	0.03037	0.02658	0.8752
0.7	0.016108	0.01349	0.8375
0.8	0.006535	0.00529	0.8095
0.9	0.001463	0.001157	0.7908

Next we consider the Linear Failure Rate Distribution given by

$F(x)=1-e^{-x-\theta\frac{x^2}{2}}$, $\overline{F}(x)=e^{-x-\theta\frac{x^2}{2}}$ and $f(x)=(1+\theta x)e^{-x-\theta\frac{x^2}{2}}$ $x>0$.

First we consider the case k=3.

$$E(U)=P[Med(X_1,X_2,X_3)>bX_4]$$

$$=\int_0^\infty P[Med(X_1,X_2,X_3)>bx]dF(x)$$

$$=\int_0^\infty\left\{\int_{bx}^\infty \frac{3!}{1!1!}F^3(y)\overline{F}^3(y)f(y)dy\right\}dF(x)$$

$$=\int_0^\infty\left\{\int_{bx}^\infty 6\left[1-e^{-y-\theta\frac{y^2}{2}}\right]\left(e^{-y-\theta\frac{y^2}{2}}\right)(1+\theta y)e^{-y-\theta\frac{y^2}{2}}dy\right\}dF(x)$$

Letting $y+\theta\frac{y^2}{2}=t$, we have

$$E(U)=\int_0^\infty\left\{\int_{\left(bx+\theta\frac{(bx)^2}{2}\right)}^\infty 6\left[1-e^{-t}\right]e^{-2t}dt\right\}dF(x)$$

$$=\int_0^\infty\left\{\int_{\left(bx+\theta\frac{(bx)^2}{2}\right)}^\infty 6e^{-t}e^{-t}dt-\int_{\left(bx+\theta\frac{(bx)^2}{2}\right)}^\infty 6e^{-2t}e^{-t}dt\right\}dF(x)$$

Letting e^{-t}=y we get

$$=\int_0^\infty\left[3e^{-bx(2+\theta bx)}-2e^{-3bx\left(1+\theta\frac{bx}{2}\right)}\right]\theta x^{\theta-1}e^{-x^\theta}dx$$

$$=\int_0^\infty\left[3e^{-bx(2+\theta bx)}\right](1+\theta x)e^{-x-\theta\frac{x^2}{2}}dx-\int_0^\infty\left[2e^{-3bx\left(1+\frac{\theta bx}{2}\right)}\right](1+\theta x)e^{-x-\theta\frac{x^2}{2}}dx\ .$$

Now by differentiating the above with respect to θ and putting $\theta=0$, we have

$$\frac{d}{d\theta}E(U)at\theta=0=\frac{-6b^2(b-1)(3+15b+19b^2)}{(2b+1)^3(3b+1)^3}$$

Squaring it we have

$$\left[\frac{d}{d\theta}E(U)at\theta=0\right]^2=\frac{36(b-1)^2b^4(3+15b+19b^2)^2}{(3b+1)^6(2b+1)^6}$$

Now we find

$$\text{Eff}=\frac{\frac{d}{d\theta}E(U)at\theta=0}{\sqrt{Var_{H_0}(U)}}$$

Here

$$\text{Eff}=\frac{\frac{d}{d\theta}E(U)at\theta=0}{\sqrt{B^*}}$$

Squaring we get

$$(\text{Eff})^2=\frac{\left(\frac{d}{d\theta}E(U)at\theta=0\right)^2}{B^*}$$

The calculated values of efficacies for various values of b are given below:

b	B^*	$\left(\frac{d}{d\theta}E(U)at\theta=0\right)^2$	$\frac{\left(\frac{d}{d\theta}E(U)at\theta=0\right)^2}{B^*}$
0.1	0.0448	0.0045	0.1004
0.2	0.199	0.013	0.0653
0.3	0.421	0.015	0.0356
0.4	0.678	0.0125	0.0184
0.5	0.953	0.0084	0.00881

0.6	1.233	0.0048	0.00389
0.7	1.511	0.0024	0.00159
0.8	1.781	0.0009	0.00051
0.9	2.039	0.0002	0.000098

Next for the case k=5,we have

$$E(U)= P[Med(X_1, X_2,..., X_5)> bX_6]$$

$$= \int_0^\infty P[Med(X_1, X_2,..., X_5)> bx]dF(x)$$

$$= \int_0^\infty \left\{ \int_{bx}^\infty \frac{5!}{2!2!} F^2(y)\overline{F}^2(y)f(y)dy \right\} dF(x)$$

$$= \int_0^\infty \left\{ \int_{bx}^\infty 30\left[1-e^{-y-\theta\frac{y^2}{2}}\right]^2 \left(e^{-y-\theta\frac{y^2}{2}}\right)^2 (1+\theta y)e^{-y-\theta\frac{y^2}{2}} dy \right\} dF(x)$$

Letting $y+\theta\frac{y^2}{2}=t,$ we have

$$\text{E(U)}= \int_0^\infty \left\{ \int_{\left(bx+\theta\frac{(bx)^2}{2}\right)}^\infty 30\left[1-e^{-t}\right]^2 e^{-2t}e^{-t}dt \right\} dF(x)$$

$$= \int_0^\infty \left\{ \int_{\left(bx+\theta\frac{(bx)^2}{2}\right)}^\infty 30e^{-3t}dt - \int_{\left(bx+\theta\frac{(bx)^2}{2}\right)}^\infty 60e^{-4t}dt + \int_{\left(bx+\theta\frac{(bx)^2}{2}\right)}^\infty 30e^{-5t}dt \right\} dF(x).$$

Letting e^{-t}=y, we have

$$\text{E(U)} = \int_0^\infty \left[10e^{-3bx\left(1+\theta\frac{bx}{2}\right)} - 15e^{-4bx\left(1+\theta\frac{bx}{2}\right)} + 6e^{-5bx\left(1+\theta\frac{bx}{2}\right)} \right] \theta x^{\theta-1}e^{-x^\theta} dx$$

$$= \int_0^{\infty}\left[10e^{-3bx\left(1+\theta\frac{bx}{2}\right)}\right](1+\theta x)e^{-x-\theta\frac{x^2}{2}}dx - \int_0^{\infty}\left[15e^{-4bx\left(1+\frac{\theta bx}{2}\right)}\right](1+\theta x)e^{-x-\theta\frac{x^2}{2}}dx$$

$$+ \int_0^{\infty}\left[6e^{-5bx\left(1+\theta\frac{bx}{2}\right)}\right](1+\theta x)e^{-x-\theta\frac{x^2}{2}}dx$$

Now by differentiating the above with respect to θ and putting $\theta = 0$ and simplifying, we have

$$\frac{d}{d\theta}E(U)at\theta = 0 = \frac{60b^3\left(3626b^4 + 2811b^3 + 953b^2 + 150b + 9 + 1331b^5\right)}{(3b+1)^3(4b+1)^3(5b+1)^3}$$

Squaring the above we have

$$\left[\frac{d}{d\theta}E(U)at\theta = 0\right]^2 = \frac{3600b^6\left(3626b^4 + 2811b^3 + 953b^2 + 150b + 9 + 1331b^5\right)^2}{(3b+1)^6(4b+1)^6(5b+1)^6}$$

Further

$$\text{Eff} = \frac{\frac{d}{d\theta}E(U)at\theta = 0}{\sqrt{Var_{H_0}(U)}}$$

Thus

$$\text{Eff} = \frac{\frac{d}{d\theta}E(U)at\theta = 0}{\sqrt{B^{**}}}$$

Squaring we get

$$(\text{Eff})^2 = \frac{\left(\frac{d}{d\theta}E(U)at\theta = 0\right)^2}{B^{**}}$$

For different values of b,we tabulate below the square of efficacies

b	B^{**}	$\left(\frac{d}{d\theta}E(U)at\theta=0\right)^2$	$\frac{\left(\frac{d}{d\theta}E(U)at\theta=0\right)^2}{B^{**}}$
0.1	0.00817	0.0037	0.4529
0.2	0.03865	0.016	0.4139
0.3	0.05606	0.019	0.3389
0.4	0.05293	0.015	0.2834
0.5	0.03966	0.0099	0.2496
0.6	0.02503	0.005	0.1998
0.7	0.01321	0.0026	0.1968
0.8	0.00535	0.00096	0.1794
0.9	0.001200	0.00019	0.1583

Next for the case k=7, we have

$$E(U)= P[Med(X_1, X_2,..., X_7)> bX_8].$$

$$= \int_0^\infty P[Med(X_1, X_2,..., X_7)> bx]dF(x)$$

$$= \int_0^\infty \left\{ \int_{bx}^\infty \frac{7!}{3!3!} F^3(y)\overline{F}^3(y)f(y)dy \right\} dF(x)$$

$$= \int_0^\infty \left\{ \int_{bx}^\infty 140\left[1-e^{-y-\theta\frac{y^2}{2}}\right]^3\left(e^{-y-\theta\frac{y^2}{2}}\right)^3(1+\theta y)e^{-y-\theta\frac{y^2}{2}}dy \right\} dF(x).$$

Letting $y+\theta\frac{y^2}{2}=t$

$$E(U)=\int_0^{\infty}\left\{\int_{\left(bx+\theta\frac{(bx)^2}{2}\right)}^{\infty}140\left[1-e^{-t}\right]^3e^{-3t}e^{-t}dt\right\}dF(x)$$

$$=\int_0^{\infty}\left\{\int_{\left(bx+\theta\frac{(bx)^2}{2}\right)}^{\infty}140e^{-3t}e^{-t}dt-\int_{\left(bx+\theta\frac{(bx)^2}{2}\right)}^{\infty}140e^{-4t}3e^{-t}dt+\int_{\left(bx+\theta\frac{(bx)^2}{2}\right)}^{\infty}140e^{-5t}3e^{-t}dt-\int_{\left(bx+\theta\frac{(bx)^2}{2}\right)}^{\infty}140e^{-6t}e^{-t}dt\right\}dF(x)$$

Letting $e^{-t}=y$ we obtain

$$E(U)=\int_0^{\infty}\left[35e^{-4bx\left(1+\theta\frac{bx}{2}\right)}-84e^{-5bx\left(1+\theta\frac{bx}{2}\right)}+70e^{-6bx\left(1+\theta\frac{bx}{2}\right)}-20e^{-7bx\left(1+\theta\frac{bx}{2}\right)}\right]\theta x^{\theta-1}e^{-x^{\theta}}dx$$

$$=\int_0^{\infty}\left[35e^{-4bx\left(1+\theta\frac{bx}{2}\right)}\right](1+\theta x)e^{-x-\theta\frac{x^2}{2}}dx-\int_0^{\infty}\left[84e^{-5bx\left(1+\frac{\theta bx}{2}\right)}\right](1+\theta x)e^{-x-\theta\frac{x^2}{2}}dx$$

$$+\int_0^{\infty}\left[70e^{-6bx\left(1+\theta\frac{bx}{2}\right)}\right](1+\theta x)e^{-x-\theta\frac{x^2}{2}}dx--\int_0^{\infty}\left[20e^{-7bx\left(1+\theta\frac{bx}{2}\right)}\right](1+\theta x)e^{-x-\theta\frac{x^2}{2}}dx$$

Now by differentiating the above with respect to 6 and putting $6=0$ we have

$$\frac{d}{d6}E(U)|_{\theta=0}=-840b^4(b-1)\frac{\left(133191b^4+256684b^6+287166b^5+4515b^2+32780b^3+330b+10\right)}{\left[(4b+1)^3(5b+1)^3(6b+1)^3(7b+1)^3\right]}$$

Squaring we get

$$\left[\frac{d}{d\theta}E(U)|_{\theta=0}\right]^2=705600b^8(b-1)^2\frac{\left(133191b^4+256684b^6+287166b^5+4515b^2+32780b^3+330b+10\right)^2}{\left[(4b+1)^6(5b+1)^6(6b+1)^6(7b+1)^6\right]}$$

Now we find

$$\text{Eff}=\frac{\frac{d}{d\theta}E(U)at\theta=0}{\sqrt{Var_{H_0}(U)}}$$

Thus

$$\text{Eff} = \frac{\frac{d}{d\theta}E(U)at\theta = 0}{\sqrt{\mathbf{B}^{***}}}$$

Squaring we get

$$(\text{Eff})^2 = \frac{\left(\frac{d}{d\theta}E(U)at\theta = 0\right)^2}{B^{***}}$$

Thus we have $(\text{Eff})^2$ for different values of b

b	B^{***}	$\left(\frac{d}{d\theta}E(U)at\theta = 0\right)^2$	$\frac{\left(\frac{d}{d\theta}E(U)at\theta = 0\right)^2}{B^{***}}$
0.1	0.005308	0.003106	0.5852
0.2	0.03771	0.01731	0.459
0.3	0.06209	0.02169	0.349
0.4	0.06186	0.01707	0.2759
0.5	0.04755	0.01079	0.2269
0.6	0.03037	0.00585	0.1926
0.7	0.016108	0.00271	0.168
0.8	0.006535	0.0009	0.1377
0.9	0.001463	0.000198	0.1353

Consider now Makeham Distribution given by

$F(x) = 1 - e^{-x-\theta\left(x+e^{-x}-1\right)}$, $\overline{F}(x) = e^{-x-\theta\left(x+e^{-x}-1\right)}$ and $f(x) = \left[1+\theta\left(1-e^{-x}\right)\right]e^{-x-\theta\left(x+e^{-x}-1\right)}$, $x > 0$

First for the case k=3,we have

$$E(U) = P[Med(X_1, X_2, X_3) > bX_4]$$

$$= \int_0^\infty P[Med(X_1, X_2, X_3) > bx]dF(x)$$

$$= \int_0^\infty \left\{ \int_{bx}^\infty \frac{3!}{1!1!} F^{2-1}(y)\bar{F}^{2-1}(y) f(y) dy \right\} dF(x)$$

$$= \int_0^\infty \left\{ \int_{bx}^\infty 6\left[1 - e^{-y-\theta\left(y+e^{-y}-1\right)}\right] e^{-y-\theta\left(y+e^{-y}-1\right)} \left[1+\theta\left(1-e^{-y}\right)\right] \right\} e^{-y-\theta\left(y+e^{-y}-1\right)} dF(x)$$

Letting $e^{-y-\theta\left(y+e^{-y}-1\right)} = t$, we have

$$\text{E(U)} = \int_0^\infty \left\{ \int_0^{e^{-bx-\theta\left(bx+e^{-bx}-1\right)}} 6t(1-t)dt \right\} dF(x)$$

$$= \int_0^\infty \left\{ \int_0^{e^{-bx-\theta\left(bx+e^{-bx}-1\right)}} 6t dt - \int_0^{e^{-bx-\theta\left(bx+e^{-bx}-1\right)}} 6t^2 dt \right\} dF(x)$$

$$= \int_0^\infty \left\{ 3e^{-2bx-2\theta\left(bx+e^{-bx}-1\right)} - 2e^{-3bx-3\theta\left(bx+e^{-bx}-1\right)} \right\} f(x) dx$$

$$= \int_0^\infty \left\{ 3e^{-2bx-2\theta\left(e^{-bx}-1\right)} \right\} \left\{ 1+\theta\left(1-e^{-x}\right) \right\} e^{-x-\theta\left(x+e^{-x}-1\right)} dx - \int_0^\infty \left\{ 2e^{-3bx-3\theta\left(e^{-bx}-1\right)} \right\} \left\{ 1+\theta\left(1-e^{-x}\right) \right\} e^{-x-\theta\left(x+e^{-x}-1\right)} dx$$

Now by differentiating the above with respect to θ and putting $\theta = 0$

$$\frac{d}{d\theta} E(U) at \theta = 0 = \frac{-3b^2(b-1)(8b+5)}{(3b+2)(3b+1)(4b+1)(b+1)(2b+1)}$$

Squaring we get

$$\left[\frac{d}{d\theta} E(U) at \theta = 0\right]^2 = \frac{9b^4(b-1)^2(8b+5)^2}{(3b+2)^2(3b+1)^2(4b+1)^2(b+1)^2(2b+1)^2}$$

Further

$$\text{Eff} = \frac{\frac{d}{d\theta} E(U) at \theta = 0}{\sqrt{Var_{H_0}(U)}}$$

Here

$$\text{Eff} = \frac{\frac{d}{d\theta}E(U)at\theta = 0}{\sqrt{B^*}}$$

Squaring we get

$$(\text{Eff})^2 = \frac{\left(\frac{d}{d\theta}E(U)at\theta = 0\right)^2}{B^*}$$

Thus we calculate $(\text{Eff})^2$ for different values of b

b	B^*	$\left(\frac{d}{d\theta}E(U)at\theta = 1\right)^2$	$\frac{\left(\frac{d}{d\theta}E(U)at\theta = 1\right)^2}{B^*}$
0.1	0.0448	0.0008	0.0179
0.2	0.199	0.0025	0.0126
0.3	0.421	0.0031	0.00736
0.4	0.678	0.0026	0.00383
0.5	0.953	0.0018	0.00189
0.6	1.233	0.0011	0.00089
0.7	1.511	0.00056	0.000371
0.8	1.781	0.00022	0.000124
0.9	2.039	0.000049	0.000024

Next for the case k=5,we have

$$E(U) = P[Med(X_1, X_2, ..., X_5) > bX_6]$$

$$= \int_0^\infty P[Med(X_1, X_2, ..., X_5) > bx]dF(x)$$

$$= \int_0^{\infty} \left\{ \int_{bx}^{\infty} \frac{5!}{2!2!} F^2(y)\overline{F}^2(y) f(y) dy \right\} dF(x)$$

$$= \int_0^{\infty} \left\{ \int_{bx}^{\infty} 30\left[1 - e^{-y-\theta\left(y+e^{-y}-1\right)}\right]^2 \left(e^{-y-\theta\left(y+e^{-y}-1\right)}\right)^2 \left[1+\theta\left(1-e^{-y}\right)\right] \right\} e^{-y-\theta\left(y+e^{-y}-1\right)} dF(x)$$

Letting $e^{-y-\theta\left(y+e^{-y}-1\right)} = t$, we have

$$= \int_0^{\infty} \left\{ \int_0^{e^{-bx-\theta\left(bx+e^{-bx}-1\right)}} 30(1-t)^2 t^2 dt \right\} dF(x)$$

$$= \int_0^{\infty} \left\{ \int_0^{e^{-bx-\theta\left(bx+e^{-bx}-1\right)}} 30t^2 dt - \int_0^{e^{-bx-\theta\left(bx+e^{-bx}-1\right)}} 60t^3 dt + \int_0^{e^{-bx-\theta\left(bx+e^{-bx}-1\right)}} 30t^4 dt \right\} dF(x)$$

$$= \int_0^{\infty} \left\{ 10e^{-3bx-3\theta\left(bx+e^{-bx}-1\right)} - 15e^{-4bx-4\theta\left(bx+e^{-bx}-1\right)} + 6e^{-5bx-5\theta\left(bx+e^{-bx}-1\right)} \right\} f(x) dx$$

$$= \int_0^{\infty} \left\{ 10e^{-3bx-3\theta\left(bx+e^{-bx}-1\right)} \right\} \left(1+\theta\left(1-e^{-x}\right)\right) e^{-x-\theta\left(x+e^{-x}-1\right)} dx - \int_0^{\infty} \left\{ 15e^{-4bx-4\theta\left(bx+e^{-bx}-1\right)} \right\} \left(1+\theta\left(1-e^{-x}\right)\right) e^{-x-\theta\left(x+e^{-x}-1\right)} dx$$
$$+ \int_0^{\infty} \left\{ 6e^{-5bx-5\theta\left(bx+e^{-bx}-1\right)} \right\} \left(1+\theta\left(1-e^{-x}\right)\right) e^{-x-\theta\left(x+e^{-x}-1\right)} dx$$

Now by differentiating the above with respect to θ and putting $\theta = 0$ we have

$$\frac{d}{d\theta} E(U) at \theta = 0 = \frac{-30b^3(b-1)\left(102b^2+83b+17\right)}{(3b+2)(3b+1)(4b+1)(5b+1)(2b+1)(5b+2)(6b+1)}$$

Squaring the expression, we have

$$\left[\frac{d}{d\theta} E(U) at \theta = 0\right]^2 = \frac{900b^6(b-1)^2\left(102b^2+83b+17\right)^2}{(3b+2)^2(3b+1)^2(4b+1)^2(5b+1)^2(2b+1)^2(5b+2)^2(6b+1)^2}$$

Further we find

$$\text{Eff} = \frac{\frac{d}{d\theta}E(U)at\theta = 0}{\sqrt{Var_{H_0}(U)}}$$

Thus

$$\text{Eff} = \frac{\frac{d}{d\theta}E(U)at\theta = 0}{\sqrt{\mathbf{B}^{**}}}$$

Squaring we get

$$(\text{Eff})^2 = \frac{\left(\frac{d}{d\theta}E(U)at\theta = 0\right)^2}{B^{**}}$$

Thus we calculate $(\text{Eff})^2$ for different values of b

b	B^{**}	$\left(\frac{d}{d\theta}E(U)at\theta = 1\right)^2$	$\frac{\left(\frac{d}{d\theta}E(U)at\theta = 1\right)^2}{B^{**}}$
0.1	0.00817	0.00056	0.0685
0.2	0.03865	0.0027	0.0699
0.3	0.05606	0.0037	0.0660
0.4	0.05293	0.0032	0.06046
0.5	0.03966	0.0023	0.0579
0.6	0.02503	0.0013	0.0519
0.7	0.01321	0.00067	0.0507
0.8	0.00535	0.00026	0.0486
0.9	0.001200	0.000055	0.0458

Next for the case k=7,we consider

$$E(U) = P[Med(X_1, X_2, ..., X_7) > bX_8]$$

$$= \int_0^\infty P[Med(X_1, X_2, ..., X_7) > bx] dF(x)$$

$$= \int_0^\infty \left\{ \int_{bx}^\infty \frac{7!}{3!3!} F^3(y) \overline{F}^3(y) f(y) dy \right\} dF(x)$$

$$= \int_0^\infty \left\{ \int_{bx}^\infty 140\left[1 - e^{-y-\theta\left(y+e^{-y}-1\right)}\right]^3 \left(e^{-y-\theta\left(y+e^{-y}-1\right)}\right)^3 \left[1+\theta\left(1-e^{-y}\right)\right] \right\} e^{-y-\theta\left(y+e^{-y}-1\right)} dF(x)$$

Letting $e^{-y-\theta\left(y+e^{-y}-1\right)} = t$, we have

$$\text{E(U)} = \int_0^\infty \left\{ \int_0^{e^{-bx-\theta\left(bx+e^{-bx}-1\right)}} 140 t^3 (1-t)^3 dt \right\} dF(x)$$

$$= \int_0^\infty \left\{ \int_0^{e^{-bx-\theta\left(bx+e^{-bx}-1\right)}} 140 t^3 dt - \int_0^{e^{-bx-\theta\left(bx+e^{-bx}-1\right)}} 140 t^4 3 dt + \int_0^{e^{-bx-\theta\left(bx+e^{-bx}-1\right)}} 140 t^5 3 dt - \int_0^{e^{-bx-\theta\left(bx+e^{-bx}-1\right)}} 140 t^6 dt \right\} dF(x)$$

$$= \int_0^\infty \left\{ 35 e^{-4bx-4\theta\left(bx+e^{-bx}-1\right)} - 84 e^{-5bx-5\theta\left(bx+e^{-bx}-1\right)} + 70 e^{-6bx-6\theta\left(bx+e^{-bx}-1\right)} - 20 e^{-7bx-7\theta\left(bx+e^{-bx}-1\right)} \right\} f(x) dx$$

$$= \int_0^\infty \left\{ 35 e^{-4bx-4\theta\left(bx+e^{-bx}-1\right)} \right\} \left\{ 1+\theta\left(1-e^{-x}\right) \right\} e^{-x-\theta\left(x+e^{-x}-1\right)} dx - \int_0^\infty \left\{ 84 e^{-5bx-5\theta\left(bx+e^{-bx}-1\right)} \right\} \left\{ 1+\theta\left(1-e^{-x}\right) \right\} e^{-x-\theta\left(x+e^{-x}-1\right)} dx$$
$$+ \int_0^\infty \left\{ 70 e^{-6bx-6\theta\left(bx+e^{-bx}-1\right)} \right\} \left\{ 1+\theta\left(1-e^{-x}\right) \right\} e^{-x-\theta\left(x+e^{-x}-1\right)} dx - \int_0^\infty \left\{ 20 e^{-7bx-7\theta\left(bx+e^{-bx}-1\right)} \right\} \left\{ 1+\theta\left(1-e^{-x}\right) \right\} e^{-x-\theta\left(x+e^{-x}-1\right)} dx$$

Now by differentiating the above with respect to θ and putting $\theta = 0$ and simplying we have

$$\frac{d}{d\theta} E(U)|_{\theta=0} = -210(b-1)b^4 \frac{\left(1744b^3 + 1574b^2 + 479b + 49\right)}{\left[(4b+1)(5b+1)(2b+1)(6b+1)(5b+2)(7b+1)(3b+1)(8b+1)(7b+2)\right]}$$

Squaring we get

$$\frac{d}{d\theta} E(U)|_{\theta=0} = 44100(b-1)^2 b^8 \frac{\left(1744b^3 + 1574b^2 + 479b + 49\right)^2}{\left[(4b+1)^2(5b+1)^2(2b+1)^2(6b+1)^2(5b+2)^2(7b+1)^2(3b+1)^2(8b+1)^2(7b+2)^2\right]}$$

Thus

$$\text{Eff} = \frac{\frac{d}{d\theta}E(U)at\theta = 0}{\sqrt{Var_{H_0}(U)}}$$

Thus

$$\text{Eff} = \frac{\frac{d}{d\theta}E(U)at\theta = 0}{\sqrt{\mathbf{B}^{***}}}$$

Squaring we get

$$(\text{Eff})^2 = \frac{\left(\frac{d}{d\theta}E(U)at\theta = 0\right)^2}{B^{***}}$$

Thus we calculate $(\text{Eff})^2$ for different values of b

b	B^{***}	$\left(\frac{d}{d\theta}E(U)at\theta = 1\right)^2$	$\frac{\left(\frac{d}{d\theta}E(U)at\theta = 1\right)^2}{B^{***}}$
0.1	0.005308	0.000399	0.07517
0.2	0.03771	0.00279	0.0739
0.3	0.06209	0.00411	0.06619
0.4	0.06186	0.00367	0.0593
0.5	0.04755	0.00255	0.0536
0.6	0.03037	0.00149	0.0491
0.7	0.016108	0.000736	0.0457
0.8	0.006535	0.00028	0.0428
0.9	0.001463	0.000059	0.0403

4.5 Comparison

Now we compare the Pitman asymptotic relative efficiency of our test with respect to the test statistic proposed by Bhadri(2005).

The null variance of the test proposed by Bhadri(2005) is given by

$$\sigma_0^2 = \frac{1}{2bk+1} + \frac{2k}{b(k-1)+1}\left(\frac{1}{bk+1} - \frac{b}{b^2k+bk+1}\right) - \frac{(k+1)^2}{(bk+1)^2} + \frac{k^2}{(b(k-1)+1)^2}\left(1 + \frac{b}{b(2k-1)+2} - \frac{2b}{bk+1}\right).$$

The null variance of the two tests and ARE are given for different values of k=3,5,7 and different values of b for the three distributions. and then obtain the Asymptotic Relative Efficiency. The same is shown in tables below

k=3 **TABLE NO.4.5.1**

b	Med Var	Median $(Eff)^2$			Min Var	Minima $(Eff)^2$			ARE(Med,Min)		
		W	LFR	M		W	LFR	M	W	LFR	M
0.1	0.0448	0.449	0.1004	0.0179	0.166	1.0051	0.0909	0.0251	0.447	1.105	0.713
0.2	0.199	0.266	0.0653	0.0126	0.135	1.056	0.101	0.0304	0.252	0.647	0.414
0.3	0.421	0.145	0.0356	0.00736	0.087	1.035	0.0970	0.0310	0.140	0.367	0.237
0.4	0.678	0.0742	0.0184	0.00383	0.051	1.012	0.0896	0.0304	0.0733	0.205	0.126
0.5	0.9532	0.0367	0.00881	0.00189	0.028	0.9892	0.0821	0.0291	0.0371	0.107	0.065
0.6	1.233	0.0170	0.00389	0.00089	0.015	0.920	0.072	0.0264	0.0185	0.054	0.034
0.7	1.511	0.0072	0.00159	0.000371	0.0066	0.9196	0.0677	0.0258	0.0078	0.0235	0.014
0.8	1.781	0.0024	0.00051	0.000124	0.0024	0.896	0.0621	0.0242	0.0027	0.0082	0.0051
0.9	2.039	0.00047	0.000098	0.000024	0.00049	0.8816	0.0571	0.0233	0.00053	0.0017	0.0010

Foot Note: Med Var: Variance for Median

Min Var: Variance for Minima

W: Weibull Distribution

LFR: Linear Failure Rate Distribution

M: Makeham Distribution

k=5 **TABLE NO.4.5.2**

b	Med Var	Median $(Eff)^2$			Min Var	Minima $(Eff)^2$			ARE(Med,Min)		
		W	LFR	M		W	LFR	M	W	LFR	M
0.1	0.00817	1.444	0.4529	0.0685	0.295	0.887	0.0603	0.0191	1.628	7.511	3.586
0.2	0.03865	1.319	0.4139	0.0699	0.185	0.875	0.0541	0.0198	1.507	7.651	3.530
0.3	0.05605	1.177	0.3389	0.0660	0.101	0.827	0.0448	0.0182	1.423	7.565	3.626
0.4	0.05293	1.096	0.2834	0.06046	0.053	0.782	0.0374	0.0163	1.402	7.578	3.709
0.5	0.03966	1.034	0.2496	0.0579	0.027	0.741	0.0314	0.0146	1.395	7.949	3.966
0.6	0.02503	0.999	0.1998	0.0519	0.013	0.7054	0.0271	0.0131	1.416	7.373	3.962
0.7	0.01321	0.984	0.1968	0.0507	0.0057	0.667	0.0233	0.0117	1.475	8.446	4.333
0.8	0.00535	0.916	0.1794	0.0486	0.0019	0.668	0.0215	0.0111	1.371	8.344	4.378
0.9	0.001200	0.917	0.1583	0.0458	0.00039	0.6307	0.0187	0.0099	1.454	8.465	4.626

Foot Note: Med Var: Variance for Median

Min Var: Variance for Minima

W: Weibull Distribution

LFR: Linear Failure Rate Distribution

M: Makeham Distribution

k=7 **TABLE NO.4.5.3**

b	Med Var	Median $(Eff)^2$			Min Var	Minima $(Eff)^2$			ARE(Med,Min)		
		W	LFR	M		W	LFR	M	W	LFR	M
0.1	0.005308	1.4537	0.5852	0.07517	0.393	0.791	0.0417	0.0148	1.838	14.033	5.079
0.2	0.03771	1.2749	0.459	0.0739	0.205	0.746	0.0320	0.0136	1.709	14.343	5.434
0.3	0.06209	1.1163	0.349	0.06619	0.101	0.685	0.0241	0.0115	1.629	14.481	5.756
0.4	0.06186	1.0049	0.2759	0.0593	0.0501	0.631	0.0188	0.0096	1.593	14.676	6.177
0.5	0.04755	0.9285	0.2269	0.0536	0.0244	0.590	0.0152	0.0078	1.574	14.928	6.872
0.6	0.03037	0.8752	0.1926	0.0491	0.0113	0.557	0.0126	0.0072	1.571	15.286	6.819
0.7	0.016108	0.8375	0.168	0.0457	0.0048	0.525	0.0106	0.00604	1.595	15.849	7.566
0.8	0.006535	0.8095	0.1377	0.0428	0.00163	0.503	0.0092	0.0056	1.609	14.967	7.643
0.9	0.001463	0.7908	0.1353	0.0403	0.00032	0.500	0.0081	0.0050	1.582	16.704	8.060

Foot Note: Med Var: Variance for Median

Min Var: Variance for Minima

W: Weibull Distribution

LFR: Linear Failure Rate Distribution

M: Makeham Distribution

Thus the test based on sub-sample median performs better than sub-sample minimum when the sub-sample sizes are large like k=5 and 7.

CHAPTER V

SMALL SAMPLE PERFORMANCE OF TESTS

5.1 INTRODUCTION

In this chapter we study the small sample performance of the U-Statistic that we have developed in the previous chapter. A Monte Carlo study was carried out to estimate critical points, the exact significance level and the power for three specific alternatives corresponding to significance levels near to $\alpha = 0.05$ and $\alpha = 0.10$ of the three tests belonging to this class based on the U-Statistics for b=0.5 and 0.9.The study is done for n=8 and 10.The simulation is based on 10,000 samples of the required size. The three alternatives to the null hypothesis H_0 of exponentiality for which the power has been estimated are H_1, the Weibull distribution of index $\theta = 2,3 and 4$ with $F(x)=1-e^{-x^{\theta}}$ and H_2, a Linear Failure Rate distribution with $\theta = 2,3 and 4$ with $F(x)=1-e^{\left(-x-\frac{1}{2}x^2\right)}$ and H_3, a Makeham distribution with $\theta = 2,3 and 4$ with $F(x)=1-e^{-x-\theta\left(x+e^{-x}-1\right)}$.

All these distributions are increasing failure rate distributions and hence also increasing failure rate average.

Section 2 gives the details of how the exact α and the cut off points and the power was estimated for different distributions. The work was carried out using MS Excel package.

Section 3 gives the Monte Carlo estimate of critical values, exact levels of significance and power for the three distributions.

5.2. Calculation of exact α

First we tabulate the null distribution. The work was carried out using the Excel programming. First we generated a sample of size 8 from Uniform (0,1) distribution that is our sample say x is $u=1-e^{-x}$ that is $e^{-x}=1-u$ that is $x=-\ln(1-u)$.

Then we compute $\binom{n}{k+1}U$ for this sample that is $\binom{8}{4}=70$ combinations that is we take the 70 combinations of these 8 samples taking 4 each at a time. Then for each of these 70 combinations we find $Med(X_1,X_2,X_3)$, $Med(X_1,X_2,X_4)$, $Med(X_1,X_3,X_4)$ and $Med(X_2,X_3,X_4)$ and compare it with bX_4, bX_3, bX_2 and bX_1 respectively. The value U can take, then varies from 0 to 4.This process is repeated for all the 70 combinations. Then the total for these 70 combinations should not exceed 280. Repeat this process for 10,000 times. Then form a frequency distribution of these 10,000 values. This gives the null distribution of $\binom{8}{4}U$ for n=8 .This is done for values of b=0.5 and b=0.9.

The process is repeated for n=10.For n=10 we have 210 combinations.

5.3. Calculation of Empirical Power

We compute the empirical power for the three distributions, namely, Weibull, Linear Failure Rate and Makeham distribution.

First consider the Weibull distribution given by the p.d.f

$$f(x,\theta)=\theta x^{\theta-1}e^{-x^{\theta}}$$

First we generate 10,000 samples of size 8 from Weibull distribution and compute $\binom{8}{4}U$ for each sample as before. The values of θ considered for the purpose are θ =2,3 and 4.

For Weibull distribution, $\overline{F}(x,\theta)=e^{-x^{\theta}}$

For θ =2 we have $\overline{F}(x,2)=e^{-x^{2}}$

Then by probability integral transformation e^{-X^2} has Uniform distribution over (0,1).

Therefore, letting $e^{-X^2}=U$, we have

$$X=(-\ln U)^{\frac{1}{2}}$$

Similarly for θ =3 we have $X=(-\ln U)^{\frac{1}{3}}$

and for θ =4 we have $X=(-\ln U)^{\frac{1}{4}}$

Thus we generate observations from Weibull distribution starting with Uniform (0,1) observations.

Following the same procedure as for the null distribution, we compute $\binom{8}{4}U$ for each sample. Then we compare this with the 5% and 10% cut off points of the null distribution. This process is

repeated for 10,000 samples. The proportion of times the test rejects H_0 gives the empirical power. This is done for b=0.5 and b=0.9.

Similar procedure is adopted for n=10 also.

For Linear Failure Rate distribution, we have

$$F(x)=1-e^{-x-\theta\frac{x^2}{2}} \qquad x>0$$

When $\theta=2$, we have $F(x)=1-e^{-x-x^2}$

Using Probability integral transformation, and letting

$$e^{-X-X^2}=1-U,$$

we have $-X-X^2=\ln(1-U)$

$$X+X^2=-\ln(1-U)$$

$$X+X^2+\ln(1-U)=0$$

Thus $X=\dfrac{-1\pm\sqrt{1-4\ln(1-U)}}{2}$

Admissible value of the root of the quadratic equation is chosen as the observation from Linear Failure Rate distribution.

Similarly for $\theta=3$ we get $X=\dfrac{-1\pm\sqrt{1-6\ln(1-U)}}{3}$

and for $\theta=4$ we get $X=\dfrac{-1\pm\sqrt{1-8\ln(1-U)}}{4}$

Following the same procedure as for Weibull distribution we compute $\binom{8}{4}U$ .Then we compare the same with the 5% and 10%

cut off points of the null distribution. This is repeated 10,000 times. The proportion of times the test rejects the H_0 gives the empirical power. This is done for b=0.5 and b=0.9.

Similar procedure is repeated for n=10.

Lastly for the Makeham Distribution, we have

$$F(x)=1-e^{-x-\theta\left(x+e^{-x}-1\right)}$$

By Probability integral transformation, we have

$$U= 1-e^{-X-\theta\left(X+e^{-X}-1\right)}$$

$$\ln(1-U)=-X-\theta\left(X+e^{-X}-1\right)$$

$$-\ln(1-U)=X+\theta\left(X+e^{-X}-1\right)$$

First for $\theta=2$ we get

$$-\ln(1-U)=X+2\left(X+e^{-X}-1\right)$$

$$=3X+2e^{-X}-2$$

$$=3X-2+2e^{-X}$$

An appropriate root of this equation gives an observation from Makeham distribution for $\theta=2$. Newton-Raptson Method is used to find roots of the equation.

Similar procedure is used to find observations from Makeham distribution for $\theta=3$ and 4.

For $\theta=3$, we have $-\ln(1-U)=X+3\left(X+e^{-X}-1\right)$

$$=4X+3e^{-X}-3$$

For $\theta=4$, we have $-\ln(1-U)=X+4\left(X+e^{-X}-1\right)$

$$=5X+4e^{-X}-4$$

Then we compute $\binom{8}{4}U$ and compare the same with 5% and 10% cut off points of the null distribution. This procedure is repeated 10,000 times. The proportion of times the test rejects H_0 gives the empirical power. The procedure is carried out for b=0.5 and b=0.9.

This process is repeated for n=10 also.

5.4. Comparison

The following table gives the Monte Carlo estimates of critical values, exact levels of significance and empirical power of our test vis-à-vis Desphande(J_b), Bhadri(2005) minima(Min) for b=0.5 and b=0.9.These are useful for comparison.

TABLE 5.4.1

b = 0.5

	α	n ▶	8	10
Exact α	0.10	J_b	0.052	0.070
		V(b,2)	0.0983	0.0986
		V(b,3)	0.0995	0.0996
		Med(b,3)	0.0823	0.0887
	0.05	J_b	0.033	0.048
		V(b,2)	0.0483	0.0497
		V(b,3)	0.0488	0.05
		Med(b,3)	0.0478	0.0465
Cut-off Points	0.10	J_b	51	73
		V(b,2)	168	316
		V(b,3)	321	775
		Med(b,3)	245	683
	0.05	J_b	52	74
		V(b,2)	181	337
		V(b,3)	356	850
		Med(b,3)	254	707

TABLE 5.4.2

Empirical powers

α	n ▶		8			10		
0.10			W	LFR	M	W	LFR	M
	J_b		0.100	0.572		0.142	0.703	
	V(b,2)		0.7037 (θ = 2)	0.2035 (θ = 1)		0.7860 (θ = 2)	0.2225 (θ = 1)	
	V(b,3)		0.6505 (θ = 2)	0.1883 (θ = 1)		0.7374 (θ = 2)	0.1889 (θ = 1)	
	Med(b,3)	θ = 2	0.4724	0.2570	0.1465	0.7121	0.2764	0.2633
		θ = 3	0.8818	0.2967	0.1063	0.9769	0.3212	0.1636
		θ = 4	0.9875	0.3270	0.1241	0.9990	0.3520	0.2260
0.05			W	LFR	M	W	LFR	M
	J_b		0.067	0.481		0.099	0.631	
	V(b,2)		0.5477 (θ = 2)	0.1238 (θ = 1)		0.6408 (θ = 2)	0.1208 (θ = 1)	
	V(b,3)		0.4968 (θ = 2)	0.1082 (θ = 1)		0.5783 (θ = 2)	0.1082 (θ = 1)	
	Med(b,3)	θ = 2	0.3478	0.1662	0.0882	0.5881	0.1785	0.1764
		θ = 3	0.7929	0.1989	0.6230	0.9462	0.2166	0.1043
		θ = 4	0.9625	0.2207	0.7340	0.9966	0.2425	0.1475

TABLE 5.4.3

b = 0.9

	α	n ▶	8	10
Exact α	0.10	J_b	0.056	0.106
		V(b,2)	0.0855	0.0941
		V(b,3)	0.095	0.0979
		Med(b,3)	0.0856	0.0804
	0.05	J_b	0.021	0.043
		V(b,2)	0.0432	0.0476
		V(b,3)	0.0498	0.0494
		Med(b,3)	0.0298	0.0481
Cut-off Points	0.10	J_b	40	59
		V(b,2)	101	193
		V(b,3)	164	409
		Med(b,3)	161	474
	0.05	J_b	41	60
		V(b,2)	105	199
		V(b,3)	172	432
		Med(b,3)	166	481

TABLE 5.4.4

Empirical powers

α	n ▶		8			10		
0.10			W	LFR	M	W	LFR	M
	J_b		0.082	0. 278		0.152	0.452	
	V(b,2)		0.3551 (θ = 2)	0.1232 (θ = 1)		0.4570 (θ = 2)	0.1517 (θ = 1)	
	V(b,3)		0.3251 (θ = 2)	0.1313 (θ = 1)		0.4046 (θ = 2)	0.1545 (θ = 1)	
	Med(b,3)	θ = 2	0.3103	0.1532	0.1437	0.3880	0.1676	0.1615
		θ = 3	0.5661	0.1645	0.1150	0.7191	0.1872	0.1721
		θ = 4	0. 7566	0.1814	0.1298	0.8996	0.2015	0.1872
0.05			W	LFR	M	W	LFR	M
	J_b		0.034	0.139		0.066	0.279	
	V(b,2)		0.2358 (θ = 2)	0.688 (θ = 1)		0.3107 (θ = 2)	0.0804 (θ = 1)	
	V(b,3)		0.2261 (θ = 2)	0.0713 (θ = 1)		0.2581 (θ = 2)	0.0760 (θ = 1)	
	Med(b,3)	θ = 2	0.1777	0.686	0.738	0.2914	0.1108	0.1116
		θ = 3	0.4008	0.7324	0.567	0.6212	0.1262	0.1201
		θ = 4	0.6169	0.841	0.631	0.8445	0.1347	0.1262

Foot Note:

J_b:Deshpande U-statistic based on the kernel

$$h_b(X_1, X_2) = \begin{cases} 1 & (X_1 > bX_2) \\ 0 & otherwise \end{cases}$$

i.e. $J_b = n(n-1)^{-1} \sum{}^{*} h_b(X_i, X_j)$

where $\sum{}^{*}$ denotes summation over $1 \le i \le n$, $1 \le j \le n$ such that $i \ne j$

V(b,k):Bhadri U statistic which is a one sample U statistic corresponding to the kernel

$$h^{k+1}{}_b(X_1, X_2, \ldots, X_{k+1}) = \begin{cases} 1 & if \ \min(X_1, \ldots, X_k > bX_{k+1}) \\ 0 & otherwise \end{cases}$$

where b is a fixed number such that $0 < b < 1$

i.e. $V(b,k)=\binom{n}{k+1}^{-1}\sum h^{*}{}_{b}(X_{i_1},...,X_{i_{k+1}})$

where summation is taken over all combinations of integers $(i_1,i_2,....,i_k)$ chosen out of integers $(1,2,....,n)$ and

$h^{*}{}_{b}(X_1,...,X_{k+1})=(k+1)^{-1}\sum_{i=1}^{k+1}h^{i}{}_{b}(X_1,...,X_{k+1})$.

for k=2 and 3.

Med(b,k):Our statistic for k=3.

W: Weibull Distribution.

LFR: Linear Failure Rate Distribution.

M: Makeham Distribution.

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